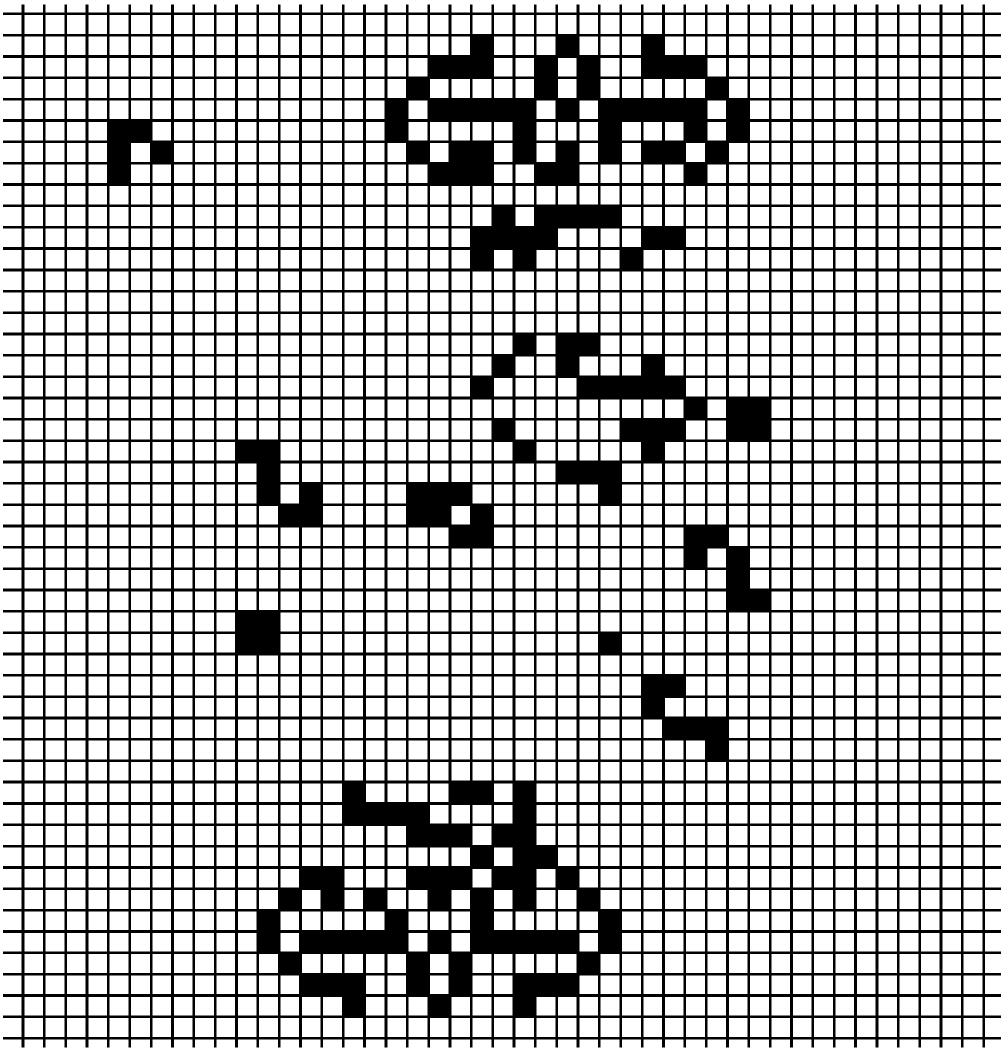
Issue 1, 2000

Paradox

THE MAGAZINE OF THE MELBOURNE UNIVERSITY MATHEMATICS AND STATISTICS SOCIETY



A period-94 glider gun in Conway's Game of Life.

Paradox

Words from the Editor ...

Welcome to the first edition of Paradox in 2000. Readers may be pleased to notice that *Paradox* is now stapled. This is all thanks to the efforts of Norman Do who, at the time of writing, volunteered to be staple boy. If that's not enough, there are also interesting things to read. This issue, we have an article by Chung Leong Li exploring the relationship between music and mathematics. Anne Thomas, a maths student at the University of New South Wales, writes about the past and present of women in mathematics. Norman Do has provided some interesting problems for your amusement. Some even have cash prizes for those who solve them. And, of course, Paradox Kid fans will be pleased to know that his adventures will continue this year.

— George Doukas, *Paradox* Editor

... and some from the President

It is often said that all students who are enrolled in a mathematics or statistics subject are automatically members of MUMS (the Melbourne University Mathematics and Statistics Society), and this is indeed the case. Many of our members, however, do not become involved in and are often not aware of the multitude of events (almost always with mandatory freebies) that are due to this society.

First, for the benefit of those who don't already know, allow me to explain a little about MUMS. Our principal activity is to organise events for the interest and enjoyment of maths and stats students. For this year, we have already lined up a barbecue, a trivia competition (or two), a comprehensive range of seminars on genuinely interesting maths and stats topics that are habitually followed by unrestrained feasting, and of course the infamous Maths Olympics alongside its younger cousin, the Schools' Maths Olympics.

Seminars are held on Friday afternoons. They are usually delivered by students, although some are given by staff and visiting academics. A wide range of topics at varying levels is presented every year, so that every student with an interest in maths or stats is sure to find a seminar that will appeal to them, and others that might trigger new interests.

The Maths Olympics is the largest event on the MUMS calendar each year, and requires somewhat more of a mention. In short, it is a chaotic 45 minutes in semester two during which Theatre A is transformed into the scene for a vicious mathematics relay involving secondary school students, university students, and staff members.

So how does one become involved (and partake in the feasting)? If you wish to receive notification of all our events by e-mail, please mail `mums@ms.unimelb.edu.au` requesting to be added to our mailing list. Otherwise, news on upcoming events is posted on our web page (`http://www.ms.unimelb.edu.au/~mums`) and our notice-board opposite the drinking fountain on the ground floor, right outside the Thomas Cherry Room. Lastly, please feel free to contact us and offer suggestions or give us feedback at any time during the year.

— Desmond Lun, MUMS President

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Divine music in a divine universe

If music be the food of love, play on. *Shakespeare*

During your Christmas holidays you may have heard a performance of the 'Hallelujah' chorus from Handel's reverent *Messiah*. It may have even played in your mind when you found out your results at school or university! Imbued with feelings of triumph or glory, it is a choral spectacle that gives you a sense of aesthetic satisfaction. Doesn't it make you wonder what it is about music such as this that strikes a chord in those who listen? Such a question may seem irrelevant to ask in a mathematics magazine, but we will see later that mathematics may be instrumental in understanding or appreciating beauty in music.

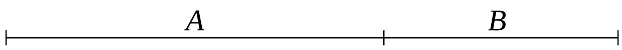
Philosophers have long sought to understand what makes some music (or the arts in general) sublimely beautiful in a branch of philosophy called aesthetics. Aestheticians ask whether there are any criteria for interpreting 'good' art, and how art, such as music, is related to how people feel, in order to get a better view of the aesthetic experience.

Psychologists, even, study the brain's physiological arousal when our feelings are stirred up while listening to music, and music therapists use music to elicit emotions in their patients.

What if mathematics were to tell philosophers, psychologists, and music therapists that there could be an underlying mathematical element in understanding the aesthetic experience in music? Many believe that mathematics is devoid of emotion, and that to apply mathematics to a study of beauty is utterly absurd.

The relationship between mathematics and music, however, extends far beyond any stretch of the imagination and needs to be extensively realised. The study of this relationship dates back to the eccentric Pythagorean cult (5th century B.C.) which classified music as a subdivision of mathematics, and believed that music and mathematics provided keys to the secrets of the universe. Leibniz, co-creator of calculus, once remarked, 'music is the pleasure the human soul experiences from counting without being aware that it is counting.'

Among the relationships between mathematics and music, the most startling is that of the golden proportion. Geometrically speaking, the golden proportion, or golden section, is what a short line is to a longer line as the longer line is to their sum.



Or: A is to B as $A+B$ is to A. If B is 1, then the equation defining their ratio is

$$\frac{1}{A} = \frac{A}{A+1},$$

which can be solved as the positive root of the quadratic equation $A^2 - A - 1 = 0$, that is, $(1+\sqrt{5})/2$. This special number has been symbolically named the Greek letter Φ^* and

[*] U.S. mathematician Mark Barr named this number Φ after the first Greek letter of the name of Phidias, who used the golden proportion in his sculpture.

represents the golden proportion. It has the decimal approximation 1.61803398... and is the only number whose reciprocal is equal to one less than itself.

Φ can also be represented by the continued fraction

$$\Phi = 1 + \frac{1}{1 + \frac{1}{1+\ldots}}$$

or by the continuous surd

$$\Phi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}.$$

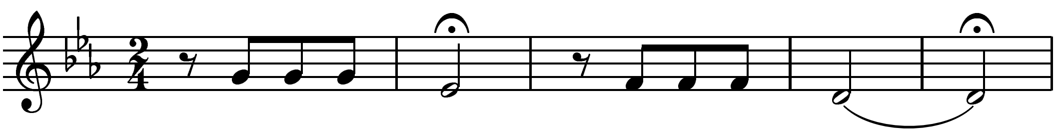
The Fibonacci sequence, $1, 1, 2, 3, 5, 8, 13, 21\ldots$, also defines the golden section. Subsequent terms are generated by adding the two previous terms: $F_{n+2} = F_{n+1} + F_n$ for $n \geq 1$. If we examine the ratio of successive terms in this sequence as we approach infinity, we find that the limit of the sequence of ratios approximates the golden ratio.

Why this number has been bestowed with the prestigious title of 'golden' will soon become apparent. In botany, it makes its presence in petals, leaves, pine cones, and fruit shapes. In zoology and biology, spirals on molluscs, rabbit breeding, and biological morphology are manifested in golden proportions. Geometric shapes such as the pentagon and the platonic solids contain golden ratios. Even in painting and sculpture, artists have used the golden ratio to create their masterpieces in visually pleasing ways. Painters who have used the golden proportion include Leonardo da Vinci, Durer, Seurat, and Dali. In architecture, the golden proportion is evident on the Parthenon in Athens, Gothic cathedrals, and even the pyramids of Egypt!

Because of its omnipresence in life, Φ has been given another complimentary name — the 'divine' proportion. The divine proportion's propensity to appear extends as far as the realms of music.

Take Handel's 'Hallelujah' chorus as an example. The entrance of solo trumpets 'King of Kings' occurs in measures 57 to 58 out of 94 measures, which is close to a divine ratio. 'The kingdom of glory ...' appears after 8/13 of the first 57 measures, and so does 'And he shall reign' which appears after 8/13 of the next 37 bars, again heralded by trumpets.

What about the all-too-famous opening motto of Beethoven's *Fifth Symphony*? If you didn't do well in your results, this is what you probably heard play in your mind:



Its positioning divides the first movement of the *Fifth Symphony* into golden proportions (see Figure 1).

It is speculated whether Mozart's music contains usage of the golden proportion. Mozart was said to have been fond of numbers, and his sister once said that Wolfgang 'talked of nothing, nothing but figures' during his school-days.

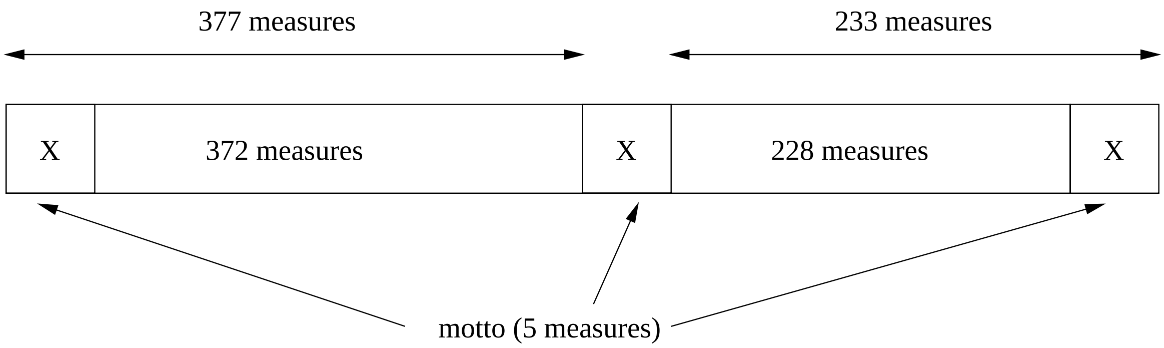


Figure 1: First movement of Beethoven's *Fifth Symphony* divided into golden proportions by the motto. (After [1].)

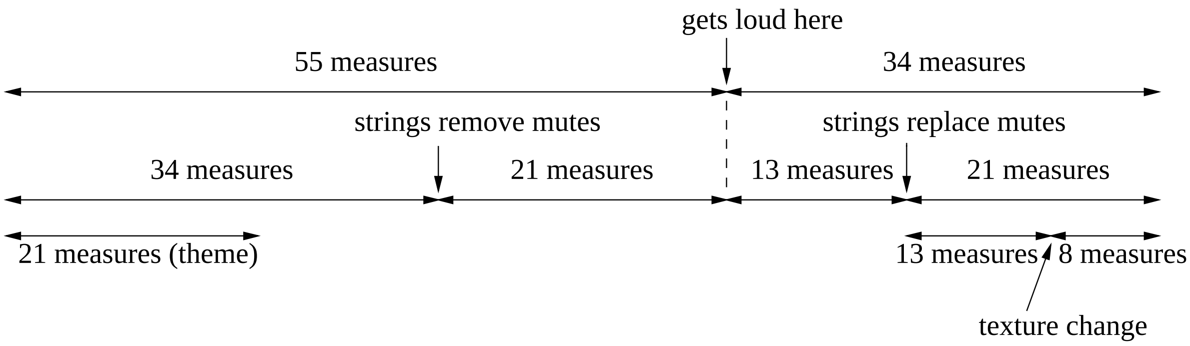


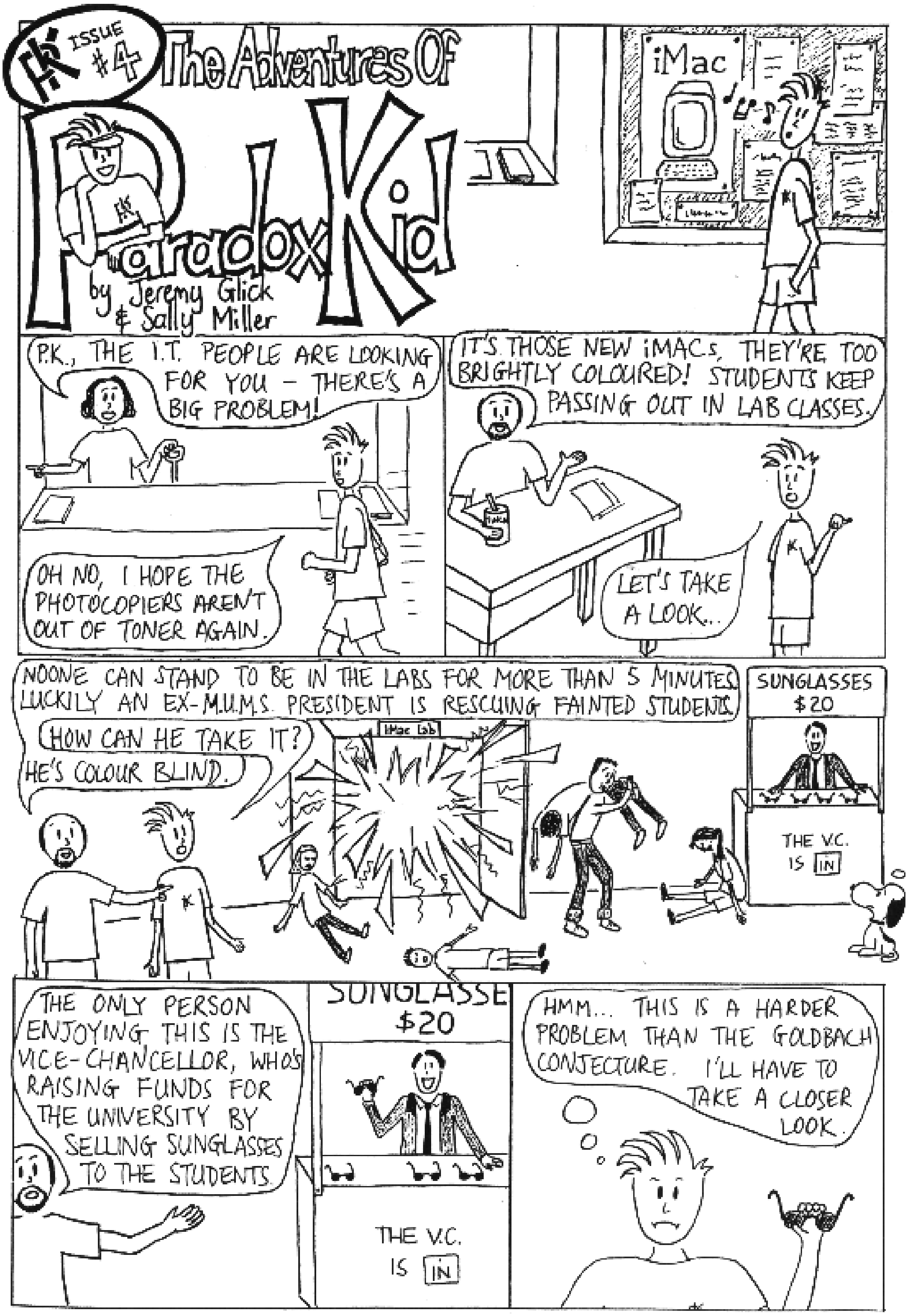
Figure 2: First movement of *Music for Strings, Percussion, and Celeste* by Béla Bartók. (After [1].)

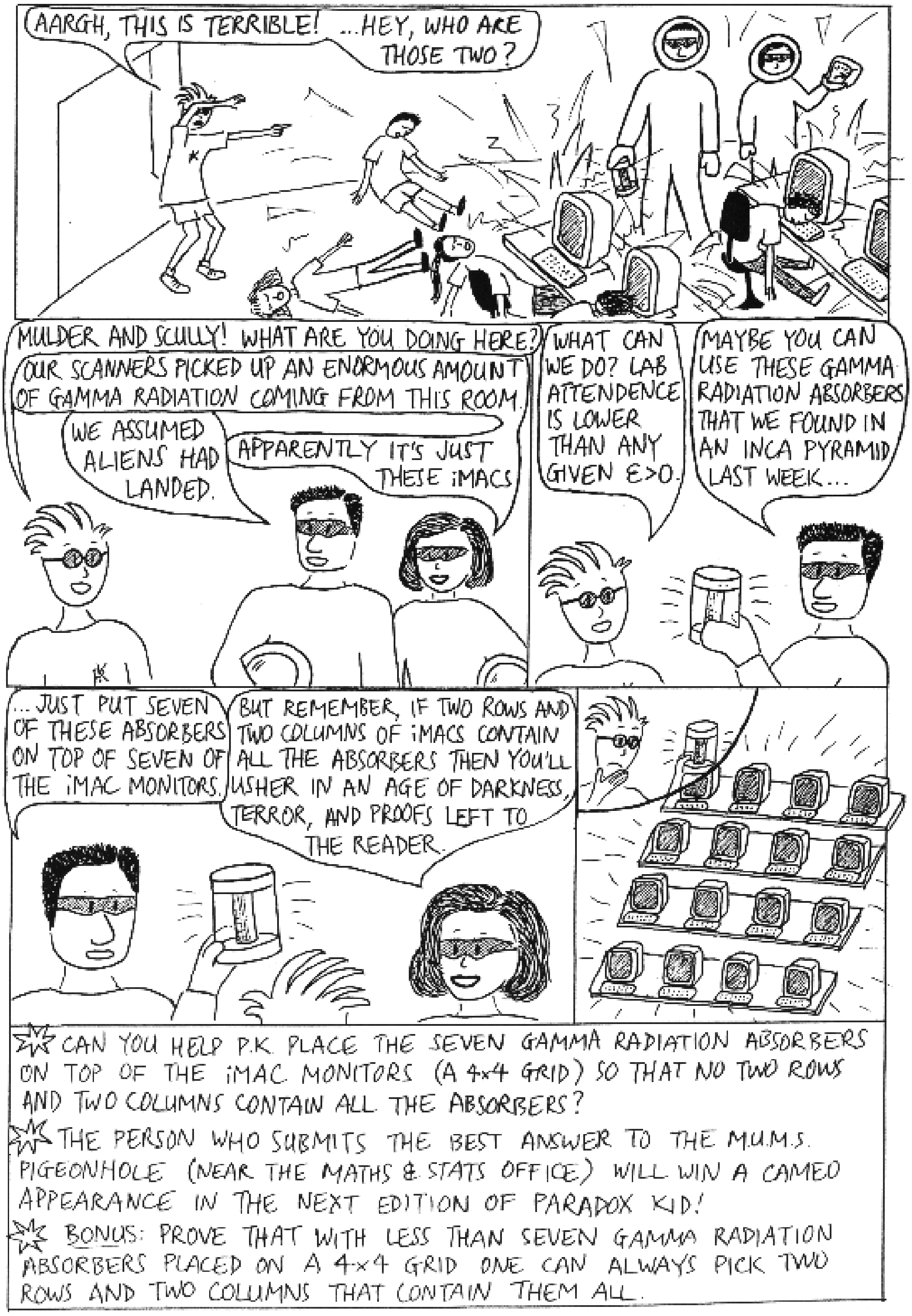
John F. Pultz mathematically analysed Mozart's *Sonata no. 1 in C major* where the exposition is 38 measures long and the development and recapitulation is 62. He says that 'a 100 measure movement could not be divided any closer to the golden proportion than 38 and 62.' Results from examining more sonatas by Mozart reveal further appearances of the golden section. So there is some evidence of the golden section in Mozart's music.

Other composers have been thought to have used the golden section, such as the impressionists Debussy and Satie. We may never know whether these composers intended to use the golden section or, whether, returning to Leibniz's comment that 'music is the pleasure the human soul experiences from counting without being aware that it is counting', they weren't aware of their usage of the golden section. Nevertheless, we know for certain that these geniuses wrote from the heart, and composed what felt right to them, even if the appearance of the golden section came coincidentally or otherwise.

One composer who consciously used the golden ratio in his compositions was the Hungarian Béla Bartók, who had accents and climaxes occur in golden proportions (see Figure 2).

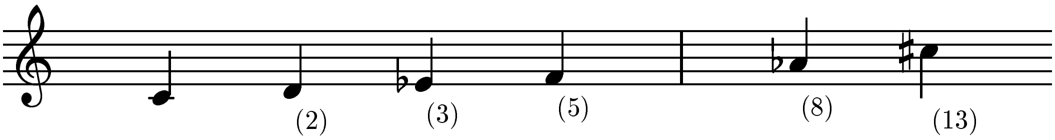
Continued page 8


PK
ISSUE #4
The Adventures Of
Paradox Kid
by Jeremy Glick & Sally Miller
iMac
P.K., THE I.T. PEOPLE ARE LOOKING FOR YOU - THERE'S A BIG PROBLEM!
OH NO, I HOPE THE PHOTOCOPIERS AREN'T OUT OF TONER AGAIN.
IT'S THOSE NEW iMACs, THEY'RE TOO BRIGHTLY COLOURED! STUDENTS KEEP PASSING OUT IN LAB CLASSES.
LET'S TAKE A LOOK..
NOONE CAN STAND TO BE IN THE LABS FOR MORE THAN 5 MINUTES. LUCKILY AN EX-M.U.M.S. PRESIDENT IS RESCUING FAINTED STUDENTS.
SUNGLASSES $20
HOW CAN HE TAKE IT?
HE'S COLOUR BLIND.
iMac Lab
THE V.C. IS IN
THE ONLY PERSON ENJOYING THIS IS THE VICE-CHANCELLOR, WHO'S RAISING FUNDS FOR THE UNIVERSITY BY SELLING SUNGLASSES TO THE STUDENTS.
SUNGLASSE $20
THE V.C. IS IN
HMM... THIS IS A HARDER PROBLEM THAN THE GOLDBACH CONJECTURE. I'LL HAVE TO TAKE A CLOSER LOOK.


AARGH, THIS IS TERRIBLE!
...HEY, WHO ARE THOSE TWO?
MULDER AND SCULLY! WHAT ARE YOU DOING HERE?
OUR SCANNERS PICKED UP AN ENORMOUS AMOUNT OF GAMMA RADIATION COMING FROM THIS ROOM.
WE ASSUMED ALIENS HAD LANDED.
APPARENTLY IT'S JUST THESE iMACS
WHAT CAN WE DO? LAB ATTENDENCE IS LOWER THAN ANY GIVEN ε>0.
MAYBE YOU CAN USE THESE GAMMA RADIATION ABSORBERS THAT WE FOUND IN AN INCA PYRAMID LAST WEEK...
...JUST PUT SEVEN OF THESE ABSORBERS ON TOP OF SEVEN OF THE iMAC MONITORS.
BUT REMEMBER, IF TWO ROWS AND TWO COLUMNS OF iMACS CONTAIN ALL THE ABSORBERS THEN YOU'LL USHER IN AN AGE OF DARKNESS, TERROR, AND PROOFS LEFT TO THE READER.
CAN YOU HELP P.K. PLACE THE SEVEN GAMMA RADIATION ABSORBERS ON TOP OF THE iMAC MONITORS (A 4×4 GRID) SO THAT NO TWO ROWS AND TWO COLUMNS CONTAIN ALL THE ABSORBERS?
THE PERSON WHO SUBMITS THE BEST ANSWER TO THE M.U.M.S. PIGEONHOLE (NEAR THE MATHS & STATS OFFICE) WILL WIN A CAMEO APPEARANCE IN THE NEXT EDITION OF PARADOX KID!
BONUS: PROVE THAT WITH LESS THAN SEVEN GAMMA RADIATION ABSORBERS PLACED ON A 4×4 GRID ONE CAN ALWAYS PICK TWO ROWS AND TWO COLUMNS THAT CONTAIN THEM ALL.

From page 5

He even developed his own Fibonacci scale:



(The numbers in parentheses indicate the number of semitones from C.)

Joseph Schillinger advocated using Fibonacci numbers in his own method of musical composition. Elsewhere in music, golden proportions are present in chords. A major sixth might consist of a C vibrating at 264 Hz and an A vibrating at 440 Hz. 264/440 reduces to 3/5, a Fibonacci ratio. Moreover, the engineering and construction of violins involves golden proportions.

It is not known exactly why golden ratios appear naturally or why they seem 'beautiful' when used in the arts — or at least in Western arts. One thing for sure is that the divine proportion pervades the universe and that its ubiquitous presence may make it feel natural for us to subconsciously expect it to occur evermore.

Although the golden proportion may not adequately explain the aesthetics of music, it does serve to link the natural harmony between mathematics, music, and the universe.

> Mighty are numbers, joined with art, resistless. *Euripides*

— Chung Leong Li

REFERENCES

[1] Hammel Garland, T. & Vaughan Kahn, C. 1995, *Math and Music: Harmonious Connections*, Dale Seymour Publications, Palo Alto.

[2] May, M. 1996, 'Did Mozart use the Golden Section?', *American Scientist*, vol. 84, no. 1, pp. 118–119.

Upcoming events

A free barbecue will be held on 4 April at 1:00 p.m. in the Old Geology courtyard. The event is predominantly for the benefit of first year students in the department, but all members of the maths and stats community are welcome. So please do come along to get some free food and to find out some more about MUMS.

A mathematically-orientated trivia competition will take place late in this semester. The competition will be a fantastic opportunity for those who know Newton's date of birth but not Tom Cruise's to parade their talent, and to earn some prizes while doing so. A hugely enjoyable and popular event in the past, look out for further publicity of it in the future.

Gauss's sister: Women and mathematics

History is more than just the deeds of men, and men are not naturally more intelligent than women. Thanks to feminism, these are generally considered statements of the bleeding obvious. Yet it is still widely, and wrongly, believed that in the history *of mathematics* men are the only players, and that men are innately better *at maths* than women. Female mathematicians have been written out of history books and are invisible in popular culture, so there are few role models for women in mathematics. Women are deterred from mathematics by the western association of men with transcendence and women with daily, bodily concerns. While open and conscious sexual discrimination has recently ceased, exclusion and lack of promotion continue to hinder women in academia.

Imagine Gauss had had a sister who was equally gifted in mathematics (apologies to Virginia Woolf who played this game with Shakespeare's sister in *A Room of One's Own*). Although it was unthinkable that a girl be sent to school, suppose that her clever brother taught her arithmetic and reading, by candlelight after her housework was done. She showed great ability and interest. But rather than being impressed, her parents scolded her. It was unnatural for a girl to mess about with numbers, and no-one would want to marry a women who was too smart. After that, when she was discovered reading her brother's books at night, her candles were taken away. She then tried to read during the day, neglecting her spinning. This time she was beaten for disobedience. When her brother received the patronage of the Duke of Brunswick, she was betrothed to a pig farmer. By the time Carl Friedrich graduated from the University of Göttingen (from which women were banned), his sister had died in childbirth. Even had she lived, had she been equally educated, publication and professorship would have been out of the question.

Given these conditions, it is at best naïve to claim that women are not as good at maths as men because there is no female equivalent to Gauss, Euler, or any of the other 'greats'. There have been innumerable such sisters, talented women who were denied education and encouragement because of their sex. They are not really imaginary, and they should be remembered, although we know none of their names.

What of the women whose names are known? There are some incredible women who did mathematics in profoundly hostile conditions. Let me sketch a few of the most outstanding [2]. These women were famous in their day, but their histories are now obscure. Women's contributions to mathematics have been systematically marginalised and trivialised. If this is not to continue, we must commemorate the work of previous generations.

According to contemporaries, Hypatia (370?-415) surpassed both contemporaries and later intellectuals in mathematical knowledge. She was educated by an unusually enlightened father, like almost all early female mathematicians. As did male philosophers of the time, she carried on discussions in the centre of Alexandria, and advised city officials. She is the first woman known to have written on mathematical subjects, although none of her works survive. For her pagan philosophy, and for being an unconventional woman, she was murdered by a Christian mob.

Maria Agnesi (1718-1799) wrote and supervised the printing of *Instituzioni Analitiche* (Foundations of Analysis), a systematic presentation of algebra, analytic geometry, calculus and differential equations, which was widely praised and used. She was an honorary professor at the University of Bologna but never lectured there. Her studies in mathematics were supported by her father. After he died, she lived in poverty and stopped doing mathematics.

Sofia Kovalevskaia (1850-1891) was considered one of the best mathematicians of her time. She proved the theorem in partial differential equations now known as the Cauchy-Kovalevskaia theorem, and did significant work on the revolution of a rigid body about a fixed point. She was a valuable intermediary between mathematicians in Western Europe and those in her native Russia, introducing Weierstrassian function theory to the latter. Kovalevskaia was initially educated in mathematics because of her father's broad-mindedness. Women in Russia were banned from the universities, and could not go abroad without permission of family or husband (like Saudi Arabia today), so she contracted a marriage of convenience with another nihilist to get to Heidelberg. Only with the express permission of each of her professors was she allowed to attend courses there. Moving to Berlin to study under Weierstrass, she was not allowed to take any courses at all. In 1874 Kovalevskaia became the first woman to receive a doctorate in mathematics, presenting not one but three doctoral dissertations to the University of Göttingen.

Emmy Noether (1882-1935) is considered the greatest female mathematician of all time and one of the greatest mathematicians (male or female) of the twentieth century. Noether's theorem is basic to the general theory of relativity and elementary particle physics. She unified the study of non-commutative algebras, and suggested to Alexandrov the introduction of group theory into combinatorial topology. Educated to be a school teacher, she went to university instead and in 1907 gained a Ph.D. in mathematics from Erlangen. This was one of the few German universities where women could enrol in courses rather than just sitting in with the permission of the professor. Until 1915, Noether worked at that university without any formal appointment, researching and supervising doctoral students. From 1915-1933 she lectured at the University of Göttingen, never being appointed to a chair (unlike her male doctoral students), and not even being paid until 1922. The Nazi regime withdrew her permission to teach in 1933 as she was Jewish, and she took up the offer of a guest professorship at Princeton.

The temptation is to look at these histories and think those were the bad old days. If women aren't equally represented in mathematics at the end of the twentieth century, it must be their own fault for being less able. There's no discrimination any more. However, the bad old days of blatant discrimination are not so long ago, and attitudes can take a long time to alter. Compared to the centuries in which women in mathematics were truly exceptional, there has been so very little time for the situation to change. When today's professors were undergraduates, Princeton did not admit women to its graduate mathematics program, and Harvard made its postgraduate women sit outside the door of the lecture theatre. Today's undergraduates have been socialised in a society which believes boys are better at maths than girls.

This belief is plain wrong. Until after puberty, girls and boys perform equally well on standard tests of mathematical ability [3]. It is when the pressure towards conventional sex roles intensifies that girls drop out of higher level courses and don't put themselves forward in class. To show that even today women are deterred from mathematics by societal pressures, let us imagine an Australian girl born in the late seventies with some mathematical talent. The ways in which she is discouraged show how the belief that maths is male territory is a self-fulfilling prophecy.

If this girl is very lucky, she will find out that she is good at maths before she finds out that she isn't meant to be, that she should hide her intelligence and focus on her weight and appearance. Suppose she wants a role model of a woman in maths, to counteract the playground taunting. The poster at the back of her classroom is titled 'Men of Mathematics', and she's never heard of any mathematician not on the poster. There aren't any movies featuring female maths whizzes, nothing like *Good Will Hunting*. She can't even fit herself into a negative stereotype, that of the nerd. If she wins some prizes, when she collects them at the university she realises with dismay that all the mathematicians on stage are men. Everyone else in the maths olympiad training session is male, and they don't let her in on their card games, seemingly their only form of social interaction. Everyone on the maths olympiad team is male too.

Despite these discouragements, she decides to study maths at university. As she progresses to harder courses, the proportion of female students drops. She is pleasantly surprised to find that a few offices in the maths building contain women who aren't secretaries. But none of these women is a professor, and they are all flat out managing work and family. Mentoring seems unlikely in these circumstances. Perhaps this girl is inspired to persevere by knowledge of the nameless thousands of talented women who did not have her opportunities, and admiration of the few women who made a name for themselves despite prevailing sexism. I hope so.

It is not nor has it ever been easy for women in mathematics. One of the tasks of feminism is to show women that their personal experience is part of a wider pattern of discrimination: that the personal is political. Another task is to rehabilitate the history of women, so that each generation does not have to start right at the beginning in fighting injustice. Further, what is 'natural' must be critiqued, for claiming that something is 'natural' is a powerful way of justifying inequality. To claim that maths history is men's history is to ignore the women who never had a chance, and to dishonour those who did contribute. To claim that men are naturally better is to trivialise women's struggles, and thus to maintain the status quo. Now, I claim that through awareness and encouragement of others, we women in mathematics need no longer feel isolated and invisible.

— Anne Thomas

REFERENCES

[1] *Association of Women in Mathematics*, [On-line]. Available: `http://www.awm-math.org`. This organisation was founded in 1971 to encourage women in the mathematical sciences. The web site includes resources for education and careers, biographies of women mathematician, links to related organisations and sites and job advertisements. Their current on-line forum is on girls and mathematics competitions, especially the

maths olympiad. The main weakness of the site is its American focus; I know of no Australian organisation for women in mathematics. A project for the future?

[2] Grinstein, Louise S. & Campbell, Paul J. (eds.) 1987, *Women of Mathematics: A Bio-bibliographic Sourcebook*, Greenwood Press, New York. This contains brief biographies of the most important women, descriptions of their work, and bibliographies of both their writings and books about them.

[3] Wertheim, Margaret 1995, *Pythagoras' Trousers: God, Physics and the Gender Wars*, Times Books, New York. Although this book focuses on physics, there is a lot of overlap with the history of mathematics. Margaret Wertheim explores the deeper reasons for the association between men and mathematical sciences, arguing that physicists are a kind of priesthood who, like the Catholic church, have managed to exclude women from their mysteries for thousands of years. She suggests that having more women in physics would change the culture of the subject, as women are more grounded.

Problems

The following are some problems for prize-money. The person who submits the best (i.e. clearest and most elegant) solution to each problem will be awarded the sum of money indicated beside the problem number, and will have their solution published in the next edition of *Paradox*. Solutions may be e-mailed to `paradox@ms.unimelb.edu.au`. (LaTeX format would be appreciated though not demanded.) If you do not have access to e-mail, drop a hard copy of your solution into the MUMS pigeon-hole near the Maths and Stats Office in the Richard Berry Building.

1. ($5) Do there exist irrational numbers a and b such that a^b is rational?

2. ($10) The sequence of positive integers $a_1, a_2, \ldots$ is such that $a_{a_n} + a_n = 2n$ for all n. Find all such sequences.

3. ($10) Two grasshoppers are sitting at the endpoints of the segment $[0, 1]$. A set of n points of the segment are marked, dividing $[0, 1]$ into $(n+1)$ intervals. A grasshopper can choose any of the marked points and jump over it to the point symmetric to his previous location, provided that this symmetric point also belongs to the segment $[0, 1]$. For one move, the grasshoppers either jump simultaneously according to this rule, or else one of them jumps and the other stays where it is. What is the least possible number of moves needed to ensure that the grasshoppers occupy locations in the same interval (that is, with no marked points between them)?

— Norman Do

The *Paradox* team would like to thank Andrew Oppenheim, Charles Kemp, and Chaitanya Rao for assisting with this issue.

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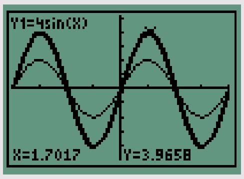

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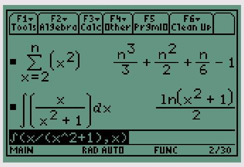
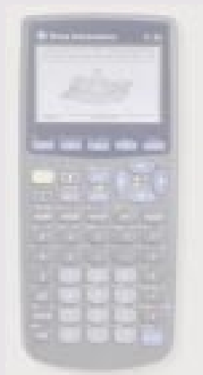


Symbolic manipulation on the TI-89

that provide significant functionality enhancements. Find the latest Flash applications at our web site: www.ti.com/calc/flash. To learn more about TI tools, Flash upgradability, and our popular educator support programs visit our web site, or call us at 1-800-TI-CARES.



The TI-89 featuring a robust Computer Algebra System (CAS), Flash electronic upgradability, and over 500k memory, is ideal for advanced calculus, differential equations, and beyond.

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