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Paradox

The Magazine of the Melbourne University Mathematics and Statistics Society

Computer-generated affine transformation of
Associate Professor Barry Hughes to a hippie.

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Paradox

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A hippie hodgepodge

University funding aplenty, mathematicians well respected, hippies roaming the land, and Jeff not so unfathomably ugly. Paradise? No, just the '70s. In this very nostalgic issue of *Paradox*, the mystery of Paradox Kid is revealed in a gripping season finale, Charles Kemp mixes mathematics and literature in blasphemy, and there are finally some problems worth spending your time on.

Even if you don't realise it, you are most probably a member of the Melbourne University Mathematics and Statistics Society (MUMS). Membership is automatic if you're enrolled in any maths/stats subject. MUMS has been organising the Maths Olympics, Friday afternoon seminars, and BBQs for many years. And, of course, *Paradox* wouldn't be possible without MUMS holding a gun to the editors' heads. Recently, APESMA (the Association of Professional Engineers, Scientists and Managers) has agreed to sponsor MUMS. MUMS will get $2 per student membership application to APESMA. So please take your 30 seconds to fill out a form and put it in the MUMS pigeon-hole; it won't cost you anything if you're an undergraduate. That's 30 seconds for a free BBQ and, who knows, maybe you'll eventually hold a copy of *Paradox* with staples.

— Jian He, *Paradox* Editor

Virile gents rise but 19 is number one

Forget the Sydney 2000 Olympics. Forget about pre-booking your tickets for the event of the millennium — because it's been and gone, baby! So if you missed out on the 1999 MUMS Maths Olympics, then you'll just have to be content with reading this post mortem of the day's proceedings and perhaps even start training for next year's competition.

It was a lazy September afternoon when 27 teams, each consisting of five die-hard mathematicians, congregated in the Richard Berry Building. Theatre A was to be the field of dreams where these Olympians would battle it out in a quest for mathematical supremacy. Usually the scene of lectures brimming with sleep-inducing tedium, the place had been transformed to accommodate competitors as well as spectators of this mathematical mayhem.

For those unenlightened of this singular event, the Maths Olympics is a team contest organised by MUMS that combines both mental and physical agility. Team members are required to take turns to ferry questions back and forth between their team-mates — who are separated, one pair on each aisle of the lecture theatre. The problems usually involve little knowledge, but rather require speed and creativity of thought. At the conclusion of the 45 minutes of competition, winners are decided, prizes are allocated, and people who overstretched whilst negotiating the hazardous stairs of Richard Berry's Theatre A boast about their injuries.

The teams for the competition came from diverse backgrounds. There was the usual rivalry between pure and applied maths staff, there were both undergrads and postgrads from a variety of faculties and a few school teams had been invited. As predicted, this year's

Maths Olympics was close, with three teams battling it out at the top end. Eventually, after many twists and turns of the most devious nature, 'Team Number Nineteen', a team consisting of second and third year maths students, emerged victorious. Much of their success was due to a highly raucous cheer squad with a dazzling array of chants helping them surpass such heavyweight teams as 'Uni Low' (five very handsome, quite virile gents) and 'The Möbius Bandits' (pure maths staff).

The results follow. The first number is the aggregate score, the second number is the number of the last question that was solved. In the event of equal scores, the second number was used to resolve ties. If this number was tied, the previous question solved was used, and so on. For example, 90/15 indicates that the team scored 90 points and that the last question they solved was question 15 (of 20).

(1)	Team Number Nineteen	(Melbourne Uni maths nerds)	90/15
(2)	Uni Low	(five very handsome, quite virile gents)	80/16
(3)	The Möbius Bandits	(pure maths staff)	80/14
(4)	Ordogonal projection	(oldies from maths, physics, engineering)	75/14
(5)	$e\mu\pi$	(cool dudes)	65/11
(6)	Caulfield Grammar School		60/13
(7)	Penny's Pentagonal Pythagorean Pals	(maths and stats tutors)	55/10
(8)	Black Tigers	(Melbourne Uni Indian Club)	50/14
(9)	Squared PEGS	(Penleigh and Essendon Grammar School)	50/11
(10)	Fred	(first year comp sci and engineering)	45/10
(11)	Catch 7π and a bit	(friends from eng, med, sci)	40/9
(12)	Neural Driftwood	(assortment)	35/9
(13)	The Statistical Anomalies	(statistics staff)	35/9
(14)	Uni High	(University High School)	35/9
(15)	MGS	(Melbourne Grammar School)	35/9
(16)	Illegal Entry	(engineering)	35/9
(17)	Interfacial continua	(applied maths staff)	30/10
(18)	Four box counters and a sheep counter	(stat mech night owls)	30/9
(19)	The Invisible Mathematicians	(Trinity College: July fast track)	20/11
(=20)	Sons of Ho	(medicine, engineering, and arts)	20/10
(=20)	Team Wira	(ex-Taylor's College and Sunway College S.J.)	20/10
(22)	All good ideas were born on a lunch break	(physics students)	20/8
(23)	A function of U is a strict subset of K	(third year maths)	20/7
(24)	Four lawyers and a nerd	(science/law and computer science)	20/6
(25)	Success	(chemical, computer and mechanical eng)	15/6
(26)	Fat Foxy's Fellas	(bunch of hacks)	15/3
(27)	Deli	(chemical engineering)	5/7

The day was a fantastic success, and thanks must go to Lawrence Ip who set the questions, Phil Swedosh who hosted the event, and Tony Wirth who organised the whole affair.

— Norman Do

Three mathematicians walk into a bar ...
(The Jokes Section)

A woman walks into a bar accompanied by a dog and a cow. The bartender says, 'Hey, no animals are allowed in here.'

The woman replies, 'These are very special animals.'

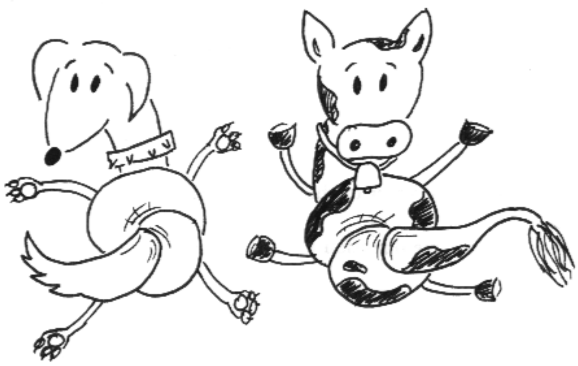
'How so?'

'They're knot theorists.'

The bartender raises his eyebrows and says, 'I've met a number of knot theorists who I thought were animals, but never an animal that was a knot theorist.'

'Well, I'll prove it to you. Ask them them anything you like.'

So the bartender asks the dog, 'Name a knot invariant.'



'Arf, arf' barks the dog.

The bartender scowls and turns to the cow asking, 'Name a topological invariant.'

The cow says, 'Mu, mu.'

At this point the bartender turns to the woman, says, 'Just what are you trying to pull?' and throws them out of the bar.

Outside, the dog turns to the woman and asks, 'Do you think I should have said the Jones polynomial?'

A proof that everyone at Woodstock had the same star sign

Try to find the fallacy in the following 'proof'.

We will show by induction that for all natural numbers, n, every group of n people has the same star sign. The conclusion follows by letting n be the number of people at Woodstock.

The statement is trivially true for $n = 1$: in any group of one person, everyone in the group — consisting only of one person — has the same star sign. Now assume that in every group of k people, everyone has the same star sign. We need to show that the statement is true for every group of $k + 1$ people.

Let G be an arbitrary group of $k + 1$ people. Suppose P and Q are members of G. Consider the group of everyone in G except P. This is a group of k people, so they must all have the same star sign. Now consider the group of everyone in G except Q. Again, this is a group of k people, so they must also all have the same star sign. Let R be a person in G other than P or Q. Since Q and R are in the group G excluding P, they have the same star sign. Likewise, since P and R are the in the group G excluding Q, they also have the same star sign. Since P has the same star sign as R, who has the same star sign as Q, it follows that P and Q have the same star sign. Thus any two people, P and Q, in G, have the same star sign, so everyone in G has the same star sign.

Hence by the principle of mathematical induction, the statement is true for all n.

Problems

The following are some problems for prize-money. The person who submits the best (i.e. clearest and most elegant) solution to each problem will be awarded the sum of money indicated beside the problem number, and will have their solution published in the next edition of *Paradox*. Solutions may be e-mailed to `paradox@ms.unimelb.edu.au`. (LaTeX format would be appreciated though not demanded.) If you do not have access to e-mail, drop in a hard copy of your solution to the MUMS pigeon-hole near the Maths and Stats Office in the Richard Berry Building.

1. ($10) Moonbeam notices that he has 25 Bob Dylan tracks, and that if he multiplies this number by 4, then deletes the zeros, he will end up with 1 — the number of harmony. He tells his friend Rufus, who replies nonchalantly, 'It doesn't matter which number you start with; if you multiply by some integer then delete some zeros in the decimal representation, then multiply by an integer, then delete some zeros and so on a few times, you can always get 1.' Moonbeam thinks Rufus should chill out. Is Rufus right? (*Bonus:* Is Moonbeam right?)

2. ($10) A dictator wants to wipe out the entire human race. A cluster bomb can destroy everyone who is an irrational distance away from where the bomb lands. How many cluster bombs does the dictator need to buy from the US to achieve his goal? (The cluster bombs are laser-guided so they can land where he wants them to; and being a typical dictator, he thinks the earth is an infinite plane).

3. ($10) In a wrestling tournament, each pair of wrestlers have exactly one fight. To please the bloodthirsty crowds, there can be no draws. A wrestler A is declared the winner of the tournament if for every other wrestler B, either A defeats B or A defeats some wrestler C who defeats B. Show that if the organisers want to rig the tournament so that there's exactly one winner, they must arrange for them to defeat every other wrestler.

4. ($10) Pawns are placed on an infinite chess board so that they form an infinite square net (along any row or column containing pawns there is a pawn, three free squares, pawn, three free squares, and so forth, with only every fourth row and every fourth column containing pawns). Show that it is not possible for a knight to tour every free square once and only once.

5. ($5) Show that if a, b, c, and d are positive real numbers then $(a/b+c+d)+(b/c+d+a)+(c/d+a+b)+(d/a+b+c)>1$. (Only solutions from first year students will be accepted for this problem.)

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Mathematics and Literature

Does mathematics have anything to say about literature? Probably not, and we'd be foolish to expect otherwise. But the opposite question is a little more interesting. Literature is supposed to reflect human experience in all its forms, and it's not asking too much to hope that it might give some insight into the psychological pull of the infinite, or the feeling of cracking a complex mathematical problem, or any number of similar topics.

A certain type of maths student might respond that mathematicians are the only people with anything at all worthwhile to say on these matters. He (let's call him Ron) might go on to claim that none of these questions is interesting, anyway.

Ron's first point has some merit. If you want to learn something about mathematics, the obvious person to ask is a mathematician. The first part of this article discusses *Surreal Numbers*, Donald Knuth's attempt at exploring mathematical concepts through fiction.

Of course, there are two ways to build a bridge between Mathematics and Literature. Knuth begins construction in the Land of Mathematics, but Sue Woolfe's *Leaning Towards Infinity*, the second text considered, is firmly planted in the Land of Literature.

As we will see, neither of these attempts is entirely satisfactory. Our elusive goal is perhaps the synthesis of these two approaches: what would happen if the two bridge segments were to meet up in the middle? (Lose the metaphor, says Ron. The union of two crappy pieces of architecture is something even worse than either piece taken individually.) No existing text measures up to this ideal, but the stories of Jorge Luis Borges (represented here by the Penguin collection *Labyrinths: Selected Stories and other Writings*) come closest.

THE MATHEMATICIAN

D. E. Knuth, *Surreal Numbers*, Addison-Wesley, 1974.

Mathematicians, to give them their due, have written valuable works of non-fiction; but very few have written novels, plays or poems about their field. Maybe they're not interested, or maybe they're not capable. (Maybe they have too much good sense to waste their time with fiction, says Ron. Actually, he doesn't. He's already given up on this article, and we'll no longer have to cope with his interruptions.)

For most people, the list of mathematicians with a literary bent starts with Lewis Carroll and ends with Charles Lutwidge Dodson. Donald Knuth is the only other candidate I've come across, and his credentials as a novelist are wafer-thin. As far as I know, *Surreal Numbers* is the only substantial work of fiction he's produced (his first published effort appeared in Mad Magazine, however, so perhaps we can expect a comic novel in the future). Knuth's 'mathematical novelette' is an attempt to present some serious mathematical ideas in an entertaining style.

It takes the form of a dialogue between Alice and Bill, two ex-students who 'turned on to pure mathematics and found total happiness'. They stumble across the Conway Stone, a large black rock with a lengthy inscription:

> In the beginning, everything was void, and J. H. W. H. Conway began to create numbers. Conway said, 'Let there be two rules which bring forth all numbers large and small.'

Alice and Bill spend the rest of the novelette developing some implications of Conway's two rules. In a postscript, Knuth reveals that his primary aim was not to teach Conway's theory, but to teach 'how one might go about developing such a theory', and thereby to 'urge the reader to try his or her own hand at exploring abstract mathematical ideas'. To a large extent, Knuth succeeds — for anyone prepared to follow the discussion through to the end (and to stop and think along the way), *Surreal Numbers* is a fine introduction to the art of creative mathematics.

Sadly, *Surreal Numbers* is not a lot of fun to read. It reads more like a textbook than a novel — an unusually interesting textbook, but a textbook nonetheless. The dialogue is clunky, and it's hard to imagine anyone reading *Surreal Numbers* for the excitement of seeing what the author will come up with next. For a better attempt at presenting mathematical ideas in a playful way, check out some of the dialogues in Douglas Hofstadter's *Gödel, Escher, Bach: An Eternal Golden Braid.* (In many ways, GEB would have been a better example for this section. It doesn't have the same emphasis on encouraging readers to figure things out for themselves, but it's a much more successful piece of writing. For anyone unacquainted with GEB, there's not a lot to say. Read it. Your life will not be complete until you do.)

Even the phenomenal GEB, however, does not escape the second charge that needs to be levelled against *Surreal Numbers.* Both Knuth and Hofstadter attempt to present some fairly complicated mathematical concepts. They rightly assume that these concepts will be unfamiliar to the average reader, and work quite hard to make them accessible. The trouble is, literature starts from a different level. It takes concepts that are known to everybody (love, power, revenge, and so on), and tries to give them a new complexion. Knuth and Hofstadter tell us things we didn't know, but literature talks about things we've known all along without realising it.

How are these approaches to be combined? The answer, I think, is to write about mathematical concepts that are known to everyone. And that's exactly what Borges, our third author, has done. First, though, a look at one novelist's attempt to write about mathematics.

THE NOVELIST

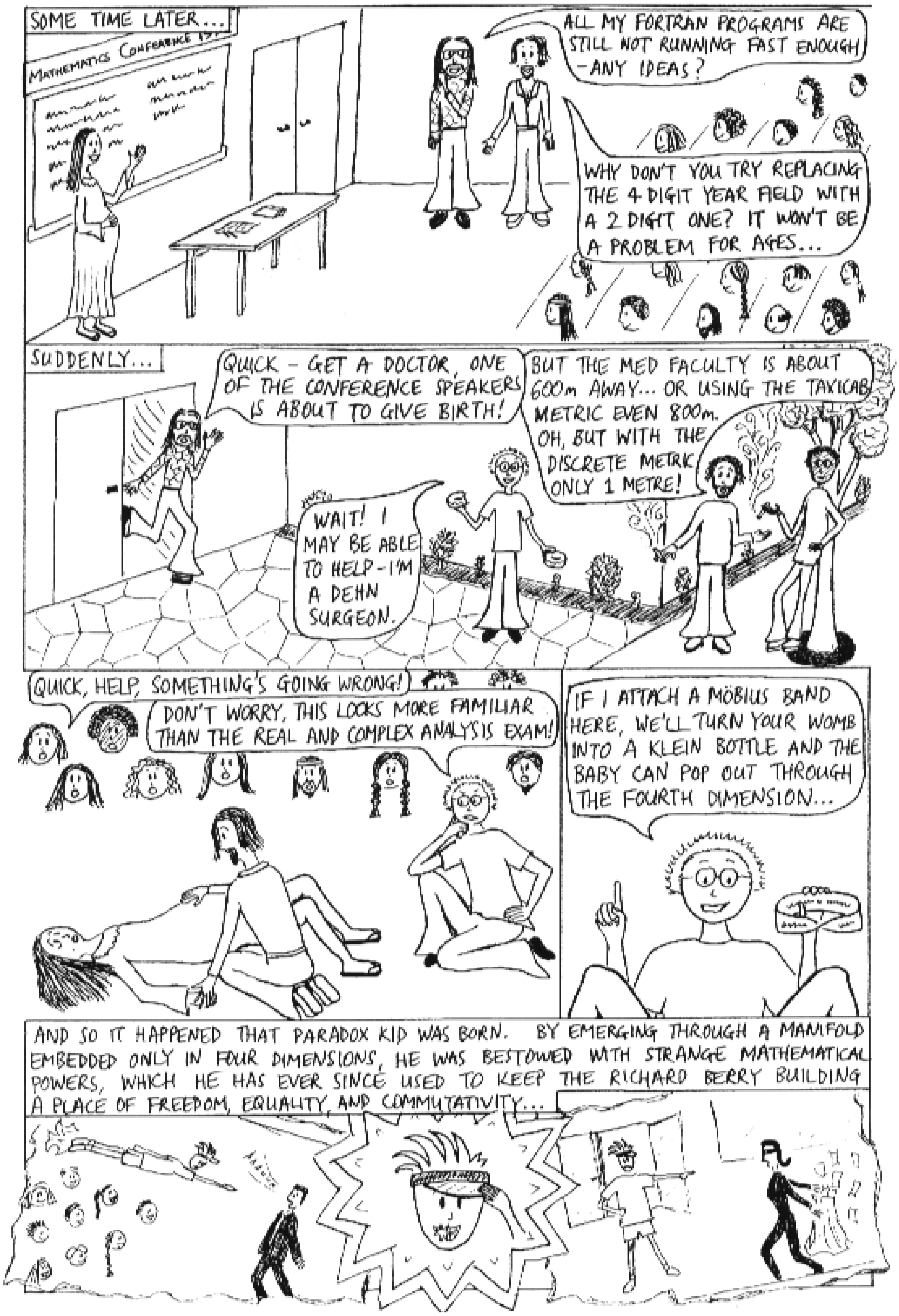
Sue Woolfe, *Leaning Towards Infinity*, Vintage Australia, 1996.*

P. A. M. Dirac was once asked how he'd enjoyed *Crime and Punishment.* His reply? 'It is nice, but in one of the chapters the author made a mistake. He describes the sun as

Continued page 10

*Two other novels worth looking at are Tom Petsinis' *The French Mathematician*, an account of the life of Galois; and Iain Pears's *An Instance of the Fingerpost*, an historical murder mystery in which mathematician John Wallis plays a major role.

ISSUE #3
The Adventures Of
Paradox Kid
by Jeremy Glick & Sally Miller
Love is a complex function of many variables
THINGS WERE DIFFERENT IN THE EARLY SEVENTIES... SCIENCE HAD PLENTY OF FUNDING, PEOPLE WONDERED WHETHER THE FOUR-COLOUR HYPOTHESIS WAS TRUE, AND PARADOX KID HAD NOT YET BEEN BORN.
IN FACT, IF NOT FOR A SERIES OF STATISTICAL ANOMALIES HE MAY NEVER HAVE BEEN BORN AT ALL!
IT ALL BEGAN AT THE MATHS OLYMPICS (SOME THINGS NEVER CHANGE)...
CRASH!
OH, I'M REALLY REALLY REALLY REALLY SORRY!
TAKE A CHILL PILL BABY, MAYBE OUR COLLISION WAS, LIKE, IN THE STARS!
HE-HE-HE!
HEY, MAYBE WE COULD, LIKE, CONVERGE LATER, AT THE SEMINAR ON FRIDAY?
I'D LOVE TO!
THAT FRIDAY...
AFTER THE SEMINAR...
FUNKY TALK, MANDELBROT!
THANKS! HEY, CHECK OUT THAT CHICK'S SMOOTH CURVES!*
YEAH, SHE'S QUITE A BABE - OUR PATHS INTERSECTED AT THE MATHS OLYMPICS...
EXACT REPLICAS OF MANDELBROT
* POLITICAL CORRECTNESS HAD NOT YET GAINED MOMENTUM
HEY, WANNA COME BACK TO MY PLACE AND TRY TO MAKE A SELF-REPLICATING MACHINE IN CONWAY'S GAME OF LIFE?
SURE, CALL ME - MY PHONE NUMBER STARTS AT THE 2542624TH DIGIT OF PI...
OH, 4260655?
WOW - YOU'RE MY KIND OF MAN!
HEY, I KNOW ANOTHER WAY TO MAKE A SELF-REPLICATING MACHINE - WANNA TRY?


SOME TIME LATER...
ALL MY FORTRAN PROGRAMS ARE STILL NOT RUNNING FAST ENOUGH —ANY IDEAS?
WHY DON'T YOU TRY REPLACING THE 4 DIGIT YEAR FIELD WITH A 2 DIGIT ONE? IT WON'T BE A PROBLEM FOR AGES...
SUDDENLY...
QUICK - GET A DOCTOR, ONE OF THE CONFERENCE SPEAKERS IS ABOUT TO GIVE BIRTH!
BUT THE MED FACULTY IS ABOUT 600m AWAY... OR USING THE TAXICAB METRIC EVEN 800m. OH, BUT WITH THE DISCRETE METRIC ONLY 1 METRE!
WAIT! I MAY BE ABLE TO HELP - I'M A DEHN SURGEON.
QUICK, HELP, SOMETHING'S GOING WRONG!
DON'T WORRY, THIS LOOKS MORE FAMILIAR THAN THE REAL AND COMPLEX ANALYSIS EXAM!
IF I ATTACH A MÖBIUS BAND HERE, WE'LL TURN YOUR WOMB INTO A KLEIN BOTTLE AND THE BABY CAN POP OUT THROUGH THE FOURTH DIMENSION...
AND SO IT HAPPENED THAT PARADOX KID WAS BORN. BY EMERGING THROUGH A MANIFOLD EMBEDDED ONLY IN FOUR DIMENSIONS, HE WAS BESTOWED WITH STRANGE MATHEMATICAL POWERS, WHICH HE HAS EVER SINCE USED TO KEEP THE RICHARD BERRY BUILDING A PLACE OF FREEDOM, EQUALITY, AND COMMUTATIVITY...

From page 7

rising twice on the same day.'

Apart from making a good story, Dirac's response shows that it's quite possible to miss the point of a novel. After finishing *Leaning Towards Infinity*, I felt disappointment more than anything else. Woolfe's novel tells the tale of Frances Montrose, a woman without formal mathematical training who discovers a new type of number, but nowhere is the reader given any hint of what these numbers might be. About all we learn is that it's difficult to calculate these numbers: at the beginning of the narrative Frances has found only one of the new numbers, and is looking for the second.

Of course, I missed the point. But I think there is a message for anyone who hopes to blend mathematics and fiction: don't introduce a new mathematical concept unless you're prepared to follow through with it. Mathematics is a rich enough source of imagery as it is.

This objection aside, Woolfe's novel is a valuable reminder that mathematics is something done by people. Perhaps this is obvious, but as an undergraduate, it's easy to treat mathematics as nothing more than a collection of abstract results. The widespread idea that these results are timeless, that they were true long before they were discovered by humans only serves to widen the gulf between the theorems and the men and women who first wrote them down.

Leaning towards Infinity never lets us forget that mathematics occurs in a human context. This is a novel about bodies, beauty and mother-daughter relationships, and mathematics is situated with respect to each of these themes. From the opening sentence, it's clear where the novel is heading: I think it all began because of the shape of my mother's breasts. Already we have geometry, motherhood and the body tied up into one neat parcel.

The novel's major concern is to chart the connections between female identity and professional ambition. As the symbol of this ambition, mathematics is never just an abstract game. It's a way of getting noticed, of moving ahead in the world, of interacting with other people. And the choice to do mathematics is never a straightforward one. Frances' daughter, for example, resents her mother doing sums 'in our time': she feels that Frances should have spent less time doing mathematics and more time looking after her children.

In many ways, Woolfe's mathematicians are all too fallible. At a conference Frances attends, the recognised experts are more interested in petty power struggles than new ideas, and they delight in jeering at Frances and crushing a young postgraduate who is bold enough to present a paper. Many of these scenes are overdrawn (for me, the novel reaches its nadir when a mathematician yells, 'I can see a nipple, I can see a nipple' during a female colleague's talk), but they do give mathematics a human dimension, if a fairly unpleasant one.

As a literary examination of mathematics, the greatest flaw of *Leaning Towards Infinity* is its inability to say anything about the nature of mathematical thought. Woolfe's version of mathematics is really a symbol of abstract thought in general. She claims she was drawn to the topic because she'd 'always been haunted by the suspicion that mathematics is a mirror metaphor of who we are', but I'm not convinced. I suspect that mathematics was chosen to contrast with her primary interest, female identity. She looked for the profession most removed from the realm of the female body, and thought she'd found it in mathematics.

THE FUTURE

Jorge Luis Borges, *Labyrinths: Selected Stories and Other Writings*, Penguin, 1970.†

After coming this far, we should have a better idea of what to expect from a fusion of literature and mathematics. An ideal work would be:

1. written with a sense of poetry; and
2. an exploration, but not an exposition, of a mathematical idea.

Who would be qualified to write this sort of book? Well, what do you get if you cross a mathematician and a novelist? The thought experiment is almost impossible to perform, but the fanciful half-breed might write a little like Jorge Luis Borges.

For a start, Borges did not write novels. He worked on a smaller scale, producing essays, poems, and short narratives. Yet, as André Maurois points out in the preface to this Penguin collection, 'they suffice for us to call him great because of their wonderful intelligence, their wealth of invention and their tight, almost mathematical style.'

Maurois has hit upon an important point. Borges does not write about mathematics, but he writes like a mathematician. In fact, the best of his stories read like good mathematical papers. Each sentence contributes something to the larger structure under construction, yet carries enough weight in its own right to repay a few moments of thought.

Many writers, however, use language with precision. What sets Borges apart is that he also seems to think like a mathematician. This might sound ridiculous — how can a man think like a mathematician unless he's grappling with mathematical problems? The answer, I think, is that Borges approaches his art in the same way that many mathematicians would if they stopped to think deeply about these issues.

Consider, for example, his answer to the question 'What is a divine mind?':

> The steps a man takes from the day of his birth until that of his death trace in time an inconceivable figure. The Divine Mind intuitively grasps that form immediately, as men do a triangle.

Borges takes a question that Ron would dismiss as nonsensical, and responds with an image of breathtaking beauty. Yet it's an image that Ron can probably relate to — if he had taken the question seriously, he might have been able to produce a similar answer himself.

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†A more extensive collection is now available: Jorge Luis Borges, *Collected Fictions*, translated by Andrew Hurley, Viking, 1998. There is some debate over whether the translations are as good as those appearing in the Penguin edition.

So Borges thinks and writes like a mathematician. What about point two on the checklist above? — does he write about mathematics? Well, almost, and this is why Borges is only an approximation to the ideal blend of the mathematician and the novelist.

Among mathematical circles, he is perhaps most famous for his treatment of the infinite. I'm sure you've heard one of the stories used to explain eternity to children.

> Imagine a rock as tall as the tallest skyscraper in the world. Every thousand years, a bird flies along and drops a seed on the rock. The rock will be worn away to nothing before eternity runs out.

The Library of Babel is essentially a more powerful version of the same idea. Borges describes a library with an unimaginably large collection of books. The library contains:

> Everything: the minutely detailed history of the future, the archangels' autobiographies, the faithful catalogue of the Library, thousands and thousands of false catalogues, the demonstration of the fallacy of those catalogues, the demonstration of the fallacy of the true catalogue, the Gnostic gospel of Basilides, the commentary on that gospel, the commentary on the commentary on that gospel, the true story of your death, the translation of every book in all languages, the interpolations of every book in all books.

Not a new idea, as Ron would undoubtedly point out. But Borges drives home the point with unprecedented force. The effect is to remind us that we can never fully wrap our minds around the infinite. We can pin it down at an analytical level — after all, Borges has described the library quite successfully — but it continues to overwhelm anyone who tries to approach it on its own terms.

The infinite, unfortunately, is the only mathematical concept that Borges considers at length. Sprinkled throughout his stories, however, are signs that he'd have had plenty to say if he'd turned his mind to other aspects of mathematics. All of the stories, of course, are worth reading in their own right. A reasonable summary of Borges' achievements is given in the introduction to the Penguin collection: Borges' work is 'one of the most extraordinary expressions in all Western literature of modern man's anguish of time, of space, of the infinite.'

To finish, two exercises, arranged in order of increasing difficulty.

1. Read Borges, if you haven't already.

2. (For extra credit.) Write your own blend of mathematics and fiction.

— Charles Kemp

Chess variants

Why should an article about chess appear in *Paradox*, which is, after all, a maths magazine? You wouldn't see an article about hexaflexagons in *Inside Chess*. (This is for the dual reasons of you not reading *Inside Chess* and the article not being there anyway). The reason is that the links between chess and maths are numerous, and skills in chess may increase spatial manipulation ability, and may also lead one's mind to interesting maths problems. For example, John H. Conway came up with the *Game of Life* while sitting over a *Go* board[‡]. This article isn't about the links between chess and maths, but rather about some of the many chess variants that can be played with a chess set or two.

SUICIDE CHESS

Rules

The pieces start in the same place as chess, and move in the same way. If you can take an opponent's piece then you *must*. If you can take more than one piece, then you can choose which to take, but you must take one. The aim of the game is to lose all your pieces. The king holds no special significance, it moves the same way as in normal chess, but you can move into check, since if you lose your king, the game continues as normal. Pawn promotion is as in normal chess. There are different beliefs as to the outcome if the player on the move cannot move a piece (say has only pawns, all of which are blocked off by the opponent's pieces). I think the best outcome in that situation is a draw, since that allows someone who is losing (as they have many more pieces on the board) to still have some hope — if they can block off their opponent's pieces then they can draw.

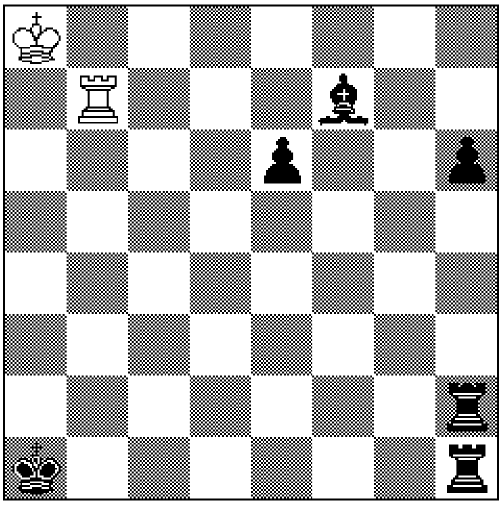
Strategies

Bishops can be a real disaster. Make every effort to lose your bishop early in the game, since otherwise it may be forced to take several of your opponent's pieces. In particular, placing a bishop on b2 (or g2) (as white) is very dangerous, since your opponent can play g6 (or b6) and force you to take the rook. Then they can move their bishop to g7 (or b7) and force you to take their bishop, then king (queen) across to f8 (c8), etc.

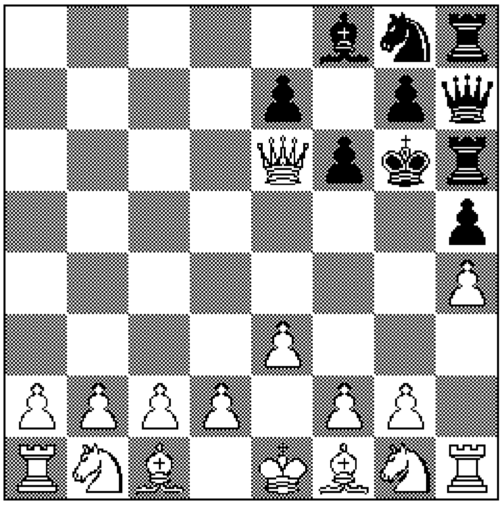
INTERLUDE

What is the shortest game of chess that ends in a stalemate (a position where the side on the move has no legal move, but is not in check)? Sam Loyd, in the late 19th century, found a ten-move stalemate that is still the record, though it has never been proved minimal. The final position is shown in the diagram overleaf — can you work out the ten white and nine black moves that lead to it?

[‡]Go is another complete information strategy game.



Suicide chess: Black to move and win.
Note that black is moving *up* the board.



Loyd's ten-move stalemate position.

TRANSFER CHESS

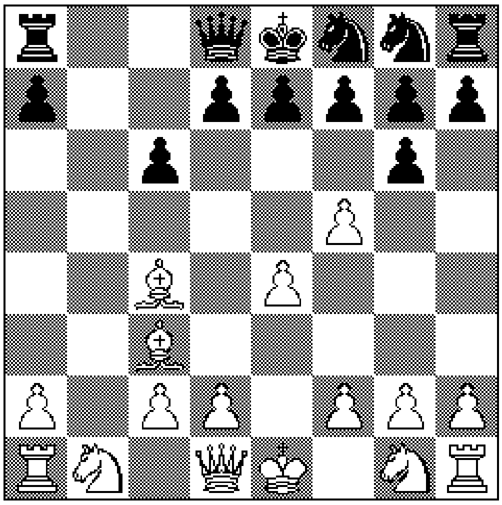
Rules

This is one of the more entertaining variants. It requires two teams of two people and two chess sets. One person from each team plays white, against the person playing black from the opposing team. Each time a piece is taken from one of the games, the piece is passed to the taker's team-mate, who can place it on their board instead of playing a move from then on. So, for example, if I take off my opponent's bishop, I pass it to my partner, who can then put it on the board later. This results in games where one board may have, say, 8 bishops on it. Players can also accumulate pieces until they want to put them on the board (one at time of course). The other rules are the same as chess, except that a pawn which reaches the eighth rank does not promote, rather it is taken off the board and passed to one's opponent. The game is won by checkmating the king on either board.

Strategies

In transfer chess, the attack is all important. It is often worth sacrificing a piece (though the piece will be given to your partner's opponent), to force your opponent's king into a dangerous position. Once this happens, if your partner can swap pieces, that allows you to continually add pieces to your board, placing your opponent in check with each one. Since the check must be addressed, your opponent may have to move the king rather than place a piece on the board, giving you, in effect, an extra piece. For this reason, knights and pawns are particularly good, since a check by either cannot be blocked (by putting an extra piece on the board). Remember to keep attacking. The main difference between chess and transfer chess strategy is that in transfer there is never an endgame to consider.

Every piece that gets taken off is still kept in play (though on the other board) so that the situation rarely arises where there are just a king and a few pawns in play. Even if that situation does arise, having an extra pawn is no good once your opponent places a queen on the board.



Transfer chess: White to move and win.
White has a knight and a pawn to be placed on the board.

INTERLUDE

The *n queens problem* is surely one of the best known mathematics and computer science problems involving the ideas of chess. It involves placing n queens on an $n \times n$ chess board so that no queen attacks any other. Finding solutions for large n, or enumerating solutions are both interesting and difficult problems. Another related problem is the $2n$ *queens problem* which involves placing $2n$ queens on an $n \times n$ chess board such that no more than 2 queens are on the same row, column or diagonal. This can be generalised to the *kn queens problem* in the obvious way.

PROGRESSIVE CHESS

Rules

Progressive chess is an excellent training game, in which the better player almost always wins. This makes it an ideal game for those wishing to show off. In progressive chess, white first makes a move, then black makes two moves, then white three, and so on. The aim is to checkmate your opponent. If you check your opponent before your allowed number of moves are over, then your turn finishes. That is, if you are up to having seven moves, and on your sixth you place your opponent in check, then your opponent begins their eight moves immediately.

Strategies

Progressive chess requires considerable calculation skill. It is important to keep searching through variations, in particular when you have five or six moves to play with, since in those situations there is almost always a mate on the board, but it is sometimes quite hard to find, since it may involve promotion, or setting up a double check.

FISCHER RANDOM

Rules

Fischer Random chess is the invention of the eccentric (OK, let's cut out the euphemisms, he's completely insane) American ex-World Chess Champion. It is the variant most likely to reach mainstream acceptance as a game in its own right, and it has many advantages over chess. The sole difference between normal chess and Fischer Random is that in the latter the rooks, knight, bishops, king and the queen start randomly placed along the back rank. The only restriction is that the two bishops be on a different colour. Also, black's set-up is the mirror image of white's. Randomly setting up the pieces like this vastly increases the complexity of the game, and is done primarily to combat the concern that chess was becoming boring, since players learnt vast amounts of opening theory, and professionals would frequently follow accepted move orders until move 15 or so. The value of learning opening theory in Fischer Random is negligible, since the chances of playing with the same set-up twice in one's lifetime are infinitesimal. This moves the focus of the game to natural calculation skills, and to intuition. This is where many people feel the focus belongs, but others are nervous since they have spent hours learning obscure opening systems that could be rendered useless by the widespread adoption of Fischer Random.

Strategies

Beats me. Fischer Random really requires intuition and calculation — often it is necessary to think for some time before playing the first move! Perhaps it becomes easier with practice, but since every game is essentially new, I doubt if this is so.

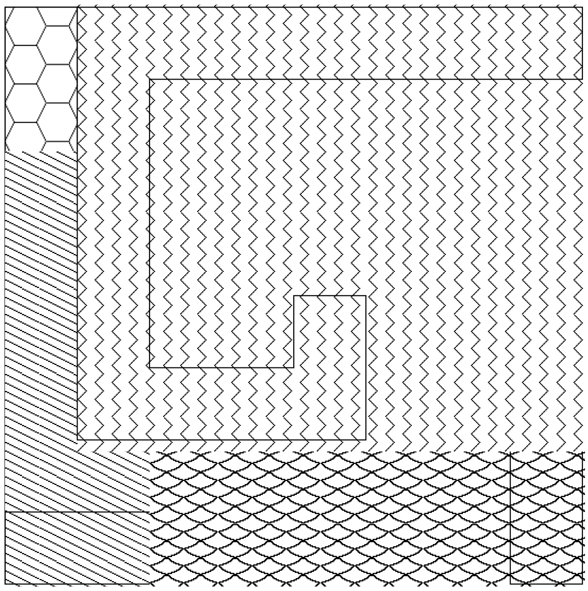
There are many other chess variants around, and many other chess inspired maths and computational problems. A good place to start looking for more information is the Internet, with Yahoo having a whole subcategory for chess variants. There are also Internet sites which give information on playing tournaments in the various variants, generally using e-mail. Not only can a lot of fun be had playing these variants, but one day some interesting maths problem might occur to you, and will vindicate your claim that you were studying while playing chess.

— Jeremy Glick

Solutions to last issue's problems

Problem 1: (\$5) Divide an 8×8 grid into four identical shapes, such that cell (8,8), cell (7,7), cell (6,6) and cell (5,5) are each in one of the pieces.

Solution: Of the 782 ways to dissect an 8×8 grid into four identical shapes, this is one which has the required property.



— Andrew Oppenheim

Problem 3: (\$5) Des has a pack of ten cards, numbered one through ten. He shuffles them, then takes the front card. It is a six. He places the six at the back of the deck, and takes the *sixth* card from the front. It is a three. He places the three at the back of the deck, and takes the *third* card. This game will loop if the card he takes is a ten, but are there any other loops possible?

Solution: A card which is never selected can only move toward the front of the deck. But in a loop a card must return to its starting position, so cards which are never selected must not move. These are in front of all cards which do get selected (otherwise they would move toward the front when a card in front of them was selected).

Thus a state which is part of a loop consists of n static cards (which are never selected) followed by $10 - n$ cards which are selected at some stage in the loop.

The dynamic cards must all have values greater than n, so as not to select any of the static cards. But there are only 10 cards to choose from, so the dynamic cards must be $n+1, n+2, \ldots, 10$. Since the 10 is in the dynamic group, and once it is selected the game loops, it must be the only card in the dynamic group.

— Hugh Allen

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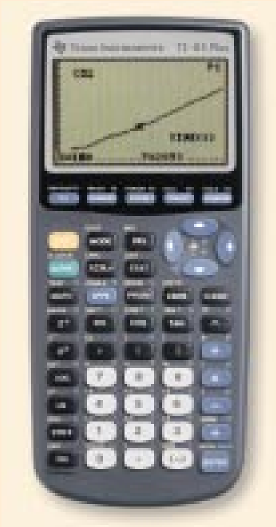

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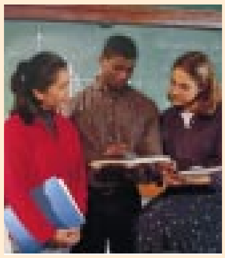

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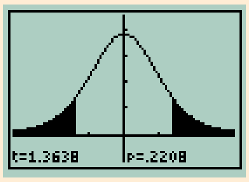

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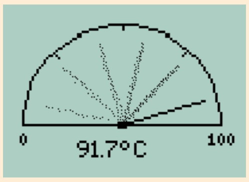

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