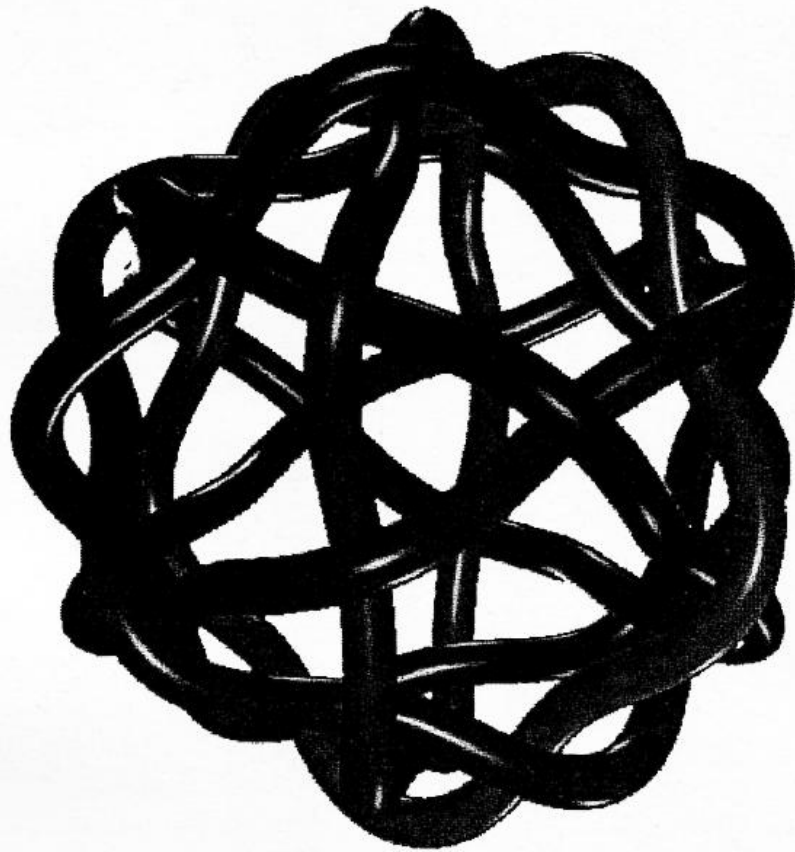

Paradox

Issue 1, 2001

THE MAGAZINE OF THE MELBOURNE UNIVERSITY MATHEMATICS AND STATISTICS SOCIETY





PRESIDENT:	Daniel Mathews d.mathews@ugrad.unimelb.edu.au
VICE-PRESIDENT:	Kuhn Ip pki@ecr.mu.oz.au
TREASURER:	Lori Trevaskis ltrevaskis@hotmail.com
SECRETARY:	Luke Mawbey lbm@ecr.mu.oz.au
EDUCATION OFFICER:	Norman Do ducco@smart.net.au
PUBLICITY OFFICER:	Jolene Koay jolz@rocketmail.com
EDITOR OF <i>Paradox</i> :	Joseph Healy j.healy@ugrad.unimelb.edu.au
1ST YEAR REPRESENTATIVE:	Elaine Miles e.miles@ugrad.unimelb.edu.au
2ND YEAR REPRESENTATIVE:	Damjan Vukcevic d.vukcevic@ugrad.unimelb.edu.au
3RD YEAR REPRESENTATIVE:	Tom Faulkner bantha@hotmail.com
HONOURS REPRESENTATIVE:	Claire Anderson cca@ms.unimelb.edu.au
POSTGRADUATE REPRESENTATIVE:	Samantha Richards samrichards85@hotmail.com
WEB PAGE:	http://www.ms.unimelb.edu.au/~mums

Paradox

EDITOR:	Joseph Healy
SUB EDITOR:	Geordie Zhang
ARTIST:	Sally Miller and Jeremy Glick
WEB PAGE:	http://www.ms.unimelb.edu.au/~paradox
E-MAIL:	paradox@ms.unimelb.edu.au
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Words from the Editor . . .

Welcome to a slightly later than planned first edition of Paradox. This year we are planning a series of articles covering some of the research done in the department, in this edition we look at some of the more pure research with an article by Professor Rubinstein. Our illustrious president has written an article on Pascal's triangle, binary numbers and the Sierpinski gasket, and in this edition of Paradox Kid we have a modern day love story.

We are very keen to hear from our readers, so if you have any ideas for articles, the magazine or activities for MUMS to do, send an email to either paradox@ms.unimelb.edu.au or mums@ms.unimelb.edu.au.

— Joseph Healy, *Paradox* Editor

Words from the President

Welcome to the latest installment of Paradox. I have always found it refreshing that a student society is able to produce something of the standard and calibre of Paradox on a regular basis – something that is genuinely entertaining (to some people), and genuinely contains real content (as in real maths – well kind of).

Paradox is just one facet of the many-sided diverse institution known as MUMS – the Melbourne University Mathematics and Statistics Society. Paradox is the literary end of the society's activities, but there is much much more. MUMS organises many events for the interest and enjoyment of maths and stats students – and even events that interest people in general!

We've just had our annual barbecue, which was a great success: if you were there, hope you enjoyed it; if not, you'll have to wait until next time for the free food. MUMS maths talks are also well under way for this year. We organise seminars on a very broad range of topics and at a wide range of levels, so if you do have an interest in mathematics or statistics there's sure to be plenty of stuff to interest you – and the free food and drink afterwards appeal to anybody's tastes! The seminars are held on Friday afternoons, usually around 4:15 pm and usually in the Thomas Cherry Room.

And of course, the major MUMS event for the year, the Maths Olympics, is already looming on the horizon. This will be held in semester 2 and promises to be a great event as always! It is a chaotic scramble and dash for mathematical glory, and just plain fun.

So, keep your eyes peeled for upcoming events and feel free to contact us with any suggestions or feedback, either by email (mums@ms.unimelb.edu.au) or by dropping into the MUMS room (G07). You can also remain up to date in what's happening in the MUMS world by visiting our noticeboard (outside the Thomas Cherry Room, opposite the drinking fountain), or our webpage (<http://www.ms.unimelb.edu.au/~mums>), or by joining our mailing list by sending your request to mums@ms.unimelb.edu.au, or by reading more Paradoxes!

— Daniel Mathews, MUMS President

Research Round-Up

The Paradox team is pleased to present the first of a series of articles on some of the research that is being conducted in the Mathematics and Statistics Department. Over the coming issues we will cover some of the branches of maths that the department is involved in and give students a taste of what happens beyond their lectures.

This issue we have an article by Professor Hyam Rubinstein, who has had an involvement in the department for over 25 years.

RECOGNISING 3-DIMENSIONAL SPACES

A surface can be described by saying that around each point, there is a small open set which can be mapped one-to-one and onto an open disk in the plane. Similarly we can define 3-dimensional surfaces (manifolds) as spaces where small open neighbourhoods of points can be mapped bijectively to an open ball in \mathbf{R}^3 . A very good source of such 3-manifolds is from knot theory. A knot is the embedded image of a circle C in \mathbf{R}^3 . However this loop can be 'knotted'. How do we classify knots, ie tell them apart? Note that if we embedded an arc in \mathbf{R}^3 , it can always be untied by pulling the ends through any knots. However this can't be done with a loop.

A key trick is to consider the **outside** of the knot, rather than the knot itself. So we take $M = \mathbf{R}^3 \setminus C$ and call this the exterior of the knot, or the knot complement. Clearly M is a 3-manifold, in fact it is an open subset of \mathbf{R}^3 . (An open set U has the property that if x is in U , then all sufficiently close points to x are also in U).

Then we can say two knots C and C' have equivalent complements, if there is a one-to-one and onto map between their complements M and M' . We require such maps to be continuous with continuous inverses. Here continuity can be viewed as requiring open sets in each space to correspond under the mapping. Such a map is a **topological equivalence** or **homeomorphism**.

A famous problem was to decide if two knots have equivalent complements, does this imply that the knots are equivalent. So for knots C and C' , we would like to have a homeomorphism of \mathbf{R}^3 to itself which takes C to C' .

This was solved by two Texas mathematicians (one ex Scottish) Cameron Gordon and John Luecke in 1987, by a very long and difficult argument. So next, how do we decide if two knot complements are equivalent. Here there is a known algorithm, but the problem is to decide whether there is a practical procedure. Various people have been working on this recently; the knot recognition problem is in the class NP, but it is also known there are many beautiful new invariants to distinguish knot complements and knots. Many of these come from ideas in mathematical physics (topological quantum field theories) and are polynomials, computed by looking at **knot projections**.

A knot projection is obtained by taking a light and shining it from behind a knot onto a screen. So the knot becomes a curve on the plane, and the 'overcrossings' and 'undercrossings' are kept in the picture, so the knot can be reconstructed from its projection.

To finish, one can think about the general problem of deciding if two 3-manifolds are equivalent or not. A classical result (the 'Hauptvermutung') states that every 3-manifold can be triangulated, ie divided up into tetrahedra, glued along their faces, edges and vertices. So a compact 3-manifold is one which can be built from finitely many such tetrahedra.

Since tetrahedra can be subdivided into smaller tetrahedra, it is difficult to compare triangulations. We do know that if two manifolds are equivalent, then any two triangulations of them can be subdivided to match exactly. However we don't know how to bound the number of subdivisions needed to achieve this.

In 1992 I studied the special case of the 3-ball. By a minimax procedure, I was able to come up with an algorithm to decide if a compact 3-manifold with 2-sphere boundary was equivalent to the 3-ball. The idea goes back to Poincare and Birkhoff. Take a 'flexible' 2-sphere and start at the boundary of the 3-ball, gradually moving the 2-sphere into the interior and then shrinking it to a point. We call this a **sweepout** of the 3-ball. If the 3-ball has some weird triangulation, we can take the sweepout for which the largest 2-sphere is as small as possible, ie the most efficient sweepout. The smallest largest 2-sphere is called the minimax sphere. (The size of a 2-sphere is measured by the number of intersections of it with the edges of the triangulation.)

It turns out that this minimax 2-sphere has a nice structure, called 'almost normal'. Wolfgang Haken (who together with Appell) solved the 4 colour mapping problem, had previously described a theory of normal surfaces. These meet every tetrahedron in small triangles (cutting off corners) or quadrilaterals (separating two opposite edges of a tetrahedron). An almost normal surface has one special piece, usually an octagon in a tetrahedron. Then there is an algorithm (based on Haken's ideas) to find this minimax 2-sphere (if it exists) and recreate the sweepout, showing the manifold is indeed the 3-ball.

The class of 3-manifolds for which the recognition problem can be solved, is called the Haken 3-manifolds. All knot complements are Haken, as are many compact 3-manifolds. However there are also probably lots of non Haken examples for which no general recognition procedure is yet known.



Professor Rubinstein is a long serving member of staff at the Department of Mathematics and Statistics, University of Melbourne. His affiliation with the department spans more than 25 years. His main areas of research include low-dimensional topology, minimal surfaces and shortest network design. You can find his homepage at <http://www.ms.unimelb.edu.au/~rubin/>.

Maths Haiku

After last years inundation of the Paradox mail box, I am pleased to present a selection of the many fine Haiku we received.

Apple π with pears
 π s with lashings of custard
 apricots and cream

Multiply and add
 Then subtract and divide them
 And the answer is ...

delightful wondrous
 captivating marvellous
 phenomenal maths

Five, seven and five
 Haiku syllables are strange
 At least they're all primes

A PROOF THAT $\sqrt{2}$ IS IRRATIONAL

Suppose rational
 Let fraction be p on q
 hcf is 1

Square both sides and so
 $\frac{p^2}{q^2} = 2$ Read: p squared on q squared is 2
 then multiply out.

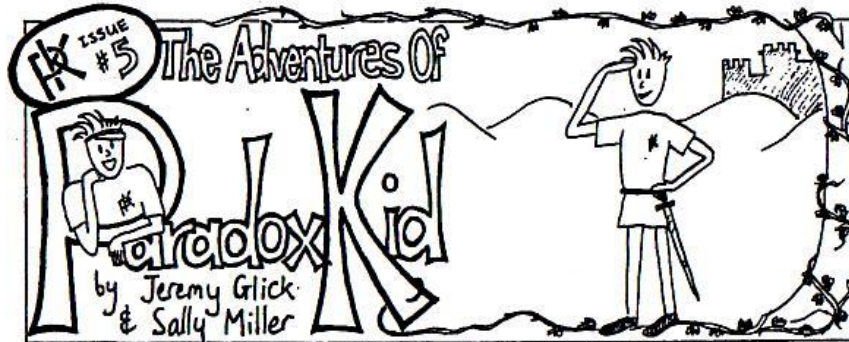
But then p's even...
 ... But then q's even! Bang! wow!
 Like freakout! Pigs fly!

AN ODE TO CONSTRUCTABILITY IN TRIUMVIRATE OF HAIKU

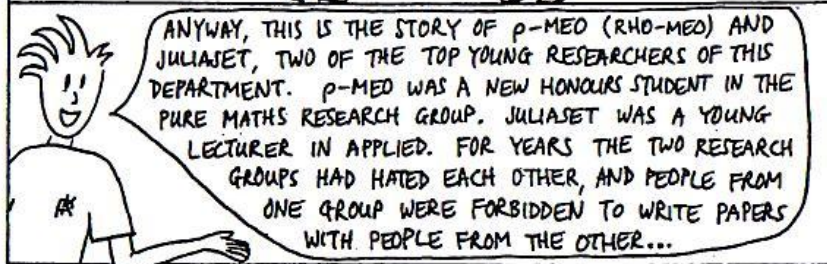
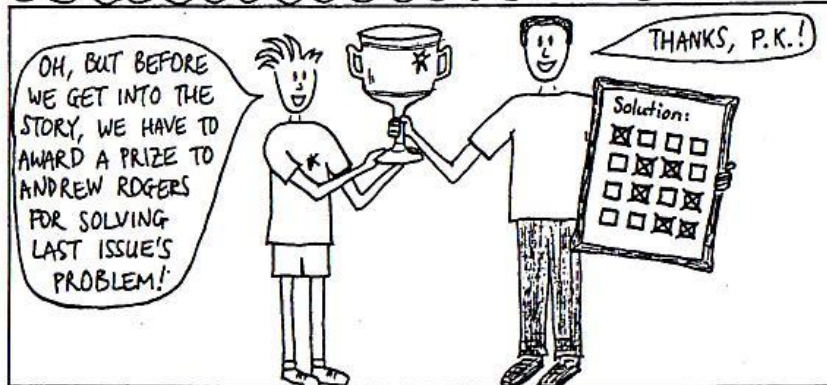
Ruler and compass
 Degree of field extension
 Must be power of 2.

Squaring the circle!
 Ha ha you stupid doofus!
 π transcendental!

Duplicating cube?
 $[Q(\sqrt[3]{2}) : (Q)]$ Read: 'Q cube root of two to Q'
 Degree three, not two!



TWO FIELDS, BOTH ALIKE IN OBSCURITY,
 IN FAIR MATHS AND STATS WHERE WE LAY OUR SCENE,
 FROM ANCIENT GRUDGE BREAK TO NEW MUTINY,
 WHERE ADMINISTRATIVE BLOOD MAKES ADMINISTRATIVE HANDS UNCLEAR.
 FROM FORTH THE FATAL RESEARCH OF THESE TWO FOES,
 A PAIR OF DOT-CROSS'D MATHEMATICIANS TAKE THEIR CAREER,
 WHOLE MISADVENTURED PITEOUS OVERTHROWS,
 DO WITH THEIR FATE BURY THEIR RESEARCH GROUPS' STRIFE...



... ONE DAY P-MEO, DESPITE ALL HIS GROUP'S WARNINGS TO THE CONTRARY, ATTENDED A SEMINAR GIVEN BY THE APPLIED GROUP. JULIASET WAS GIVING THE SEMINAR THAT DAY.

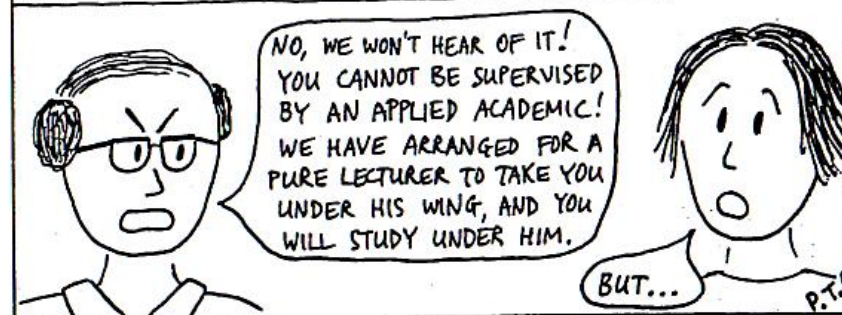
HE FELL IN LOVE WITH HER RESEARCH, AND AS SOON AS HE ASKED A QUESTION AFTER HER TALK, SHE FELL IN LOVE WITH HIS MATHEMATICAL INTELLECT TOO. THEY KNEW THEY'D HAVE TO WORK TOGETHER.



SO, MANY EVENINGS P-MEO WOULD CLIMB INTO JULIASET'S OFFICE VIA THE BALCONY AND THEY WOULD RESEARCH TILL DAWN.



THE TIME WAS APPROACHING WHEN P-MEO WOULD HAVE TO CHOOSE A SUPERVISOR. HE DESPERATELY WANTED TO BE SUPERVISED BY JULIASET, BUT HIS GROUP WOULDN'T HEAR OF IT.





SO ALL WAS GOING TO PLAN: THE PURE STAFF NO LONGER THOUGHT ABOUT COERCING P-MEO TO STAY IN THEIR GROUP, AND JULIASET HAD ONLY TO READ HER EMAIL AND THEN WAIT FOR THE 30 DAY EVALUATION PERIOD TO BE OVER BEFORE SHE COULD BEGIN TO RESEARCH WITH HER YOUNG PROTÉGÉ. UNFORTUNATELY...

OH BUMMER, TINCAN* IS DOWN AGAIN. I GUESS I WON'T BE ABLE TO CHECK MY EMAILS FOR A WHILE...

*THE MATHS EMAIL SERVER

HEY JULIASET, DID YOU HEAR? P-MEO INSTALLED TETRIS ON HIS COMPUTER! THAT'S JUST SHOCKING, HEY? IT REMINDS ME OF THE TIME...

OH NO, POOR P-MEO! WHAT HAS HE DONE?! HOW CAN I EVER WORK AGAIN KNOWING THAT HE GAVE UP HIS CAREER RATHER THAN WORK WITH SOMEONE OTHER THAN ME... I'M GOING TO INSTALL TETRIS TOO!

30 DAYS LATER P-MEO EMERGES FROM HIS TETRIS-INDUCED DAZE. HE RUNS STRAIGHT TO JULIASET'S OFFICE TO COLLABORATE, BUT...

NOOOO...

...AND SO P-MEO INSTALLED THE FULL VERSION OF TETRIS ON HIS COMPUTER TOO, AND THE MELBOURNE UNI MATHS AND STATS DEPARTMENT LOST TWO OF ITS FINEST YOUNG RESEARCHERS.

SEEING THEIR LOSS, THE TWO RESEARCH GROUPS ENDED THEIR FEUD AND PUBLISHED MANY FINE PAPERS TOGETHER.

SO REMEMBER, DESPITE WHAT WE LEARN FROM SET THEORY, THE WHOLE IS GREATER THAN THE SUM OF THE PARTS. SO COLLABORATE FREELY AND FORM UNIONS WITH PEOPLE EVEN IF YOUR FIELDS ARE DISJOINT. FINALLY, STAY AWAY FROM TETRIS BEFORE YOUR UPCOMING EXAMS!

Fluid Flow in the Figure-Eight Knot Complement

Heating Chose in Topological Semigroups

THE END

Adventures with Pascal's triangle and Binary Numbers

Some of us have probably seen the following array of numbers before:

$$\begin{array}{c}
 1 \\
 1\ 1 \\
 1\ 2\ 1 \\
 1\ 3\ 3\ 1 \\
 1\ 4\ 6\ 4\ 1 \\
 1\ 5\ 10\ 10\ 5\ 1 \\
 \vdots
 \end{array}$$

Where, as one can see, each entry is obtained by adding the two entries directly above it – Pascal's triangle as it's known.

All sorts of fun stuff occurs when one plays around with Pascal's triangle. For instance, let's look at the parity of the numbers in this triangle (ie, whether they're even or odd) – as one does, of course! Let's count how many odd and even numbers there are in each row.

	Number of Odd Entries	Number of Even Entries
1	1	0
1 1	2	0
1 2 1	2	1
1 3 3 1	4	0
1 4 6 4 1	2	3
1 5 10 10 5 1	4	2
1 6 15 20 15 6 1	4	3
⋮	⋮	⋮

I'm particularly interested in the number of odd entries in each row, because they turn out to be quite odd. (Sorry about that one, I just couldn't help it). We obtain the following sequence of numbers:

$$1, 2, 2, 4, 2, 4, 4, 8, 2, 4, 4, 8, 4, 8, 8, 16, 2, 4, 4, \dots$$

If you look at that one for a while, it turns out to be quite interesting. But we'll delay that for a bit. Let's turn our mind to something equally obscure... remember binary numbers from school? Yes that's right, those numbers which only used the digits 0 and 1 (not 2 through 9), and where the place value of each digit was not 1, 10, 100, 1000 and so on, but instead 1, 2, 4, 8, etc.

Let's write out the first few numbers in binary, starting from zero, just to get the hang of it:

$$0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, \dots$$

Hmmmm, well writing out binary numbers is one way to spend a rainy day. But now let's consider the number of 1's in the binary representation of each number.

But what happens now? The whole process starts again, TWICE! That is, the two odd entries at either side start entirely new versions of the original triangle, in exactly the same pattern! And they don't intersect for a while because they are so far apart – they can only spread down like the original triangle. In fact, they are just so far apart (you can check it if you like) so that, at the next power-of-two-minus-one, $2^{n+1} - 1$, they are just about to intersect and the row is all odd again! So the odds wipe themselves out again and the doubling process starts all over again, so on and so forth.

What has this got to do with binary numbers? Well the power of 2 gives us a clue. We can actually write an equation out of my last two paragraphs of rambling, believe it or not. First, let the number of odd entries in the n 'th row be $f(n)$. We already know because of our wipe-out theory that at every power of 2 there are only 2 odd entries, so $f(2^k) = 2$ for all possible k . Because of the way that the triangle replicates itself twice, if we go, say x steps beyond the "doubling point" 2^n , then the point we get to is a point x steps into the original triangle, copied twice. So the number of odd entries in row number $2^n + x$ is twice the number of odd entries in row number x . That is,

$$f(2^n + x) = 2f(x).$$

Excellent! But we have to remember that this doesn't work for all x , because if we go too far past 2^n the triangle will have wiped itself out again. We can check that it only works for x between 0 and $2^n - 1$. Phew!

Now to bring the binary numbers into it. When a number is written in binary form, we are basically writing it as a sum of powers of 2. For example,

$$\begin{aligned} 10_2 &= 2^1 \\ 1101_2 &= 2^3 + 2^2 + 2^0 \end{aligned}$$

So, if we think about it, we can use the f formula on these powers of 2 because, for instance,

$$\begin{aligned} f(2^3 + 2^2 + 2^0) &= f(2^3 + \text{other stuff}) \\ &= 2f(\text{other stuff}) \end{aligned}$$

(You can check for yourself that the other stuff is always within the right limits). We can now use this idea to figure out f of some numbers.

$$\begin{aligned} f(10_2) &= f(2^1) = 2 \text{ (as } f \text{ of a power of 2 gives 2)} \\ f(1101_2) &= f(2^3 + 2^2 + 2^0) \\ &= f(2^3 + (2^2 + 2^0)) \end{aligned}$$

$$\begin{aligned}
 &= 2f(2^2 + 2^0) \\
 &= 4f(2^0) \\
 &= 8
 \end{aligned}$$

And so, we can see, for every power of 2 in the sum, we have a factor of 2! But the number of powers of 2 we write out is just the number of digits in the binary representation. So we have a connection! To see this more clearly, take any old binary number with, say, m 1's in it.

$$\begin{array}{l}
 \overbrace{1101011101 \dots 101}^{m \text{ 1's}} \rightarrow \overbrace{2^{\text{stuff}} + 2^{\text{other stuff}} + \dots + 2^{\text{more stuff}}}^{m \text{ terms}} \\
 \rightarrow f(2^{\text{stuff}} + 2^{\text{other stuff}} + \dots + 2^{\text{more stuff}}) \\
 \rightarrow 2f(2^{\text{other stuff}} + \dots + 2^{\text{more stuff}}) \\
 \rightarrow (m \text{ times}) \\
 \rightarrow 2^m
 \end{array}$$

And so we have it, O intrepid mathematical adventurers! This proves the result.

Daniel Mathews

Paradox Problems

The following are some maths problems for which prize money is offered. The person who submits the best (i.e. clearest and most elegant) solution to each problem will be awarded the sum of money indicated beside the problem number. Solutions may be emailed to paradox@ms.unimelb.edu.au or you can drop a hard copy of your solution into the MUMS pigeon-hole near the Maths and Stats Office in the Richard Berry Building.

1. *Just Enough Petrol* (\$5)

There are n identical cars on a circular track. Among all of them, they have just enough petrol for one car to complete a lap. Show that there is one car which can complete a lap by collecting petrol from the other cars on its way around.

2. *Towns and Pubs* (\$10)

There are exactly n towns and n pubs, no three of which lie on a straight line. No matter how the towns and pubs are positioned, can you always join each town to a different pub with a straight line road so that none of the roads intersect?

3. *A Number Puzzle* (\$5)

Can you complete the blanks below to make each sentence true?

The number of times the digit 0 appears in this puzzle is _

The number of times the digit 1 appears in this puzzle is _

The number of times the digit 2 appears in this puzzle is _

The number of times the digit 3 appears in this puzzle is _

The number of times the digit 4 appears in this puzzle is _

The number of times the digit 5 appears in this puzzle is _

The number of times the digit 6 appears in this puzzle is _

The number of times the digit 7 appears in this puzzle is _

The number of times the digit 8 appears in this puzzle is _

The number of times the digit 9 appears in this puzzle is _

4. *What Mathematicians Talk About* (\$10)

Two integers, m and n , each between 2 and 100 inclusive, have been chosen. The product, mn , is given to mathematician X . The sum, $m + n$, is given to mathematician Y . Their conversation is as follows.

X : I don't have the foggiest idea what your sum is, Y .

Y : That's no news to me, X . I already knew that you didn't know.

X : Aha, NOW I know what your sum must be, Y !

Y : And likewise X , I have surmised your product!

Find the integers m and n .

WINNERS FROM YESTERYEAR

The following people won prize money by solving Paradox Problems from last year. E-mail to collect your money!

Issue 1, Problem 1 - (\$5) Greg Duck

Issue 1, Problem 2 - (\$10) Toby Ord

Issue 2, Problem 1 - (\$5) Thomas Taverner

– Norman Do (norm@ms.unimelb.edu.au)