Issue 3, 2002

Paradox

THE MAGAZINE OF THE MELBOURNE UNIVERSITY MATHEMATICS AND STATISTICS SOCIETY

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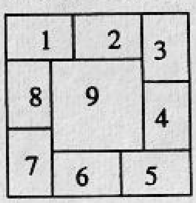
Cover Guide



1. SIERPINSKI, Waclaw.
2. CAUCHY, Augustin.
3. GAUSS, Carl Freidrich.
4. AL-KHOWARIZMI.
5. EULER, Leonhard
6. FERMAT, Pierre de.
7. HAMILTON, William Rowan.
8. TURING, Alan.
9. RAMANUJAN, Srinivasa.

Words from the Editor ...

Welcome to the third and final issue of *Paradox* for 2002, the magazine of the Melbourne University Mathematics and Statistics Society (MUMS). As the university exam period looms near, I'm sure that many of you will be busy cramming. And if so, then what better way to procrastinate than with a copy of your favourite mathematical magazine? This issue promises to be as brimming with jocularity and mathematical nuggets as our previous issues have been.

I'm sure that many of you are aware that Federation Square has just recently been opened to the public. To celebrate the event, *Paradox* features an article on the mathematical tiling used in its design. There is also an interview with staff member Professor Peter Taylor, the Chair of Operations Research in the Department of Mathematics and Statistics. Or perhaps you may wish to peruse the article about the largest number in the world?! This issue also sees the return of *Knotman*, our very own comic strip hero in his greatest adventure yet! And, as always, there are some problems for you to try your hand at, with cash prizes up for grabs.

We are always interested to hear from our readers, so if you have any comments or contributions, please email us at `paradox@ms.unimelb.edu.au`.

— Norman Do, *Paradox* Editor

Paradox

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... and some from the President

This issue of *Paradox* sees us very much at what is often described as the "business end" of the year ... people are spending less time at the café or pub, and more time worrying about assignments, study, exams, vacation work, and other academic related paraphernalia. It can be pretty scary how quickly the year passes — it seems like only yesterday we were welcoming the new first years at the MUMS BBQ in Orientation Week ...

This year's been pretty exciting for MUMS. We've run two BBQs — the aforementioned one in Orientation Week, and then another one towards the end of first semester where we went through about 25 kilograms of sausages! We also had a trivia night in first semester which was enjoyed by all in attendance — even if we did think that the capital of Turkey was Istanbul! We've seen the biggest issue of *Paradox* ever (Issue 2, totalling 32 pages!), almost weekly seminars, a new MUMS t-shirt, the Schools and University Maths Olympics, and an honours information session. We've also got another trivia night coming up in the U-Bar on Thursday at 4:30, so get your teams in as soon as possible!

With exams coming up I'm sure we're all about to move into an entirely different level of procrastination! (Lucky for you a new issue of *Paradox* is out ...) So good luck with your game of Freecell, Minesweeper, or whatever else is your preferred time waster! Oh yeah, and good luck for exams!

— Luke Mawbey, President of MUMS

Thanks

The *Paradox* team would like to thank the following people for their fantastic contributions to this issue: Priscilla Brown, Stephen Farrar, Jolene Koay, Daniel Mathews and Geordie Zhang.

Everybody in Tasmania has the same age

Theorem: In any group of people, everybody has the same age.

Proof: Let $S(n)$ denote the proposition that in any group of n people, everybody has the same age. We will proceed by mathematical induction.

Step 1 In any group that consists of just one person, everybody in the group has the same age, because after all, there is only one person. Therefore, the statement $S(1)$ is true.

Step 2 Now to use induction, we assume that $S(k)$ is true for some number and use that fact to show that $S(k+1)$ is true as well. So let us assume that in any group of k people, everybody has the same age.

Step 3 Let G be an arbitrary group of $k+1$ people; we just need to show now that every member of G has the same age. In other words, we only need to show that if P and Q are any two people in the group G, then they have the same age.

Step 4 So consider everybody in the group G except for P. These people form a group of k people, so they must all have the same age (using the induction hypothesis which tells us that $S(k)$ is true). Now consider everybody in the group G except for Q. Again they form a group of k people, so they must all have the same age by the same reasoning.

Step 5 Consider some person R in the group G other than P or Q. Then Q and R both belong to the group of people not containing P. And we deduced that everybody in this group had the same age, so Q and R have the same age. Similarly, P and R both belong to the group of people not containing Q. So Q and R also have the same age. Since Q and R are the same age, and P and R are the same age, it follows that P and Q are the same age.

Step 6 Since any two people in G have the same age, everybody in G has the same age. Since G was an arbitrary group of $k+1$ people, we deduce that the statement $S(k+1)$ is true.

Step 7 We have now shown that the statement $S(1)$ is true and that if $S(k)$ is true, then $S(k+1)$ is also true. Thus, by induction, the statement $S(n)$ is true for all positive integers n. In particular, in any group of people, everybody has the same age.

Corollary: Everybody in Tasmania has the same age.

PARTY PEOPLE AND THE LARGEST NUMBER IN THE WORLD

A Party Problem

Suppose you're at a party where there are six people including yourself and you notice that there are three people in the room, all of whom know each other. Hardly a surprising observation, one must admit. But at another party involving six people, you notice that this time there are three people in the room, all of whom do not know each other. Following that, you decide to have your own party and wonder:

Party Problem: How many people do you need at a party to guarantee that there are three people all of whom know each other or three people all of whom do not know each other?

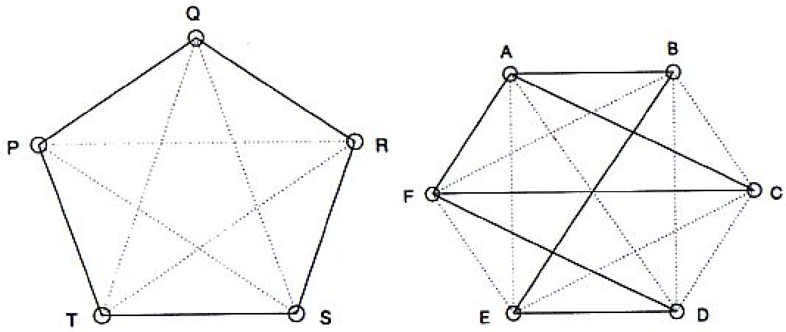


Figure 1: Two graphs which represent parties of five people and six people, respectively.

This problem can be answered by using ideas from the realm of mathematics known as graph theory. We can represent the people at the party by points and connect people who know each other by a blue line segment. We can also connect people who do not know each other by a red line segment and the result is called a two-coloured graph. For example, in Figure 1, on the left we have a graph representing a party with five people where person P knows person Q (because there is a blue line segment between them), person P does not know person R (because there is a red line segment between them), etc.[1]

An observation we can make about this graph is that there are no blue triangles and no red triangles. Partywise, this corresponds to the fact that there do not exist three people all of whom know each other and there do not exist three people all of whom do not know each other. So to guarantee that three such people do exist, we will need at least six people at our party. If we now consider the graph in Figure 1 on the right, it is clear that there exists a blue triangle corresponding to the fact that persons A, C and F all know each other.[2] In fact, it is well known amongst graph theorists and not too difficult to prove that six people is the minimum number of people required.

Party Theorem: In a party of six people, there is guaranteed to be three people all of whom know each other or three people all of whom do not know each other!

Ramsey Theory

After considering the party problem above, the question that is on your lips is probably:

Harder Party Problem: How many people do you need at a party to guarantee that there are four people all of whom know each other or four people all of whom do not know each other?

Or perhaps we can even ask the more general question:

General Party Problem: How many people do you need at a party to guarantee that there are m people all of whom know each other or n people all of whom do not know each other?

The answer to this problem we can designate by the expression $R(m, n)$ as a shorthand. So the party theorem above corresponds to the fact $R(3,3) = 6$. It turns out that to guarantee that there exist four people all of whom know each other or four people all of whom do not know each other at a party, you need 18 people. And we can use our shorthand to write this fact as

[1] Since Paradox appears in black and white, a blue line will be represented by a solid line, while a red line will be represented by a dotted line.

[2] Actually, there is also a red triangle corresponding to the fact that persons B, C and D all do not know each other.

	m=3	m=4	m=5	m=6	m=7	m=8	m=9	m=10
n=3	6	9	14	18	23	28	36	40-43
n=4	9	18	25	35-41	49-61	55-84	69-115	80-149
n=5	14	25	43-49	58-87	80-143	95-216	116-316	141-442
n=6	18	35-41	58-87	102-165	109-298	122-495	153-780	167-1171
n=7	23	49-61	80-143	109-298	205-540	1-1031	1-1713	1-2826
n=8	28	55-84	95-216	122-495	1-1031	282-1870	1-3583	1-6090
n=9	36	69-115	116-316	153-780	1-1713	1-3583	565-6588	1-12677
n=10	40-43	80-149	141-442	167-1171	1-2826	1-6090	1-12677	798-23581

Figure 2: Ramsey Numbers

$R(4,4) = 18$. These numbers are generally known as Ramsey[3] numbers and the study of such party problems is known as Ramsey theory[4]. The table in Figure 2 shows them for various values of m and n. For most cases, an exact answer is not currently known, so the range in which the corresponding Ramsey number is known to lie has been given. An observation that we can make from the table is that $R(m,n) = R(n,m)$ which is true by the symmetry of the problem.

It is surprising that even for small values of m and n, the exact value of $R(m,n)$ is unknown. One of the greatest contributors to Ramsey theory was the highly eccentric mathematician Paul Erdős.[5] When talking about the difficulty of Ramsey theory, he liked to tell the following story:

"Suppose that aliens invade Earth and decide that they will blow up the planet unless we can work out the correct value for $R(5,5)$. Then our best tactic would be to gather mathematicians from around the world, obtain as much computing power as possible and use the brute force approach of trying all 10^{200} of the specific cases, which might take years. However, if the aliens decide to ask for the correct value of $R(6,6)$, then our best survival strategy would be to gather all of the military from around the world and attack the

[3] Frank Plumpton Ramsey (1903-1930) was a brilliant English mathematician who published works in philosophy, economics and logic. As is common in the history of mathematics, Ramsey's career shone brightly but briefly since he died at the tender age of 26.

[4] Advice: DO NOT try to impress people at parties with your knowledge of Ramsey theory!

[5] Paul Erdős (1913-1996) was a Hungarian mathematician who is famous for being a prolific solver of problems, with over 1500 papers to his name. Throughout his life, Erdős spent his time travelling from one university to another, staying in the homes of mathematician friends such as Ron Graham (see below). In fact, Erdős had no permanent residence and few material possessions, owning only a half empty suitcase. This allowed him to dedicate all of his efforts to mathematics, even resorting to caffeine pills to allow him to concentrate on mathematics for as long as possible.

aliens before they could attack us!"

Such is the difficulty of these seemingly simple party problems.

Arrow Notation for Large Numbers

If asked to think of a large number, many people might come up with a million, or perhaps a billion, or perhaps even a trillion.[6]

Nevertheless, these numbers are dwarfed by a googol[7], which is equal to 10^{100} whose decimal representation consists of a 1 followed by one hundred 0's. A googol is a humongous number — much larger than the number of elementary particles in the known universe, which is about 10^{80}. The term was introduced by the American mathematician Edward Kasner who asked his nine year old nephew Milton Sirotta to invent a word for such a large number. However, not long afterwards, another mathematician coined the term googolplex to represent the even larger number whose decimal representation consists of a 1 followed by a googol of 0's. We can still write this number as a tower of exponents in the simple form $10^{10^{100}}$.[8]

Could it be possible that there is a use for numbers even larger than a googolplex, so large that simple towers of exponents will not suffice to express them? The renowned computer scientist Donald Knuth thought so, and in 1976 proceeded to invent arrow notation to represent them. Here's how it works:

$$\begin{aligned}
m \uparrow n &= \underbrace{m \times m \times \cdots \times m}_{n} = m^n \\
m \uparrow\uparrow n &= \underbrace{m \uparrow m \uparrow \cdots \uparrow m}_{n} = \underbrace{m^{m^{\cdot^{\cdot^{m}}}}}_{n} \\
m \uparrow\uparrow\uparrow n &= \underbrace{m \uparrow\uparrow m \uparrow\uparrow \cdots \uparrow\uparrow m}_{n}
\end{aligned}$$

[6]Unfortunately, what we mean by these numbers depends on whether we are adopting the American system (where a billion represents a thousand million, a trillion represents a thousand billion, etc.) or the British system (where a billion represents a million million, a trillion represents a million million million, etc.). See *Paradox 2000* — Issue 2 — The Billionist Manifesto.

[7]The name of the ever popular internet search engine Google is a play on the word googol. It reflects the company's mission to "organise the immense, seemingly infinite, amount of information on the web".

[8]Remember that when you want to find the value for a tower of exponents, you always start calculating from the top. For example, $3^{3^3} = 3^{(3^3)} = 3^{27}$ which is different from $(3^3)^3 = 27^3$.

and the pattern continues when more than four arrows are used. One thing to remember when using arrow notation is that we evaluate the arrows from right to left. For example:

$$\begin{aligned} 3 \uparrow\uparrow\uparrow 3 &= 3 \uparrow\uparrow 3 \uparrow\uparrow 3 \\ &= 3 \uparrow\uparrow (3 \uparrow\uparrow 3) \\ &= 3 \uparrow\uparrow 3^{3^3} \\ &= 3 \uparrow\uparrow 7625597484987 \\ &= \underbrace{3^{3^{\cdot^{\cdot^{3}}}}}_{7625597484987} \end{aligned}$$

This is a phenomenally huge number, unimaginably larger than a googolplex!

Graham's Number

For a while, *The Guinness Book of Records*[9] contained an entry for the largest number. Surely, as most children would know, no such largest number exists because you can always just "plus one". The entry actually contained the largest number ever used in a mathematical proof, for which Graham's number was the undisputed world champion. And the number came about from considering a party problem from Ramsey theory like the one we looked at earlier. The problem can be informally stated as follows:

Graham's Problem: Given a party of n people, consider every possible committee that you can form from them. Assign each pair of committees to one of two groups. What is the smallest value of n that will guarantee that there are four committees in which all pairs are in the same group and all the people belong to an even number of committees?

At the time that the problem was stated, it was not clear whether or not there was a finite value of n which satisfied the conditions of the problem. In 1977, the mathematicians Ron Graham[10] and Bruce Rothschild showed

[9]The idea for *The Guinness Book of Records* was hatched in 1951 by Sir Hugh Beaver, then the managing director of the Guinness brewery. He realised that there was a need for such a reference book to settle the common disputes that arose between people in pubs.

[10]Ronald Graham is an American mathematician who is a true renaissance man, delving into other areas

that such a value of n did exist by showing that it had to be less than Graham's number, which they constructed as follows.

- Using our arrow notation, let us begin with the number $G_1 = 3 \uparrow\uparrow\uparrow\uparrow 3$.
- Construct the number $G_2 = 3 \uparrow\uparrow \cdots \uparrow 3$ where there are G_1 arrows.
- Construct the number $G_3 = 3 \uparrow\uparrow \cdots \uparrow 3$ where there are G_2 arrows.
- Continue constructing these numbers in a similar fashion, where each number G_n is of the form $3 \uparrow\uparrow \cdots \uparrow 3$ and there are G_{n-1} arrows.
- Graham's number is the 64th number in the sequence, namely G_{64}.

Recall our phenomenally huge example from earlier which was $3 \uparrow\uparrow\uparrow 3$. The beginning of the construction of Graham's number begins with $G_1 = 3 \uparrow\uparrow\uparrow\uparrow 3$ which outdoes our example by a whole arrow and thus, is crazily more humongous. And with each construction of $G_2, G_3, G_4, \ldots$ the number gets unimaginably more humongous. So you begin to see that Graham's number is mind boggling in size, indescribable in terms of anything physical, indescribable in terms of our standard notation for large numbers!

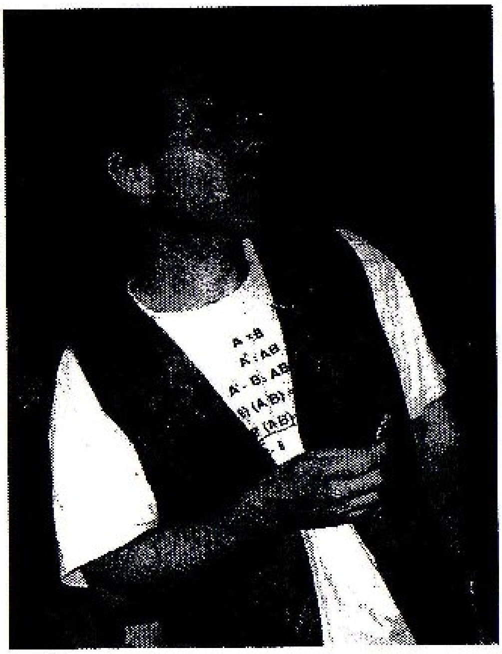
Now remember that the reason why Graham's number was constructed was to show that the answer to Graham's problem was finite. To this day, the smallest known upper bound for the answer to Graham's problem is the enormous Graham's number. But the most remarkable thing is that Ramsey theory experts strongly believe, and yet are unable to prove, that the answer to Graham's problem is in fact 6 (which is, indeed, less than Graham's number)!

— Norman Do

such as computer encryption and being a renowned juggler and trampolinist. As a student Graham formed a small circus troupe in order to pay for his college expenses, is a former president of the International Jugglers Association, and appeared on stage with Cirque du Soleil. Furthermore, he is an expert in table tennis, the boomerang and ten pin bowling (with two perfect games to his credit) and has taught himself to speak Mandarin and Japanese. Currently, Graham is on the editorial boards of over 40 mathematics and computer science journals, is the treasurer of the prestigious National Academy of Sciences and continues to do mathematics.

Peter Taylor

Professor Peter Taylor was born in Nottingham, England, and his family moved to Adelaide, Australia, when he was five years old. He attended St Peter's College and commenced a Bachelor of Science at The University of Adelaide. In 1979, he completed undergraduate majors in pure and applied mathematics, followed by an honours year in applied mathematics. Afterwards, he began to work for the Australian Public Services in Canberra.



He returned to The University of Adelaide in 1983, taking up a position as a tutor in the Department of Applied Mathematics as well as commencing his PhD in applied probability, which he received in 1987. He started lecturing at The University of Adelaide in 1988, and later at the University of Western Australia in the years 1989-1990. He returned to The University of Adelaide again in 1991, this time as an Associate Professor. There he remained until the end of 2001, when he came to The University of Melbourne to take up the positions of Professor and Chair of Operations Research in the Department of Mathematics and Statistics. While in Adelaide, he was also the co-director of the Teletraffic Research Centre, which provided consulting for industries on various aspects of telecommunication systems and networks.

Peter's main interests include stochastic modelling, Markov chains, estimation in networks, queueing theory and matrix analytical methods. Outside his academic realm, Peter leads an active life. He loves the outdoors and is a frequent participant in orienteering and rogaining competitions. He was the South Australian rogaining champion and came in the top five in the national competition. In addition, he enjoys playing cricket, and has gone bushwalking in Patagonia. He has also travelled across six continents to more than 30 countries, for conferences and for leisure. In addition, Peter is a very talented compere — those who participated in the Schools and the University Maths Olympics would have had the chance to experience his witty and humorous style of hosting.

When I asked Peter for his advice for the younger generation, he commented that studying mathematics as an undergraduate is most definitely rewarding, as it provides the foundation for many other fields. So it is important for the university and the departments to encourage students to study mathematics subjects. Moreover, there are definitely careers for mathematicians out there, in a great variety of areas. However, usually mathematical ability alone is not enough in the professional world, so maths students should try to gain other skills as well during their education, such as communication, people and writing skills. In conclusion, Peter said that the reason why he is still in mathematics, and the single reason why students should do mathematics, is because "maths is fun".

— Geordie Zhang

A Crazy Theorem!

Theorem: Every positive integer can be unambiguously described in fourteen or fewer words.

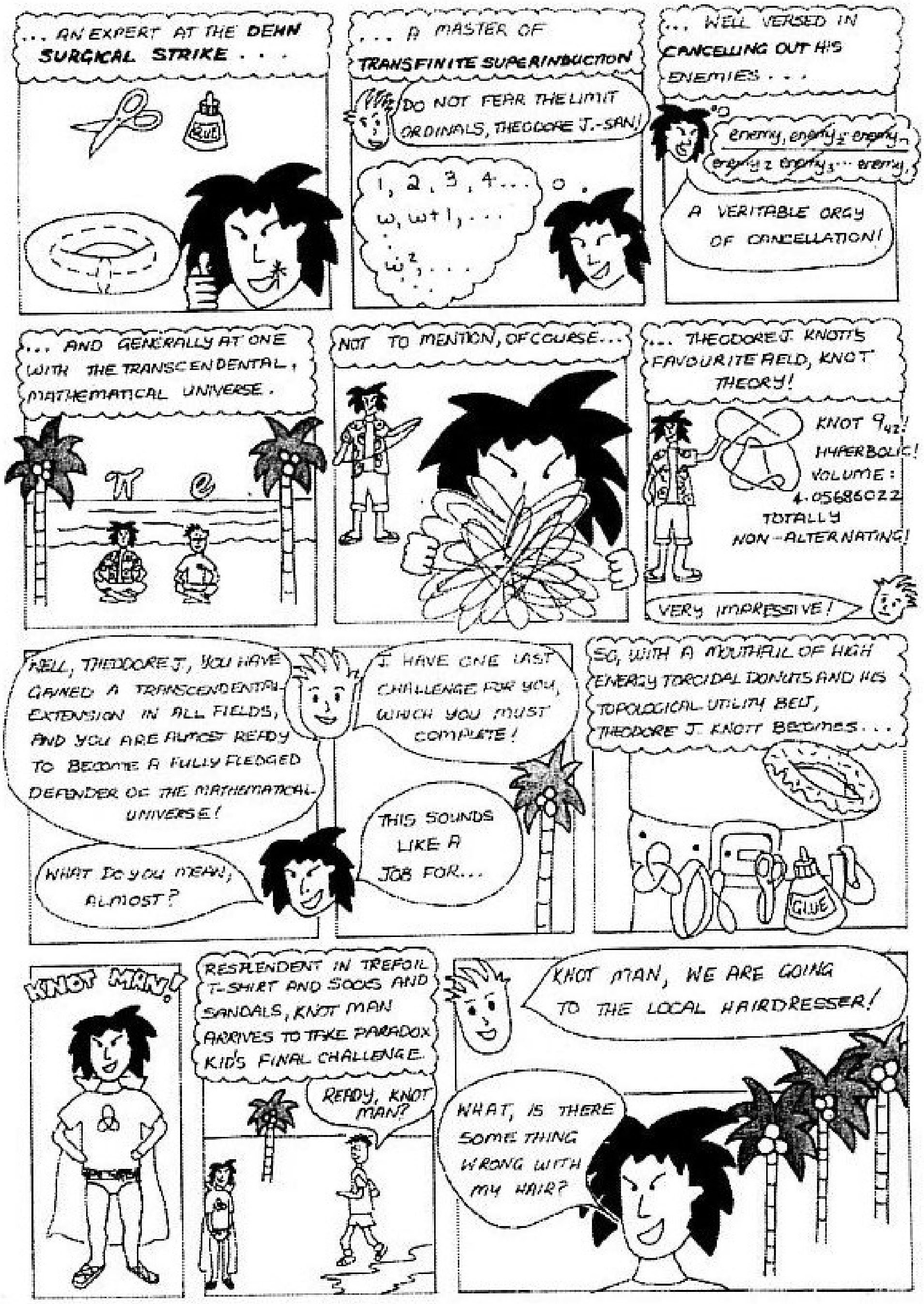
Proof: Suppose that there is some positive integer which cannot be unambiguously described in fourteen or fewer words. Then out of all such numbers, there must a smallest one which we will call n.

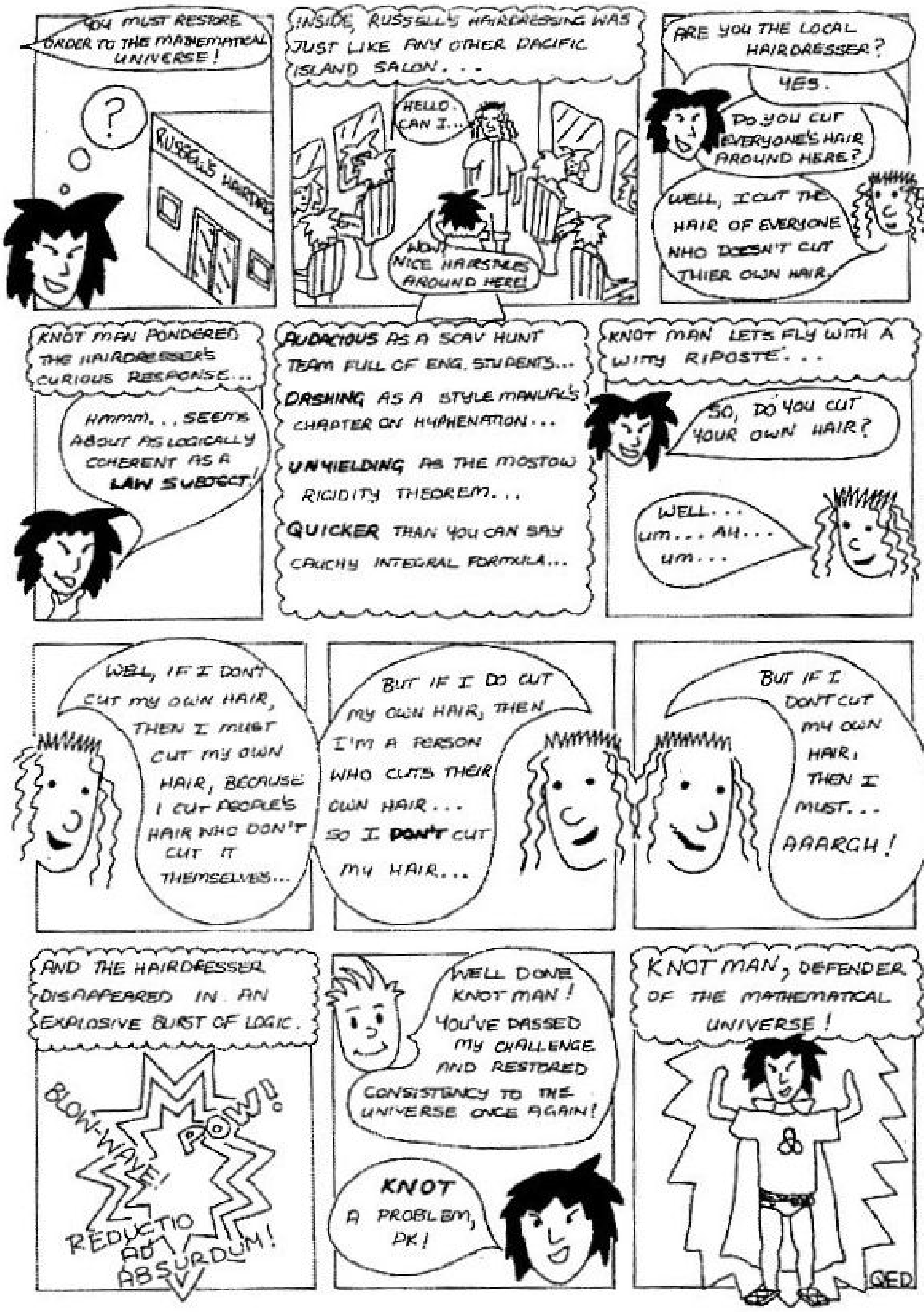
But now n is "the smallest positive integer that cannot be unambiguously described in fourteen or fewer words". This is a complete and unambiguous description of n in fourteen words, contradicting our assumption that n did not have such a description!

Since the assumption of the existence of a positive integer that cannot be unambiguously described in fourteen or fewer words led to a contradiction, it must be an incorrect assumption.

Therefore, all positive integers can be unambiguously described in fourteen or fewer words!


KNOT MAN
ISSUE #2
By DANIEL MATHEWS & PRISCILLA BROWN
AFTER HIS LATEST BOUT OF SAVING THE UNIVERSE, THEODORE J. KNOTT HAS DECIDED TO TAKE SOME TIME OUT TO RECUPERATE, ON A TROPICAL ISLAND PARADISE...
... WHERE IT SO HAPPENS THAT ONE-TIME SUPERHERO AND MATHEMAGICIAN **PARADOX KID** IS ENJOYING HIS RETIREMENT.
THEODORE J.?
PK!
PK, I LOVED YOUR WORK! IT'S BECAUSE OF YOU THAT THE MATHS DEPARTMENT IS SUCH A FREE ASSOCIATIVE GROUP!
OH, THANKS THEODORE J.! LET'S GO GET SOME SNACKS OF THE GENUS-1 MATHS VARIETY!
SO THEODORE J. AND PK, AFTER A HEARTY DOSE OF GENUS-1 DONUTS, REGALED EACH OTHER OF THEIR RESPECTIVE ADVENTURES.
...AND THEN ONE TIME THERE WAS THIS INSANE, PSYCHOTIC PHYSICS STUDENT!...
SOUNDS DERANGED! I'LL BET HE DIDN'T HAVE ANY FIXED POINTS!
BUT THEODORE J. KNOTT HAD SOMETHING TO ASK OF THE LEGENDARY PK...
...SO NOW, LIFE IS AS SMOOTH AS A **RIEMANNIAN MANIFOLD**!
PK?
YES, WHAT IS IT THEODORE J.?
I WANT TO LEARN MORE IN THE WAYS OF DEFENDING THE MATHEMATICAL UNIVERSE! CAN YOU HELP ME?
BY GÖDEL, IT'S BEEN LONGER THAN THE PROOF OF FERMAT'S LAST THEOREM, BUT I'LL GIVE IT A TRY!
THE GREAT PK THUS TOOK THEODORE J. KNOTT UNDER HIS TUTELAGE...
THEODORE J. KNOTT, I WILL SHOW YOU HOW TO OBTAIN
MORE POWER THAN A QUINTIC (5)
MORE BRAINS THAN THE BASKET BEHIND A GUILLOTINE, AND
MORE SEXUAL ALLURE THAN A HYPERBOLIC ATTRACTOR!
PK TAUGHT THEODORE J. TO BECOME INVULNERABLE TO ALL **DIFFERENTIAL ATTACKS**...
FEEL THE e^x, BE THE e^x, THEODORE J.-SAN!
$\frac{d}{dx}$
$\frac{d}{dx}$
$\frac{d}{dx}$
$\frac{d}{dx}$


... AN EXPERT AT THE DEHN SURGICAL STRIKE . . .
GLUE
... A MASTER OF TRANSFINITE SUPERINDUCTION
DO NOT FEAR THE LIMIT ORDINALS, THEODORE J.-SAN!
1, 2, 3, 4... ω, ω+1, ... ω², ...
... WELL VERSED IN CANCELLING OUT HIS ENEMIES . . .
A VERITABLE ORGY OF CANCELLATION!
... AND GENERALLY AT ONE WITH THE TRANSCENDENTAL, MATHEMATICAL UNIVERSE.
π e
NOT TO MENTION, OF COURSE...
... THEODORE J. KNOTT'S FAVOURITE FIELD, KNOT THEORY!
KNOT 9_{42}! HYPERBOLIC! VOLUME: 4.05686022 TOTALLY NON-ALTERNATING!
VERY IMPRESSIVE!
WELL, THEODORE J, YOU HAVE GAINED A TRANSCENDENTAL EXTENSION IN ALL FIELDS, AND YOU ARE ALMOST READY TO BECOME A FULLY FLEDGED DEFENDER OF THE MATHEMATICAL UNIVERSE!
I HAVE ONE LAST CHALLENGE FOR YOU, WHICH YOU MUST COMPLETE!
WHAT DO YOU MEAN, ALMOST?
THIS SOUNDS LIKE A JOB FOR...
SO, WITH A MOUTHFUL OF HIGH ENERGY TOROIDAL DONUTS AND HIS TOPOLOGICAL UTILITY BELT, THEODORE J. KNOTT BECOMES . . .
GLUE
KNOT MAN!
RESPLENDENT IN TREFOIL T-SHIRT AND SOCKS AND SANDALS, KNOT MAN ARRIVES TO TAKE PARADOX KID'S FINAL CHALLENGE.
READY, KNOT MAN?
KNOT MAN, WE ARE GOING TO THE LOCAL HAIRDRESSER!
WHAT, IS THERE SOMETHING WRONG WITH MY HAIR?


YOU MUST RESTORE ORDER TO THE MATHEMATICAL UNIVERSE!
?
INSIDE, RUSSELL'S HAIRDRESSING WAS JUST LIKE ANY OTHER PACIFIC ISLAND SALON...
HELLO. CAN I...
WOW! NICE HAIRSTYLES AROUND HERE!
ARE YOU THE LOCAL HAIRDRESSER?
YES.
DO YOU CUT EVERYONE'S HAIR AROUND HERE?
WELL, I CUT THE HAIR OF EVERYONE WHO DOESN'T CUT THIER OWN HAIR.
KNOT MAN PONDERED THE HAIRDRESSER'S CURIOUS RESPONSE...
HMMM... SEEMS ABOUT AS LOGICALLY COHERENT AS A LAW SUBJECT!
AUDACIOUS AS A SCAV HUNT TEAM FULL OF ENG. STUDENTS...
DASHING AS A STYLE MANUAL'S CHAPTER ON HYPHENATION...
UNYIELDING AS THE MOSTOW RIGIDITY THEOREM...
QUICKER THAN YOU CAN SAY CAUCHY INTEGRAL FORMULA...
KNOT MAN LETS FLY WITH A WITTY RIPOSTE...
SO, DO YOU CUT YOUR OWN HAIR?
WELL... UM... AH... UM...
WELL, IF I DON'T CUT MY OWN HAIR, THEN I MUST CUT MY OWN HAIR, BECAUSE I CUT PEOPLE'S HAIR WHO DON'T CUT IT THEMSELVES...
BUT IF I DO CUT MY OWN HAIR, THEN I'M A PERSON WHO CUTS THEIR OWN HAIR... SO I DON'T CUT MY HAIR...
BUT IF I DON'T CUT MY OWN HAIR, THEN I MUST... AAARGH!
AND THE HAIRDRESSER DISAPPEARED IN AN EXPLOSIVE BURST OF LOGIC.
BLOW-WAVE!
POW!
REDUCTIO AD ABSURDUM!
WELL DONE KNOT MAN! YOU'VE PASSED MY CHALLENGE AND RESTORED CONSISTENCY TO THE UNIVERSE ONCE AGAIN!
KNOT A PROBLEM, PK!
KNOT MAN, DEFENDER OF THE MATHEMATICAL UNIVERSE!
QED

HOW TO MAKE YOUR OWN FEDERATION SQUARE[1]

Melbourne seems to have a strange love for weird ways of tiling its buildings. You may have noticed that RMIT's Storey Hall on Swanston Street and the new development in Federation Square both feature funky walls covered with tiles in an apparently random fashion. Some mathematicians get pretty excited about these buildings. Let's take a look why ...

A *tiling* is a way of arranging tiles on a flat surface, so that there are no gaps. We do not require the tiles to all be the same shape, just that there be only a finite number of shapes. As you can imagine, tiling has been around a long time, probably since tiles were first invented. And probably, people soon discovered that by using a repeating pattern they could tile areas, no matter how large. Were it not for the fact that infinity was invented after tiles, people would probably have said that they knew how to tile an infinite plane. And since there isn't really any reason why you'd want to do more than tile an infinite plane, most people stopped thinking about tiles and moved on to carpet.

A Definition Or Two

The artist Maurits Cornelius Escher designed some ingenious tilings, one of which appears in Figure 3. Most tilings, like this one, are obtained by repeating the same pattern over and over, and are called *periodic*. Here is a way to test whether or a tiling is periodic. Take a piece of translucent paper and trace the tiling onto the paper. Now, if you can shift the paper (without rotating it) to a new location and the tracing coincides, then the tiling is periodic. Otherwise, we call the tiling nonperiodic. The Escher tiling is periodic. Tiling the bathroom floor with squares is periodic. The tiling of Federation Square is nonperiodic.[2]

[1] For less than $427 million!

[2] Some people incorrectly call the tiling of Federation Square *aperiodic*. It is not, as we shall see later.

Figure 3: A tiling of the plane by M.C. Escher.

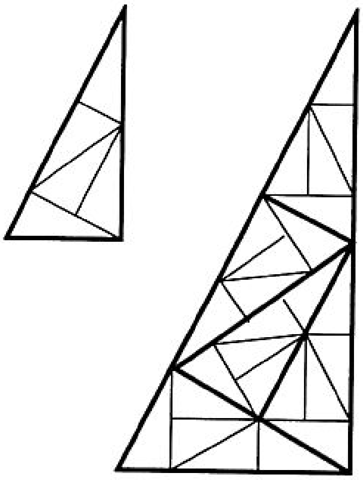


Figure 4: The first and second stages of the Pinwheel tiling.

A Nonperiodic Tiling

The tiling at Federation Square is known as the Pinwheel tiling. This tiling uses only one shape of tile, a right-angled triangle whose side lengths are in the ratio 1 to 2 to $\sqrt{5}$. The tiling is constructed in a number of stages.

Begin by taking 5 small triangles and gluing them together as shown in the left side of Figure 4. We shall call the large triangle produced a stage-one-triangle. Next take 5 stage-one-triangles and glue them together in the same manner. Call this the stage-two-triangle. Stage two is shown in the right side of Figure 4. Next take 5 stage-two-triangles and glue them together to form a stage-three-triangle. Then take 5 stage-three triangles and glue them together, ... By continuing this process for a while we can tile any area, no matter how large. Furthermore, the tiling is nonperiodic.

Aperiodic Tiling

It may seem as though we cheated a bit in the pinwheel tiling. After all, those triangles will tile the plane in a periodic fashion if we just arrange them differently. The real challenge is to find a set of tiles that will *only* tile nonperiodically. Such a tiling would then be called *aperiodic*.

For a long time no one believed that aperiodic tilings existed, mostly because no one could think of one. Then in 1964 Robert Berger shocked the world (or at least the mathematical world) by constructing a set of over 20000 tiles which would tile the plane only nonperiodically. He later found a much smaller set of 104 tiles which would do the same thing. Raphael M. Robinson then reduced the number to 24, and Roger Penrose[3] reduced the number to four, and then to two! (No one has yet found an aperiodic tiling involving only 1 tile ... I believe.)

The RMIT building known as Storey Hall is tiled with Penrose's 2-tile aperiodic tiling. The two tiles, shown in Figure 5, are rhombi[4], one with angles 72° and 108°, the other with angles 36° and 144°. If we only allow the tiles to join along edges where the colours match[5] then an interesting thing happens: the rhombi do no tile periodically (although they would

[3] Roger Penrose, along with his father L. S. Penrose, invented the famous "Penrose Staircase" that goes round and round without getting higher. Escher depicted the staircase in "Ascending and Descending" (1960).

[4] Rhombi is the plural of rhombus.

[5] Since *Paradox* appears in black and white, the two colours are grey and grey.

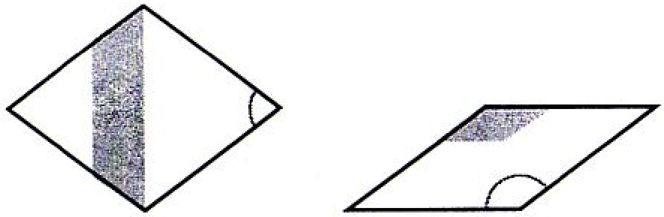


Figure 5: The Penrose tiles.

normally), and instead they tile the infinite plane nonperiodically! The Penrose tiling is shown in Figure 6.

In fact it is probably misleading to call it *the* Penrose tiling, because there is more than one. When you start joining the rhombi together, you often have a choice of how you want to join them. So it is possible to construct many different Penrose tilings.

Penrose tilings have many, many interesting properties. The most arresting is that there are infinitely many Penrose tilings. In fact, there are uncountably many, which means that there is no way we could assign numbers to the tilings (e.g. Penrose tiling number 1, Penrose tiling number 2, ...) and number them all. There are a truly unfathomable number of different Penrose tilings.

Another property is that if you take a finite part of a Penrose tiling, then you can find that same pattern infinitely many times in any Penrose tiling. For example, the RMIT building is tiled with part of a Penrose tiling. This same pattern occurs infinitely many times in any Penrose tiling of the infinite plane.

The last property that we will mention concerns the coloured lines we drew on the tiles. These lines seem to meander aimlessly across the tiling, sometimes joining up with themselves (we call this closing), other times not. It has been proved that whenever a curve closes up, it forms a shape with five-fold symmetry. Furthermore, the region inside the curve has five-fold

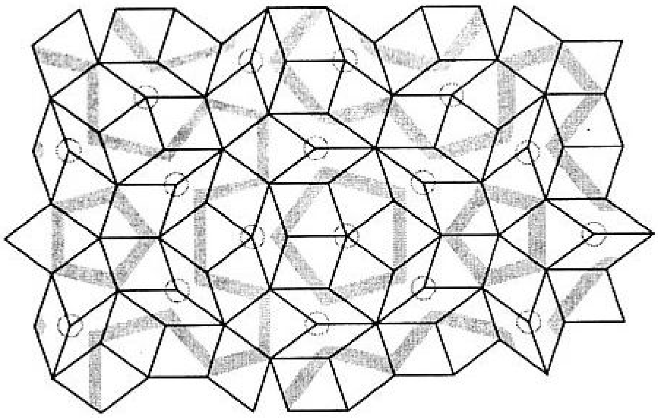


Figure 6: The Penrose tiling.

symmetry. Furthermore, in any Penrose tiling at most two curves do not close. Furthermore, in most tilings *all* curves close.

Finally, it is worth mentioning that this may have some applications, apart from tiling walls in strange ways. In the early 1980s, a strange alloy of aluminium and manganese was discovered, which had a crystal structure with five-fold symmetry. There is no periodic crystal with five-fold symmetry, and so this crystal must have an aperiodic structure. Which begs the question: did humans invent aperiodic tilings, or did nature invent them long ago and we are only just catching up?

— Stephen Farrar

Maths Olympics

The Maths Olympics have always been fun and exciting for those involved and this year was no different, with some fantastic teams entering. A team from University High School (Named "Uni High B") managed to answer 24 out of 25 questions correctly!

The University Maths Olympics saw the entry of two teams which were both expected to do extremely well - "Fermat's Last Lemma" and "*Knotman* and the Mathemagicians" - the latter being a team comprised of former participants at the International Mathematics and Phyiscs Olympiads. Unfortunately for *Knotman*, his mathemagicians had become a little rusty in their old age, and so were unable to conquer the forces of Fermat! This year also saw an entry from RMIT — "Fox Force Five — Just don't call us Aero!" As expected, Melbourne University once again proved its superiority over this second rate institution!!! :)

The teams of lecturers have never won a Maths Olympics and 2002 was no different. However, the staff this year did themselves proud, and were placed fourth overall.

We'd like to thank everyone that supported the Maths Olympics this year - the markers, the sponsors, the audiences, the competitors, the teachers, our official photographer Zaeem, and of course, our fantastic host, Professor Peter Taylor.

The final results are given below:

Schools Maths Olympics

Place	Team	School	Score
1st	Uni High B	University High School	255
2nd	PEGS Navy Blue	Penleigh & Essendon Grammar	175
3rd	Scotchies	Scotch College	150
4th	Pythagoreans	Melbourne High School	140
5th	Camberwell Grammar	Camberwell Grammar	130

University Maths Olympics

Place	Team	Score	Last Question Answered
1st	Fermat's Last Lemma	170	22
2nd	Uni High B	170	21
3rd	Knotman and the Mathemagicians	155	20
4th	Springloaded Live	135	21
5th	We Have Already Voted	135	19

Maths Jokes

A mathematician, a physicist, and an engineer were travelling in Scotland when they saw a black sheep through the window of the train.

"Aha," says the engineer. "I see that Scottish sheep are black."

"Hmmm," says the physicist. "You mean that some Scottish sheep are black."

"No," says the mathematician. "All we know is that there is at least one sheep in Scotland, and that at least one side of that sheep is black!"

$$111111111^2 = 12345678987654321$$

A biologist, a physicist and a mathematician were sitting in Brunetti's sipping their Italian style hot chocolates. Across the street they saw a man and a woman entering a building. Ten minutes later they reappeared together with a third person.

"They have multiplied," said the biologist.

"Oh no, an error in measurement," the physicist sighed.

"If exactly one person enters the building now, it will be empty again," the mathematician concluded.

$$1,741,725 = 1^7 + 7^7 + 4^7 + 1^7 + 7^7 + 2^7 + 5^7$$

A mathematician, an engineer, and a chemist were walking across the South Lawn when they saw a pile of cans of VB. Unfortunately, they were the old-fashioned cans that do not have the ring pull at the top. One of them proposed that they split up and find can openers. The chemist went to his lab and concocted a magical chemical that was able to dissolve the can top in an instant and evaporate the next instant so that the beer would not be affected. The engineer went to his workshop and created a new HyperOpener that was able to open 25 cans per second.

They went back to the pile with their inventions and found the mathematician finishing the last can of beer. "How did you manage that?" they asked in astonishment. The mathematician answered, "Well, I assumed they were open and went from there."

Paradox Problems

The following are some maths problems for which prize money is offered. The person who submits the best (i.e. clearest and most elegant) solution to each problem will be awarded the sum of money indicated beside the problem number. Solutions may be emailed to `paradox@ms.unimelb.edu.au` or you can drop a hard copy of your solution into the MUMS pigeonhole near the Maths and Stats Office in the Richard Berry Building.

1. (\$5) Using each of the digits 1 to 9 exactly once, form a 9-digit integer with the property that its first digit is divisible by 1, the number formed from its first two digits is divisible by 2, the number formed from its first three digits is divisible by 3, and so on. Find all such numbers.

2. (\$5) Colour the squares of a 6×6 checkerboard black and white in the following way: Each square must have an odd number of black neighbours (where a neighbour is a square that is adjacent either vertically or horizontally). How many ways are there of doing this?

3. (\$5) In a cryptarithm, numbers are represented by replacing their digits by letters; a given letter consistently represents the same digit and different letters represent different digits. Leading zeroes are not permitted. Solve the following cryptarithm:

   $$\text{ONE} + \text{ONE} = \text{TWO}$$
   $$\text{SEVEN is prime}$$
   $$\text{NINE is a perfect square}$$

4. (\$10) In a triangle ABC, $AB = 20$, $AC = 21$, and $BC = 29$. The points D and E lie on the line segment BC, with $ED = 8$ and $EC = 9$. Calculate $\angle DAE$.

5. (\$10) A group of people attends a party. Each person has at most 3 acquaintances in the group, and if two people do not know one another, then they have a mutual acquaintance in the group. What is the maximum number of people present?

   If, in addition, the group contains three mutual acquaintances (i.e. three people each of whom knows the other two), what is the maximum number of people?

Solutions to Last Issue's Paradox Problems

Here are solutions to some of the maths problems posted in the last issue of *Paradox*. Congratulations go to the following people for submitting correct solutions:

Problem 1: ($5) Thomas Taverner
Problem 3: ($5) Hugh Allen
Problem 4: ($10) Andrew Barr

Please e-mail `paradox@ms.unimelb.edu.au` to pick up your prizes or drop by the MUMS room in the Richard Berry Building.

1. ($5) In a traditional LCD display some numbers, when viewed upside down, are images of other numbers. For example, [1][9][9][5] becomes [5][6][6][1] . The first few numbers which can be read upside down are [1], [2], [5], [6], [8], [9], ... What is the millionth number that will appear in this sequence?

   Solution: The only one-digit numbers which are images of others when viewed upside down are [0], [1], [2], [5], [6], [8] and [9]. So we can think of each of these as representing the digits 0, 1, 2, 3, 4, 5 and 6 in a base 7 system. In base 7, the millionth number is $1,000,000$ (base 10) $=$ $11,333,311$ (base 7). So the millionth number in this sequence will be [1][1][5][5][5][5][1][1] after substituting [1] for 1 and [5] for 3.

2. ($5) A cylindrical hole six centimetres long has been drilled straight through the centre of a solid sphere. What is the volume remaining in the sphere?

   Solution: Let R be the radius of the sphere. By Pythagoras' Theorem, the radius of the cylindrical hole will then be $\sqrt{R^2-9}$. The altitude of the spherical caps at each end of the cylinder will be $R-3$. To determine the residue after the cylinder and caps have been removed, we add the volume of the cylinder which is $6\pi\sqrt{R^2-9}$, to twice the volume of the spherical cap, and subtract the total from the volume of the sphere, $4\pi R^3/3$. The volume of the cap is obtained by the following formula, in which a stands for its altitude and r for its radius: $\pi a(3r^2+a^2)/6$.

   When this computation is made, all terms obligingly cancel out except

36π — this is the volume of the residue in cubic centimetres. In other words, the residue is constant regardless of the hole's diameter or the size of the sphere!

3. ($5) Luke's calculator has a ten digit display but only the last four digits are working. He enters a four digit number into the calculator and squares it. To his surprise the display remains the same. What was the number?

   Solution: The answer is 9376 since $9376^2 = 87909376$ and this is the only answer. If n is a number with k digits and the last k digits of n^2 are equal to n, then we say that n is an automorphic number. The first few automorphic numbers are $1, 5, 6, 25, 76, 376, 625, 9376, 90625, \ldots$. For every positive integer k, there is at least one automorphic number with k digits. In fact, there are two "infinite" automorphic numbers:

   $$\ldots 56259918212890625 \text{ and } \ldots 740081787109376$$

   which, when we take the last k digits, will give us an automorphic number with k digits. For example, 2890625 is an automorphic number with 7 digits and we can check this by noting that $2890625^2 = 8355712890625$.

4. ($10) The words of a mathematical problem are numbered in alphabetical order. Then the first word of the problem is written in the position denoted by 1, the second word in the position denoted by 2, etc. The result is:

   "FIVE RANDOM ORDER IS EIGHT THAT NUMBERS SIX ONE SQUARE FOUR ARE THE WHAT A WRITTEN DIGIT IS RESULTING NUMBER PROBABILITY AND THREE IN DOWN THE THE"

   Solve the (mathematical) problem!

   Solution: By appropriately rearranging the words, we obtain the question:

   "THE NUMBERS ONE, THREE, FOUR, SIX AND EIGHT ARE WRITTEN DOWN IN RANDOM ORDER. WHAT IS THE PROBABILITY THAT THE RESULTING FIVE DIGIT NUMBER IS A SQUARE?"

   which satisfies the conditions of the problem. Note that there are $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ different five digit numbers whose digits are 1, 3, 4, 6, and 8. However, of these there are only five which are squares — namely 16384, 31684, 36481, 38416 and 43681. So the required probability is simply $\frac{5}{120} = \frac{1}{24}$.

5. **($10)** After a mutual and irreconcilable dispute among Red, Black and Blue, the three parties have agreed to a three-way duel. Each man is provided a pistol and an unlimited supply of ammunition. Instead of simultaneous volleys, a firing order is to be established and followed until only one survivor remains. Blue is a 100 percent marksman, never having missed a bull's-eye in his shooting career. Black is successful two out of three times on the average, and you, Red, are only a 1/3 marksman. Recognising the disparate degrees of marksmanship, everyone agrees that you will be the first, Black second, and Blue will come last in the firing order.

   Your pistol is loaded and cocked. At whom do you fire?

   Solution: At nobody. Fire your pistol in the air, and you will have the best chance of all three of the duellists. Certainly you don't want to shoot at Black. If you are unlikely enough to hit him, Blue will polish you off on the next shot. Suppose you aim at Blue and hit him. Then Black will have first shot against you and his overall probability of winning the duel will be $\frac{6}{7}$, yours $\frac{1}{7}$ — not too good. (You are invited to confirm Black's winning probability of 6/7 by summing the infinite geometric series: $\frac{2}{3} + \frac{1}{3}\frac{2}{3}\frac{2}{3} + \frac{1}{3} + \frac{2}{3}\frac{1}{3}\frac{2}{3}\frac{2}{3} + \cdots$.) But if you deliberately miss, you will have the first shot against either Black or Blue on the next round. With probability $\frac{2}{3}$, Black will hit Blue, and you will have an overall winning probability of $\frac{3}{7}$. With $\frac{1}{3}$ probability, Black will miss Blue, in which case Blue will dispose of his stronger opponent, Black, and your overall chance against Blue will be $\frac{1}{3}$.

   Thus by shooting in the air, your probability of winning the three-way duel is $\frac{25}{63}$ (about 40%). Black's probability is $\frac{8}{21}$ (about 38%), and poor Blue's is only $\frac{2}{9}$ (about 22%).

Melbourne University Mathematics and Statistics Society

Trivia Night!

To celebrate the end of the semester we are holding an absolutely

FREE Trivia Night!

That's right folks, there is NO entry fee!!

There are many great prizes to be won: the top THREE teams will all receive prizes, and in addition there will be EXTRA prizes for special events on the night!

Thursday, 31 October
4:30pm – 6:00pm
U-Bar, Union House

Entry is open to everyone, in teams of five people. If you don't have a team, don't worry! Just turn up, and we'll organise one for you.

To enter a team, please send an email with your team's details (team name, team members) to:
mums@ms.unimelb.edu.au