Paradox

THE MAGAZINE OF THE MELBOURNE UNIVERSITY MATHEMATICS AND STATISTICS SOCIETY
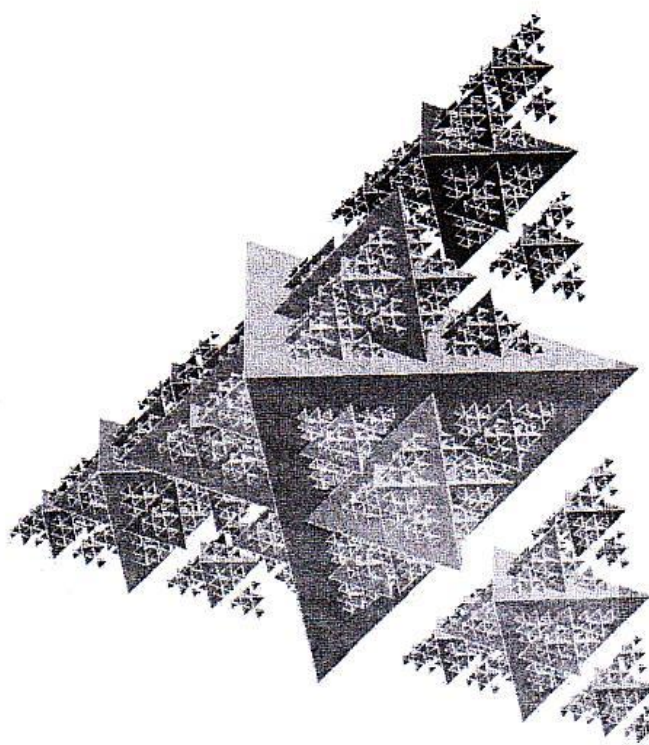
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PRINTED:	May, 2001

Words from the Editor ...

Welcome to the second issue of paradox for 2001. In this issue, we have an article on Claude Shannon a mathematician who died earlier this year after being one of the most significant researchers of the 20th century. We also have an article on the "Doomsday Algorithm", a quick method for finding the day of any given date.

I would like to thank Prof. Rubenstein and Daniel Mathews for their articles in the last issue and Geordie Zhang for his contributions to this one, also Norman Do for the ever present and much loved Paradox Problems.

— Joseph Healy, *Paradox* Editor

Words from the President ...

Welcome to this special Discovery Day edition of Paradox! Paradox is the magazine of the Melbourne University Maths & Stats Society, or MUMS. Foremost in the mind of all MUMSies on Discovery Day is one of the premier events of the day – the Schools Maths Olympics, a battle of intellects, agility, stamina and brawn in which school teams compete for the title of Olympic Champion!

As part of Discovery Day we will also be holding a seminar for interested school students entitled "Adventures in Maths", similar to seminars that we hold fortnightly throughout the year, at 3:30 in Theatre A. And, of course, we will be running a table in the maths department from which you can get free lollies simply by answering a totally-easy maths question!

Stay tuned for our major event of the year, the University Maths Olympics, which is coming soon...

— Daniel Mathews, MUMS President

The Doomsday Algorithm

Occasionally one needs to know the day of the week of a given date. This normally has you reaching for a calendar or diary and searching for that date. While that is generally fine for this year or either the previous or next year, what happens when you need to know the day of a general date, say the day of the week of July 20, 1969, the day a man first walked on the moon?

This is where the sinister sounding "Doomsday Algorithm" can come to the rescue. It allows you to simply and in your head calculate the day of any given date.

The first thing to remember is that the day of the week that the last day of February falls on is called the **Doomsday** for that year. This year, not being a leap year, the last day of February was the 28th and a Wednesday.

All of the months have a doomsday and the day of the week of this day is the same for all months in the same year. For the even months except Februrary the doomsday is just the date with the same number as the month, e.g. 10th October or 12th December.

For the odd months, you will need to remember that the doomsday of January is either the 31st or the 32nd of January. Some of the more observant of you might have noticed that January has only 31 days. In a leap year, the doomsday of February is the 29th, one day later than normal. This also pushes back the doomsday for January. The doomsday of March is the 7th of March. This will also have to be remembered.

To remember the other odd months, a useful mnemonic is 'I work 9-5 at the 7-11'. This associates the 9th and 5th months to each other and the 7th and 11th months. The doomsday for May is the 9th and for September it is the 5th. The same is true of the other pair, 7 and 11. The 7th of November and the 11th of July are doomsdays.

A summary is below

Month	January	February	March	April	May	June
Doomsday	31/32	28/29	7	4	9	6

Month	July	August	September	October	November	December
Doomsday	11	8	5	10	7	12

So knowing all of the above, one can find the day of any date given the doomsday for that year. Calculating the day of the doomsday for a year is the next part of the algorithm. The century day is the doomsday of the zeroth year in that century, example 1900's century day was Wednesday, and 2000's was a Tuesday.

The century days of centuries since 1500* are:

			1500
1600	1700	1800	1900
2000	2100	2200	2300
Tuesday	Sunday	Friday	Wednesday

To find the doomsday for a year, take the century day for that century and add the number of times 12 goes into they year, then the remainder of this, then the number of times 4 goes into the remainder.

So for our date of the Moon landing, July 20, 1969. We first work out the doomsday for 1969:

The century day for the 1900's was Wednesday. 69 divided by 12 is 5 remainder 9 and 9 divided by 4 is 2 remainder 1. This means we add $5 + 9 + 2 = 16$ days onto Wednesday, which gives us Friday.

So Friday is our doomsday for 1969. The doomsday in July is the 11th as July is an odd month and using the '9-5 and 7-11' mnemonic.

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*Before the 1500's, Pope Gregory had not reformed the calendar and this method will not work correctly. The year of reformation varies between 1583(Poland) and 1923(Greece). Britain and the American colonies adopted the new calendar in 1752.

The 20th of July is 9 days after the 11th so adding 9 days to Friday gives us Sunday. Checking in an old diary or on computer shows this to be correct.

With a little practice, this becomes very quick. To avoid having to go through the process of finding the doomsday every time, remember it for common years. This year (2001) it is a Wednesday.

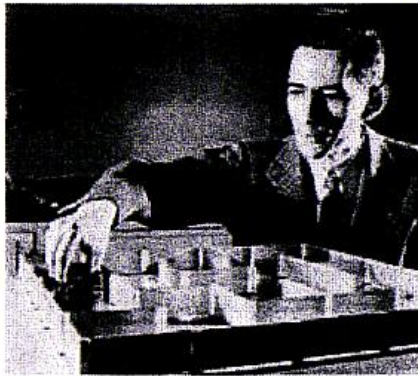## Claude E. Shannon - Father of Information Theory

Look around the world today, and you will see many high-tech products marked "digital". It is an undeniable characteristic of the information age which we live in. CDs, DVDs, the Internet – all these modern day commodities are based on digital technology. Yet all of this would not have been possible if it was not for the man who started the digital revolution – Claude Elwood Shannon.

Shannon was born on 30 April 1916 in Petoskey, Michigan, USA, and grew up in Gaylord, Michigan. His father, Claude Elwood Shannon Sr., was a judge in Gaylord. His mother, Mabel Catherine Wolf Shannon, was a language teacher and the principal of Gaylord High School for many years. At an early age Shannon showed signs of scientific inclination. At school his best subjects were mathematics and science, and at home he would construct radios, model airplanes, and various other gadgets. His most ambitious creation was a telegraph system to a friend's house half a mile away. These were early indicators of what came to be a most fulfilling career. Shannon's childhood hero was Thomas Edison, who he later learned was a distant cousin.

After graduating from Gaylord High School in 1932 and completing a Bachelor of Science in Electrical Engineering and Mathematics from the University of Michigan in 1936, Shannon entered the Massachusetts Institute of Technology to pursue an S.M. (Master of Science) in Electrical Engineering. While at MIT Shannon also completed a Ph.D. in mathematics on a Bolles Fellowship, where he used mathematics to tackle a problem in theoretical genetics. However this work was not as well known as his Masters thesis, and was only published in 1993 in "The Collected Works of Claude Elwood Shannon", a volume published by the IEEE. He received both his S.M. and Ph.D. in 1940.

In 1941, Shannon began a 31-year affiliation at the AT&T Bell Labs in New Jersey. He published his most influential work, "A Mathematical Theory of Communication", in the Bell System Technical Journal in 1948. This paper gave birth to a new field of science known as Information Theory, which forms the foundation of telecommunication today. Information theory is quantitative analysis of communication capacity and noise tolerance of a communications channel. Shannon saw that fundamentally communication is about "reproducing at one point either exactly or approximately a message selected at another point." The information content of a message, he theorised, consists simply of the number of 1s and 0s it takes to transmit it. By using this model, Shannon could construct mathematical theorems that determined the maximum capacity of a given communication channel to send reliable signals. "A Mathematical Theory of Communication" was hailed as the Magna Carta of the information age. Its impact far reaches into a diversity of

fields including computer science, genetic engineering and neuroanatomy. Moreover his theory was extremely prescient for his time. It was not until the invention of high-speed integrated circuits in the 70s that the full implications of Shannon's work could be realised and utilised. Consequentially, his discoveries are as relevant today as when he made them. "It was truly visionary thinking," said Arun Netravali, president of Lucent Technologies' Bell Labs. "As if assuming that inexpensive, high-speed processing would come to pass, Shannon figured out the upper limits on communication rates. First in telephone channels, then in optical communications, and now in wireless, Shannon has had the utmost value in defining the engineering limits we face."

Another influential paper of Shannon, published in 1949, is his "Communication Theory of Secrecy Systems", produced from his work during WWII. In this paper he described the application of information theory to encryption algorithms, by modelling the encryption process as induction of noise in a communication channel. Again Shannon's work was ground breaking. This paper was one of the first explorations into the field of cryptography, and it has been generally regarded as the piece of work that transformed cryptography from an art to a science. The transatlantic conferences held by Churchill and Roosevelt during WWII would not have been possible without Shannon's discovery. Later on, this publication led to his appointment as a consultant on cryptographic matters to the United States Government.

While at Bell Labs, Shannon was also renowned for his eclectic interests and abilities. A favourite story describes him juggling while riding a unicycle down the halls of Bell Labs. His lifelong fascination with coordination and balance led him to build many models and machines that he describes as his "toys". These include several chess-playing machines, a motorised pogo stick, a unicycle with an eccentric wheel (so the rider moved up and down when riding, to keep the rider steady while juggling), THROBAC (Thrifty ROman numerical BAckward looking Computer - a calculator that performs all the arithmetic operations in the Roman numerical system), a rocket-powered Frisbee, a machine that solved the Rubik's Cube, and a "mind-reading machine". His home in Winchester, Massachusetts, was filled with some 30 musical instruments, including five pianos and various other instruments from a bassoon to a balalaika (which he bought during a trip to Russia). Some of the other inventions of Shannon include Theseus (1950), an electronic mouse that made its way around a maze by recalling whether it has seen its previous position before. It was one of first experiments in artificial intelligence and machine learning.

For the great number of accomplishments it is not surprising for Shannon to be decorated by numerous awards, which includes the Alfred Noble American Combined Engineering Societies Prize (1940), National Medal of Science (1966), the IEEE Medal of Honour

(1966), and the Kyoto Prize for Basic Science (1985). He was a member of the American Philosophical Society, the National Academy of Sciences, the Royal Society of London and the Leopoldina Academy. He also received honorary degrees from Yale, Michigan, Princeton, the University of Edinburgh, the University of Pittsburgh, Northwestern, Oxford, the University of East Anglia, Carnegie-Mellon, Tufts and the University of Pennsylvania. In addition, he was a visiting professor at MIT in 1956, became the Donnor Professor of Science in 1958, and a Professor Emeritus in 1978.

It is undoubted that Shannon's discoveries place him as one of the most influential scientists of the twentieth century. Although his name might not be as well known as the likes of Einstein, his pioneering accomplishments have definitely left their mark in the scientific world. Inventions like the Internet, which have become an inseparable part of modern society, would not exist without Shannon. Yet all Shannon started out to do was to attempt to reduce telephone connection noise. As according to the Bell-Labs obituary, Shannon's large number of "useless inventions" bears out the claim that he was motived by curiosity rather than usefulness. In his words - "I just wondered how things were put together".

Professor Shannon died on 24th February 2001 at the Courtyard Nursing Care Centre in Medford, Massachusetts, after a long struggle with Alzheimer's disease. He was 84.

Two male mathematicians are in a caf. The first one says to the second that the average person knows very little about basic mathematics. The second mathematician disagrees, and claims that most people can cope with a reasonable amount of maths. The first goes off to the toilets, and in his absence his companion calls over the waitress.

He tells her that in a few minutes, after his friend has returned, he will call her over and ask her a question. All she has to do is answer one third x cubed. She repeats, "One thir - dex cue"? He repeats, "One third x cubed". "One thir dex cubed?" Yes, that's right, he says. So she agrees, and goes off mumbling to herself, "One thir dex cubed..."

The first guy returns and the second proposes a bet to prove his point that most people do know something about basic maths. He says he will ask the blonde waitress an integral, and the first laughingly agrees. The second man calls over the waitress and asks, "What is the integral of x squared?".

As instructed, the waitress says "One third x cubed," and while walking away, turns back and adds over her shoulder, "Plus a constant."

After the earth dries out, Noah tells all the animals to 'go forth and multiply'. However, two snakes, adders to be specific, complain to Noah that this is one thing they have never been able to do, hard as they have tried. Undaunted, Noah instructs the snakes to go into the woods, make tables from the trunks of fallen trees and give it a try on the tabletops.

The snakes respond that they don't understand how this will help them to procreate whereupon Noah explains: "Well, even adders can multiply using log tables!"

Maths Haiku

AN INTRODUCTION TO LEBESGUE INTEGRATION...

Countable subset
G-delta μ-measurable
Yeah! Lebesgue's the man!

TOPOLOGY

Group presentation
Quotient space by inclusions
Van Kampert is cool

Continuity:
Open pre-image open
Or by epsilons.

A Mobius strip
Is not orientable
Idea for boob tube.

Orientable:
Bug walking along surface
Not turned upside-down

PARADOX PROBLEMS

The following are some maths problems for which prize money is offered. The person who submits the best (i.e. clearest and most elegant) solution to each problem will be awarded the sum of money indicated beside the problem number. Solutions may be emailed to paradox@ms.unimelb.edu.au or you can drop a hard copy of your solution into the MUMS pigeon-hole near the Maths and Stats Office in the Richard Berry Building.

1. ($5) Six people, named A, B, C, D, E, F, are on a train travelling on the Dandenong line. They are one each from Caulfield, Malvern, Armadale, Toorak, Hawksburn, and South Yarra. The following facts are known:

   (a) A and the man from Caulfield are physicists.

   (b) E and the woman from Malvern are mathematicians.

   (c) The person from Armadale and C are engineers.

   (d) B and F are veterans of the Gulf war, but the person from Armadale has never served in the military.

   (e) The person from Hawksburn is older than A.

   (f) The person from South Yarra is older than C.

   (g) At Toorak, B and the man from Caulfield get off.

   (h) At Oakleigh, C and the man from Hawksburn get off.

   Match the names of the people with their professions and their suburbs.

2. ($5) A ten foot pole is dropped into a milling saw and randomly cut into three shorter poles. What is the probability that these three pieces can be used to form a triangle?

3. ($5) Ten checkers are placed in a row on a table. A move consists of lifting a checker, passing it over the next two checkers to its right or to its left, and placing the lifted checker atop the next checker after those two. How can one manage to arrange five stacks, each of two checkers, that are equally spaced?

4. ($10) Show that New Year's Day falls more frequently on a Sunday than on a Saturday.

PARADOX SOLUTIONS

1. *Just Enough Petrol* ($5)

   There are n identical cars on a circular track. Among all of them, they have just enough petrol for one car to complete a lap. Show that there is one car which can complete a lap by collecting petrol from the other cars on its way around.

Solution: This problem yields easily to a proof by induction. We can see that the result is true for $n = 1$. Now suppose that we have proven the result for k cars and then consider $k + 1$ cars on the track. Then there is a car A which can reach the next car B. (If no car could reach the next one, then there would not be enough fuel altogether for one complete lap.)

So let us empty the petrol in B into the petrol tank of A. The problem is now reduced to the case of k cars. But by the induction step, we known that with k cars, there is one which can complete a lap of the track. This car will also be able to get around the track in the case of $k + 1$ cars by travelling to car A where there will be enough petrol to travel to car B, and for the remaining part of the lap, the car will have the same amount of petrol as it did in the case of k cars. So by induction we have proven that there is always a car which can complete a lap of the track.

2. *Towns and Pubs* ($10)

There are exactly n towns and n pubs, no three of which lie on a straight line. No matter how the towns and pubs are positioned, can you always join each town to a different pub with a straight line road so that none of the roads intersect?

Solution: Firstly, we can calculate that there are $n!$ ways of joining the n towns and the n pubs when the roads are allowed to intersect. So out of all of these possible pairings, there must be one which uses the shortest total length of road. We will now prove that this configuration has no intersecting roads.

Suppose that in this configuration the roads from town X to pub x and from town Y to pub y intersect. Then if we replace these two roads by the roads joining X to y and Y to x, the total road length becomes shorter (which can be shown using the triangle inequality). This contradicts the fact that we had a configuration with the shortest total length of road. So it can't be true that this configuration has roads which intersect.

3. *A Number Puzzle* ($5)

Solution: The answer to this problem is shown below, with an alternative answer given in parentheses. These were the only two possible solutions.

The number of times the digit 0 appears in this puzzle is 1 (1)

The number of times the digit 1 appears in this puzzle is 7 (11)

The number of times the digit 2 appears in this puzzle is 3 (2)

The number of times the digit 3 appears in this puzzle is 2 (1)

The number of times the digit 4 appears in this puzzle is 1 (1)

The number of times the digit 5 appears in this puzzle is 1 (1)

The number of times the digit 6 appears in this puzzle is 1 (1)

The number of times the digit 7 appears in this puzzle is 2 (1)

The number of times the digit 8 appears in this puzzle is 1 (1)

The number of times the digit 9 appears in this puzzle is 1 (1)

Well done to the following people who all managed to solve this problem: Sittiboon Abhinorasaeth, Nicholas Groves, Geoffrey Kong, Sam Leong, Andrew Oppenheim, James Plunkett, Ian Preston, Leila Varghese. However, the prize money goes to Sebastian Saliba who was the first to submit a correct entry.

4. *What Mathematicians Talk About* ($10)

Two integers, m and n, each between 2 and 100 inclusive, have been chosen. The product, mn, is given to mathematician X. The sum, $m + n$, is given to mathematician Y. Their conversation is as follows.

X: I don't have the foggiest idea what your sum is, Y.

Y: That's no news to me, X. I already knew that you didn't know.

X: Aha, NOW I know what your sum must be, Y!

Y: And likewise X, I have surmised your product!

Find the integers m and n.

Solution: You should be able to verify that the two solutions $m = 4, n = 13$ and $m = 4, n = 61$ are both valid.

WINNERS

The following people won prize money by solving Paradox Problems from the last issue. E-mail me to collect your money!

Problem 1 - ($5) Ian Preston
Problem 2 - ($10) Geoffrey Kong
Problem 3 - ($5) Sebastian Saliba
-Norman Do (norm@ms.unimelb.edu.au)