# Paradox

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THE MAGAZINE OF THE MELBOURNE UNIVERSITY MATHEMATICS AND STATISTICS SOCIETY

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| COVER: | The picture is wrong in a few ways. For instance, the positions of the faces on the dice are not standard. |

# Words from the Editor

Welcome to this late issue of Paradox. As usual, you can expect facetious jokes, hilarious stories and philosophical quotes here. In addition, our list of interesting sequences and problems shall keep you occupied during swot vac.

A morbid feature of this issue is a list of imaginary epitaphs for certain mathematicians, followed by a more grave compilation of actual last words, some of which are rather witty. This macabre theme accentuates the fact that there is a shortage of contributions to this magazine. If the readers do not keep new material coming, Paradox itself may meet an untimely end. To prove this mathematically, see our lighthearted article on the statistics of article-writing.

— James Wan

# Words from the President

As we draw near to the twilight hours of the calendar year, it is with the deepest pain and utmost regret that I solemnly make the following declaration:

MUMS is at war.

Even now, the enemy rallies without our gates. Their numbers are great, but finite. And one by one, we will whittle them down; together, we will annihilate the forces of *mathematical ignorance.*

Already, our elite army of Maths Olympians advance to meet the enemy's legions of foul number-letter-swapping t33n13-b0pp3rz. Our Puzzlehunt-ing masterminds are enumerating plans to counter their innumerate strategists. Ultimately, with our deftly dealt arsenal of mathematical *pun*-ishment, we will triumph against their anti-numeracy propaganda and ideology.

These are interesting times, and MUMS needs you to make a stand against the oppressive hand of mathematical ignorance. So, come to the MUMS Trivia Night/Arvo on the last Friday on this semester and find out how you can make a difference. Remember: we can all make this world a better place for the nerdy little kid with thick rimmed glasses in primary school who'll grow up to become an accountant.

— Yi Huang

# Paradoxical Survey

## Introduction

Of late, as some of our more astute readers may have noticed, our poor editor James Wan has struggled to find/coerce people to write Paradox articles for him. I, being a budding statistician in the making, hence took it upon myself to kill two birds with one stone, by coming up with a statistical model on why people are willing/unwilling to write Paradox articles, and then making an article out of it myself.

## Method

The easiest way for me to collect data was clearly to send out an email to the MUMS committee and our various other minions and make them fill out a small survey of questions, some serious, some possibly slightly less so. Then, the statistical package of choice for students (i.e. it is free), R, would be used to fit a linear model.

## Questions

The following are the questions in the survey that I sent out, and a few thoughts on each of them.

**Question 1: What is the probability that you would write an article for the next edition of Paradox?**

This would be our response variable. A response variable is the dependent variable and is modelled by a linear combination of other covariates. One particular smart-arse gave me $\log(\zeta(\cosh(\Gamma(1/\mathrm{erfc}(\pi)))))$. Its true value is something very very close to zero. Later he told me that he believes the true value lies somewhere around $e^{-e^{10^6}}$. I rounded it to 0.

**Question 2: What is your gender? 1- Male, 2- Female**

Pretty self explanatory. Personally, I did not expect any difference between genders.

**Question 3: What year at uni are you in?**

What I expected here was the level to peak at around the third year level, and then drop off as students get into post-grad years. As a result, I fitted a

quadratic term for this.

**Question 4: Your "Primary" maths major? 1- Pure, 2- Stats, 3- Applied, 4- OR, 5- Don't do maths (aka noob):**

Wasn't really sure what to expect here. I did expect it to be a fairly significant factor though. One person who responded 5 also felt the need to change the question so instead of saying "aka noob", it said "not a freak". Weirdo.

**Question 5: Are you a committee member? 1- Executive 2- Non-Exec 3- No**

You would expect that committee members are more likely to write articles. Some sort of guilt/duty thing.

**Question 6: Has James Wan threatened you with physical violence? 1- Yes, 2- No, 3- No, but I have dreamt of it**

James, being the current editor of Paradox, is not known for his fierce and intimidating nature, but you never know...[1]

**Question 7: Do you know who Hans Hahn is? 1- Yes, 2- No, 3- A German Beer maker?**

Hans Hahn is a German mathematician from the early 20th century. This question was designed to test mathematical general knowledge, or to test if you have taken the third year maths subject Linear Analysis, and studied some of his theorems. But Hans Hahn's name just resonates strangely within my heart and I simply could not resist putting this into the survey. Response 3 was put in as a bit of a joke, but quite a few people put in down...[2]

**Question 8: On a scale of 0-10, rate your guilt level if you don't write an article**

How guilty do us MUMS people actually feel? Not very.

**Question that was cut: What's the number of editions since your last contribution?**

This is a question that I would have dearly liked to put it as I think its actually one of the most important factors. However I came upon the problem of people who had never contributed. What value does one assign to that then? As

[1] A few more maths degrees and I'll be rich enough to hire hitmen. – Ed.

[2] Legends say that a certain mathematician insisted on having "Hahn-Banach beer" in a pub.

a result, in despair, I left it out.

## Results

| Prob | Gender | Year | Major | Comm. | Violence | Hahn | Guilt |
|---|---|---|---|---|---|---|---|
| 0.03 | 1 | 3 | 2 | 2 | 2 | 2 | 1 |
| 1 | 1 | 4 | 1 | 1 | 3 | 1 | 10 |
| 0.00314 | 1 | 4 | 1 | 1 | 3 | 1 | 0.00314 |
| 0.31416 | 1 | 4 | 1 | 2 | 2 | 3 | 2 |
| 0.1 | 2 | 4 | 1 | 2 | 2 | 3 | 1 |
| 0.1 | 1 | 3 | 1 | 1 | 2 | 1 | 1 |
| 0.1 | 2 | 4 | 1 | 2 | 2 | 3 | 1 |
| 0 | 1 | 7 | 1 | 1 | 1 | 1 | 0 |
| 0.1 | 1 | 1 | 3 | 2 | 2 | 2 | 0 |
| 0.2 | 1 | 6 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 6 | 1 | 2 | 2 | 2 | 7 |
| 0.2 | 2 | 6 | 4 | 2 | 2 | 2 | 3 |
| 0 | 1 | 3 | 1 | 2 | 2 | 2 | 4 |
| 0.1 | 1 | 6 | 2 | 2 | 2 | 2 | 2 |
| 0.3 | 1 | 3 | 1 | 1 | 2 | 2 | 8 |
| 0.4 | 2 | 2 | 1 | 1 | 2 | 2 | 10 |
| 0.0001 | 1 | 4 | 3 | 2 | 2 | 3 | 0 |
| 0.1 | 1 | 2 | 2 | 1 | 3 | 1 | 2 |
| 0.1 | 1 | 10 | 1 | 2 | 2 | 1 | 5 |
| 0 | 2 | 2 | 5 | 1 | 2 | 2 | 0 |
| 0.02 | 1 | 2 | 5 | 2 | 1 | 1 | 1 |

## Analysis

I started with the following full linear model:

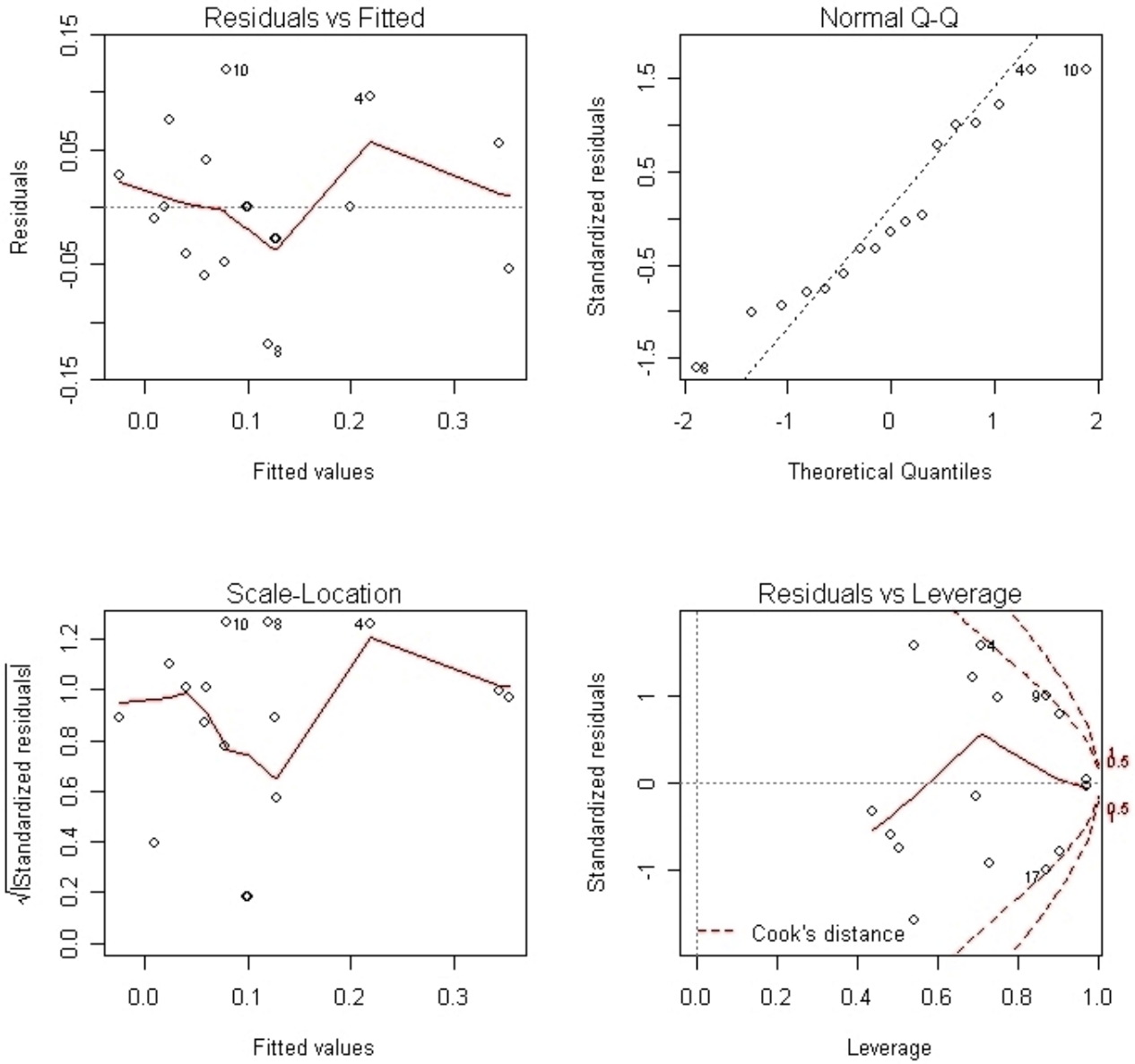
$$Prob = \mu + Gender_i + \alpha * Year + \beta * Year^2 + Major_j +$$

$$Committee_k + Violence_l + Hahn_m + \gamma * Guilt + \epsilon$$

where $\epsilon$ is a normally distributed error with mean zero and variance $\sigma^2$, and $\mu$, $\alpha$, $\beta$ and $\gamma$ are constants.

## Diagnostics

After a brief look at some of the residual plots it was clear that some of the data points were outliers and needed to be removed. The obvious choices were to remove data points 2 and 20 as they were the ones with abnormal values. After this, the underlying assumptions of the general linear model seemed to hold up reasonably well considering the lack of data.




For those without statistics majors I'll try to briefly give a bit of an explanation of the above image. The main underlying assumption that causes problems in

the general linear model is the assumption that errors are normally distributed with zero mean and constant variance. The simplest diagnostic to look at is then the first of the diagnostics. Residuals can be thought of as error terms. What we essentially want are these error terms to be evenly spread around the zero line. What we have is reasonably good. A Normal Q-Q plot is effectively a test to see how close the errors are to the normal distribution. Ideally what we want is all the dots to sit on the dotted line. Again, it is fairly close. The scale-location graph is a graph of standardised residuals. What we're looking for is for most of them to be around the 1 value, which they are. The last graph is rather complicated to explain, and personally, I don't use it much so I'm just going to ignore it.

## Variable Selection

Built into R is a lovely function called *step* which runs through your model and by adding or removing terms selects what it considers to be the best model according to a criteria called the AIC. To be honest, I don't really know how it works exactly, but it makes model selection very easy.

So our final model chosen using the step function is shown below (note: diagnostics were again run, and deemed OK).

## The Final Model

$$Prob = \mu + Gender_i + \alpha * Year + \beta * Year^2 + Major_j + Committee_k + Hahn_l + \epsilon$$

And the regression estimates are:

```
Coefficients:
             Estimate
(Intercept)  0.358322
Gender2     -0.083743
Year        -0.138118
I(Year^2)    0.014386
Major2       0.002591
Major3      -0.163281
Major4       0.255407
Major5       0.188639
Committee2  -0.308269
Hahn2        0.289085
Hahn3        0.483496
```

The Greek constants in the above formula are simply equal to the corresponding coefficient estimate. So for example, if someone is in 2nd year, you would then add $-0.138118 * 2 + 0.014386 * 2^2$ in the calculation for the probability. For the variables without subscripts these are *factor* variables. You add the corresponding value for the level of the factor if you fit into that category. For example, if you are doing major 3, then you need to add $-0.163281$. Note that the first level of all factors is set to zero by default.

One last full example in case this isn't clear. Say you are a male, 2nd year student, majoring in statistics, not on committee and you think Hans Hahn is a German beer maker. Then the estimate of your probability of writing a paradox article is:

$$\hat{p} = 0.358322 + 0 - 0.138118 * 2 + 0.014386 * 2^2 + 0.002591 - 0.308269 + 0.483496$$

which is around $0.3$.

## Conclusions

I think the main conclusion is that MUMS people are mostly smart arses, as can be seen from some of the various numbers provided in the survey. Though some of the results, i.e. being a committee member greatly increases the probability of writing an article, were clearly proven, the fact that the question on Hans Hahn was one of the most significant factors does not lead one to think that many took this survey particularly seriously. Also, the expectation calculated from the probability gives 4 or 5 articles, yet only 2 were submitted. This clearly shows that the committee members have no idea how to answer a survey. Additionally I really didn't have enough data to do a proper study, but still, it does give some pointers to James with regards to which people he should harass/coerce into writing articles for next time.[3]

— Han Liang Gan

Puzzle:
You missed a maths lecture yesterday and you want to know if there's a test today. There are two students outside the lecture theatre; one always lies and one always tells the truth, but you don't know which is which. What single yes-or-no question can you ask one of them so that you'll know if there's a test?

[3] Yeah right. – Ed.

# Fun and Frustration with Sequences

The following are some sequences (mathematical or otherwise) arranged roughly in order of difficulty. The aim is to find the next term in the sequence. The most correct answers and some explanations are given on the next page.

- 2, 6, 12, 20, 30, ?
- 1, 3, 6, 10, 15, 21, ?
- 1, 1, 2, 3, 5, 8, 13, 21, ?
- 0, 1, 2, 7, 20, 61, ?
- 6, 1, 10, 2, 14, 6, 18, 24, ?
- 1, 4, 9, 18, 35, ?
- 1, 4, 9, 61, 52, ?
- R, T, Y, U, I, ?
- 1, 5, 7, 13, 25, 45, 83, 153, ?
- O, T, T, F, F, S, S, E, ?
- 1, 1, 2, 5, 14, ?
- J, Y, F, Y, M, H, A, L, M, ?
- 1, 11, 21, 1211, 111221, ?
- 2, 3, 3, 5, 10, 13, 39, 43, ?
- 2, 4, 6, 30, 32, 34, 36, 40, 42, 44, 46, ?
- 1, 3, 7, 12, 18, 26, 35, ?
- 4, 2, 5, 2, 6, 10, 3, ?
- 8, 5, 6, 3, 4, 1, 4, ?

— Sam Chow

# Solutions:

- 42. Add 4, 6, 8, etc to get to the next term.
- 28. These are the triangular numbers, that is, the sum of the first $n$ whole numbers.
- 34. The ubiquitous Fibonacci sequence.
- 182. Obtained by $2\times$ last term $+ 3\times$ second last term.
- 22. It's just two sequences interspersed (6, 10, 14, 18, ...) and (1, 2, 6, 24, ...).
- 68. Given by $2^n + n - 2$.
- 63. The squares of integers, written backwards.
- O. Consecutive letters on a standard keyboard.
- 281. Add the last 3 terms to get the new term.
- N. The first letter of the words One, Two, Three, ...
- 42. The Catalan numbers. To find out more about these amazing numbers, see the 3rd edition of Paradox in 2007 (available online).
- Y. The first and last letter of the months: JanuarY, FebruarY, ...
- 312211. The Look and Say Sequence. Beginning with 1, we simply read out the last term to get to the next term. So the second term is 11 as there is 1 1. The third term is 21 as there are 2 1s. The fourth term is 1211, and so on.

  It is interesting to see that this sequence can only involve the digits 1, 2 and 3. The number of digits in each term of the sequence (1, 2, 2, 4, 6, 6, 8, 10, 14, ...) is asymptotic to a constant times $\lambda^n$, where $\lambda$ is Conway's constant. Conway proved that $\lambda$ is the unique positive real root of an integer polynomial of degree 71.
- 172. Add 1, times by 1, add 2, times by 2, ...
- 50. The Eban sequence, which contains only the numbers that do not have 'e' in their spelling.

- 45. One of Hofstadter's sequences, the sequence and the difference between adjacent terms contain all whole numbers.
- 7. The decimal expansion of $\pi$, $3.14159265\ldots$, with 1 added to each decimal.
- 5. This sequence describes the notes of Pachelbel's Canon in D. The notes are D' (higher), A, B, F#, G, D, G, and then A.

# Quotes

"Lisez Euler, lisez Euler, c'est notre maitre a tous." – (Read Euler, read Euler, he is the master of us all) attributed to Laplace

"It is easier to square the circle than to get round a mathematician." – de Morgan

"First that a negative quantity has no logarithm; secondly that a negative quantity has no square root; thirdly that the first non-existent is to the second as the circumference of a circle is to the diameter." – de Morgan

"At every stage in business, a disaster can occur. These kinds of considerations can make business a little less satisfying than proving a good theorem. For instance, once Pythagoras had his theorem down, he didn't have to worry about people finding a better one, or producing a cheaper one, or some kid swallowing a triangle and gagging on the hypotenuse." – Sam Savage

"A theory has only the alternative of being wrong. A model has a third possibility – it might be right but irrelevant." – Manfred Eigen (not related to eigenvalues)

"If a religion is defined to be a system of ideas that contains unprovable statements, then Gödel taught us that mathematics is not only a religion, it is the only religion that can prove itself to be one." – John Barrow

"Many very smart people are lousy at mathematics and never quite get over their failure at something so important." – R. Schank

"It is a safe rule to apply that, when a mathematical or philosophical author writes with a misty profundity, he is talking nonsense." – Alfred North Whitehead

"Experimentalists think that it is a mathematical theorem while the mathematicians believe it to be an experimental fact." – Poincaré, on the Gaussian curve

"Never express yourself more clearly than you think." – Niels Bohr

# Maths Jokes

A certain pure mathematician had a wife who, while intelligent, was not into mathematics. However, by continued practice, she learnt to distinguish between the conversations of algebraists and analysts. So when he had guests to dinner who were talking about mathematics, if they were analysts, she would introduce at a suitable pause in the conversation: "But what happens at the boundary?" Whereas, if they were algebraists, she would say: "But do the roots lie in the field?" By this means she was always able to impress his visitors by her knowledge of mathematics.

$\infty$

Admin to maths department: We've cut off funding for your string theory.

$\infty$

Q: Does maths need new axioms?

A: What do you mean by "does", "maths", "need", and "axioms"?

$\infty$

In theory, there is no difference between theory and practice. In practice, there is no relationship between theory and practice.

$\infty$

Philosophy is a game with objectives and no rules. Mathematics is a game with rules and no objectives.

$\infty$

Q: How many mathematicians does it take to replace a light bulb:

A: $0.\dot{9}$.

Q: How many lecturers does it take to replace a light bulb?

A: The answer is intuitively obvious.

Q: How many numerical analysts does it take to replace a light bulb?

A: 3.9967 (after six iterations).

Q: How many topologists does it take to replace a light bulb?

A: It really doesn't matter, since they'd rather knot.

Q: How many arts majors does it take to replace a light bulb?

A: No number of arts majors can replace the usefulness of a light bulb.

Q: How many analysts does it take to replace a light bulb?

A: Three: one to prove existence, one to prove uniqueness and one to derive a non-constructive algorithm to do it.

Q: How many professors does it take to replace a light bulb?

A: One, with two co-authors, eight research students, two programmers, three post-docs and a secretary to help him.

Q: How many graduate students does it take to replace a light bulb?

A: One, but it takes nine years.

Q: How many administrators does it take to replace a light bulb?

A: None: what was wrong with the old one?

Q: How many logicians does it take to replace a light bulb?

A: None. They can't do it, but they can easily prove that it can be done.

Q: How many classical geometers does it take to replace a light bulb?

A: None. You can't do it with a straight edge and a compass.

Q: How many Bourbakists does it take to replace a light bulb?

A: Replacing a light bulb is a special case of a more general theorem concerning the maintenance and repair of an electrical system. To establish upper

and lower bounds for the number of personnel required, we must determine whether the sufficient conditions of Lemma 2.1 (Availability of personnel) and those of Corollary 2.3.55 (Motivation of personnel) apply.

If and only if these conditions are met, we derive the result by an application of the theorems in Section 3.1123. The resulting upper bound is, of course, a result in an abstract measure space, in the weak-* topology.

$\infty$

Did you hear about the statistician who was thrown in jail? He now has zero degrees of freedom.

$\infty$

Old mathematicians never die – they just decay.

Old mathematicians never die – they just become irrational.

$\infty$

Q: How does a mathematician put an elephant into a refrigerator?

Complex analysis: put the refrigerator at the origin and the elephant outside the unit circle, and perform inversion.

Analysis: apply the Banach-Tarski Paradox to the refrigerator.

Set theory: $\{\text{refrigerator}\} = \{\text{elephant}\}$.

Algebra: show that the parts of the elephant can be put into the refrigerator, and show that the refrigerator is closed under addition.

Topology: the elephant is homeomorphic to a smaller elephant.

$\infty$

Definition:

Mathenary (*n*): someone who solves Paradox problems for money.

$\infty$

There's a big calculus party, and all the functions are invited. $\ln(x)$ is talking to some trig functions, when he sees his friend $e^x$ sulking in a corner.

$\ln(x)$: "What's wrong?"

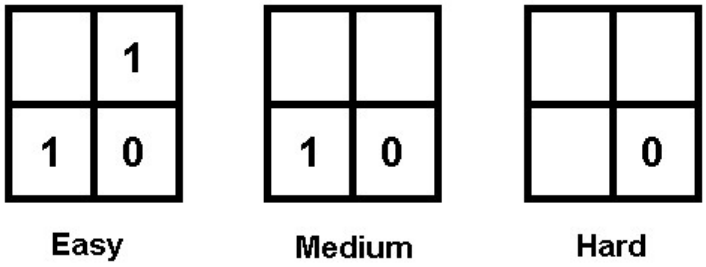
$e^x$: "I'm so lonely!"

$\ln(x)$: "Well, you should go integrate yourself into the crowd!"

$e^x$ looks up and cries, "It won't make a difference!"

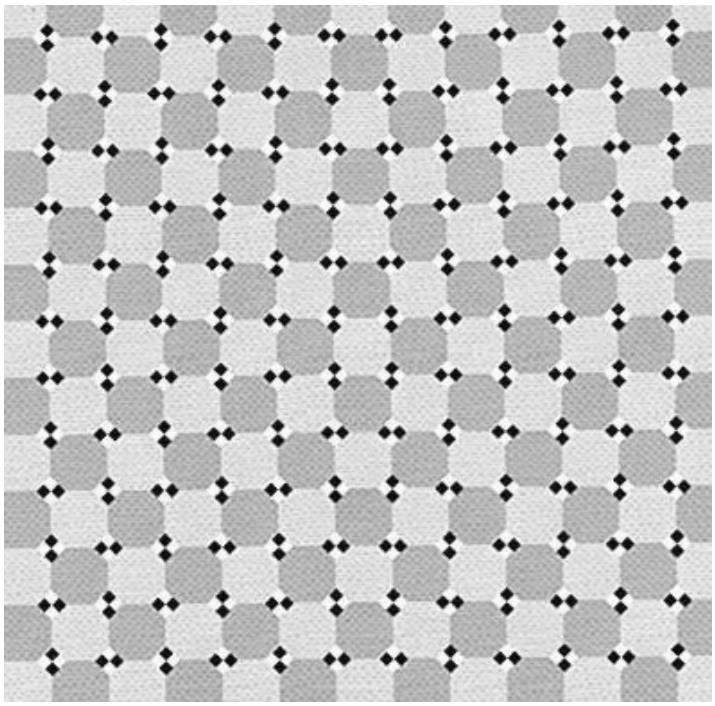
$\infty$

**Binary Sudoku**




Special thanks to xkcd.com for this idea. The easy one has a rating of 101.

$\infty$



# Mixed Histories

Here is a series of (fictitious) epitaphs/epigrams for mathematicians of disparate fame, with a little indulgence in humour in mathematics. Some of these rather appropriately (or inappropriately) relate to the mathematician's area of research.

- Argand, Jean Robert (1768 – 1822) (quintuple-pun epitaph):
  It is plane to imagine i cannot really be at this point.
- Hiern, William Philip (botanist and mathematician):
  Here floats Hiern, having taken the last of his leaves.
- Sylvester, James Joseph (1814 – 1897):
  Unfortunately my dialytic method was not the precursor to renal dialysis, I 'kid' you not!
- von Lindemann, Carl Louis Ferdinand (1852 – 1939):
  I have transcended to a 'Pi in the Sky'.
- Ampère, André-Marie (1775 – 1836):
  Don't be shocked, I am currently lying here.
- Bernoulli, Johann (1667 – 1748):
  This is probably my steepest descent since the branchistochrone.
- Bertrand, Joseph Louis (1822 – 1900): [4]
  I am now in the prime of my after-life.
- Hermite, Charles (1822 – 1901):
  Nothing askew here, except this hermit, so I determined.
- Legendre, Adrien-Marie (1752 – 1833):
  Unarguably this state is a necessary condition.
- Markov, Andrey (1856 – 1922):
  The Markov chain was pulled for the last time.

[4] Bertrand's postulate: for every integer $n > 1$ there is a prime between $n$ and $2n$.

- Poisson, Siméon-Denis (1781 – 1840):
  A cognitive dissonance; my ashes are in a non-Poisson distribution.
- Riccati, Jacopo Francesco (1676 – 1754):
  Riccati = 0, my generalised state equation.
- Salmon, George (1819 – 1904):
  Now I have ascended to the highest plane.
- Thomson, William (Lord Kelvin) (1824 – 1907):
  I wonder, no longer, how many degrees it is on the other side.

— David Halprin

# Morbid Histories

The following are truthful last words.

Archimedes, while doing geometry, to the invading soldier who killed him, "Stand away, fellow, from my diagram!" Archimedes also requested to have a sphere inscribed in a cylinder to be placed on his tomb, for working out their surface areas and volumes were his fondest discoveries.

Newton, "I don't know what I may seem to the world. But as to myself, I seem to have been only a boy playing on the seashore and diverting myself in now and then finding a smoother pebble or prettier shell than the ordinary, whilst the great ocean of truth lay all undiscovered before me."

Galois, "Don't cry, I need all my courage to die at 20."

Descartes, "My soul, thou has long been held captive. The hour has now come for thee to quit thy prison, to leave the trammels of this body. Then to this separation with joy and courage!"

Cauchy: "Jesus, Mary, and Joseph."

Laplace: "What we know is trifling, what we know not is immense." Another version is: "man follows only phantoms."

Euler: "I die."

Feynman, " I'd hate to die twice, it's so boring."

Thomas de Lagny, when asked what the square of 12 was (in an attempt to make him relaxed), "144."

Hilbert, on his tomb: We must know. We will know.

Ludolph van Ceulen computed the first 35 decimals of $\pi$. He was so proud of this that he had them inscribed on his tombstone.

Simon Stevin, who ingeniously proved the law of the equilibrium on an inclined plane, had the diagram used in the proof inscribed on his tombstone. A nice account of the simple physics involved can be found at

www.lhup.edu/˜dsimanek/museum/unwork.htm#stevinprob

— James Wan

## True Stories

G. H. Hardy gave Ramanujan a rating of 100 on his own scale of "pure-talent". Hardy's own rating was 25. Littlewood received 30 and Hilbert 80.

$\infty$

Every day Poincaré bought bread from his local baker. Though the bread was supposed to weigh 1 kg, Poicaré found that months of record keeping produced a nice normal distribution with a mean of only 950 g. Poincaré called the police and the baker was promptly told to behave himself.

A year later Poincaré again visited the police: though his own loaves had indeed grown larger, the baker, he declared, was still short-shifting his other customers. The police again confronted the baker. How, the man asked, could Poincaré possibly know that he was always given the largest loaf?

Poincaré then showed the police his record for that year: a bell curve truncated on the left side.

$\infty$

Bertrand Russell once claimed that, given $1 + 1 = 1$, he could prove any other statement. A challenger asked, "Prove you are the Pope." Russell thought for

a while and then proclaimed, "I am one. The pope is one. Therefore the Pope and I are one."

$\infty$

Laplace presented a copy of his great work, Mécanique Céleste, to Napoleon, who studied it assiduously. He then sent for Laplace and said to him: "You have written a large book about the universe without once mentioning the author of the universe."

Laplace replied: "Sire, I had no need of that hypothesis."

Lagrange later observed, "It is a beautiful hypothesis just the same. It explains so many things."

$\infty$

F. N. Cole proved in 1903, after spending every Sunday for 3 years, that $2^{67} - 1 = 193707721 \times 761838257287$. He presented this at an American Mathematical Society meeting in a talk aptly named "On the factorisation of large numbers." Without uttering a word, he calculated by hand both sides of the equal sign and was greeted with applause.

$\infty$

In Toulouse, France there is a large statue of Fermat. A sign reads "The Father of Differential Calculus". Seated in front of Fermat's is a partially nude Muse, showing her ample appreciation of Fermat's mental faculties.

$\infty$

The mathematician Christos Papakyriakopoulos proved fundamental results for 3-manifolds while working almost in isolation. Apparently, he lived for 25 years in the same hotel room, with all of his belongings inside his original luggage.

$\infty$

The younger brother of Niels Bohr, Harald Bohr, was a mathematician and also a soccer player. He won a silver medal at the 1908 Olympics. When his doctoral dissertation was examined, there were more soccer fans wishing to attend this than there were mathematicians.

Bohr collaborated with Hardy, and always put 'Proving the Riemann hypothesis' as the first item on their agenda. Once, before a dangerous journey to Denmark, Hardy sent to postcard to Bohr claiming that he had succeeded in doing so, for he was confident that God would not grant him the honour of dying and leaving people with the idea that he had taken the proof to his grave.

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Paul Cohen (Fields Medallist, 1966) once fell asleep in his office, slept through the class he was supposed to teach, woke up, complained to his colleague about how lazy the students are, and went back to sleep.

$\infty$

André Weil was once asked to write a letter of recommendation. He wrote: "This man is better than anyone you now have, anyone you ever have had, or anyone you ever will have." The man got the job.

# Solutions to Problems from Last Edition

We had a number of correct solutions to the problems from last issue. Below are the prize winners. The prize money may be collected from the MUMS room (G24) in the Richard Berry Building.

Stephen McAteer and Adrian Khoo may each collect $2 for solving problem 1.

Nicolas Warren, Richard Neilsen, Kate Mulcahy and Damian McLeod may each collect $2 for solving problem 3.

Josh Howie may collect $4 for solving problems 1 and 3.

Tharatorn Supasiti may collect $4 for solving problem 5.

Navin Ranasinghe and Will Mischlewski may each collect $7 for solving problem 8.

Christopher Chen may collect $9 for solving problem 3 and 8.

Jensen Lai may collect $11 for solving problems 1, 3 and 8.

1. Find a quintic polynomial $P(x)$ if $x^3|P(x)$ and $(x-1)^3|P(x)-1$.

Solution: let $P(x) = x^3(ax^2 + bx + c)$. Note that $x^3|P(x+1) - 1$, so we expand out $P(x+1) - 1$ and solve. The answer is $6x^5 - 15x^4 + 10x^3$.

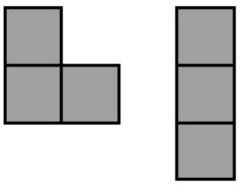
A note regarding the solution of the problem, from our reader Stephen McAteer, reads: 'Elegance is overrated... Here's Brute-Force.'

2. Show that $n^4 + 4^n$ is a prime if and only if $n = 1$.

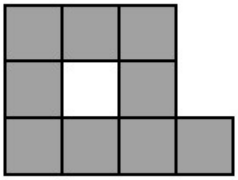
   Solution: clearly when $n = 1$ we have a prime.

   Now suppose $n^4 + 4^n$ is a prime and $n > 1$. If $n$ is even then the expression is also even, so $n$ is odd, and let $n = 2m + 1$. Then the expression becomes $(2m+1)^4 + 4 \cdot (2^m)^4$. Using the identity $x^4 + 4y^4 = (x^2 - 2xy + 2y^2)(x^2 + 2xy + 2y^2)$, we can factorise our expression. As each factor is bigger than 1, the expression is composite.

3. What shape can be made with either 2 of the first figure and 1 of the second figure, or 1 of the first figure and 2 of the second figure?



   Solution:



4. Show that for an integer $n$, $n + 3$ and $n^2 + 3$ can't both be perfect cubes.

   Solution: suppose they are both cubes, then their product, $n^3+3n^2+3n+9$ is also a cube, and differs from $(n+1)^3$ by 8. There are only 2 pairs of cubes that differ by 8 (0 and 8, $-8$ and 0), and it is quickly checked that $n$ cannot exist.

5. Given 3 parallel lines, construct (using ruler and compass) an equilateral triangle such that each vertex lies on a line.

   Solution: imagine that we already have such a triangle constructed. Using its centroid as the centre, rotate the entire figure by 120 degrees. Now

3 of the points of intersection between the 3 original lines and their images are the vertices of the triangle.

As rotations about different centres differ by a translation, we can simply rotate the initial configuration about any point by 120 degrees, and 3 of the intersection points must give an equilateral triangle.

6. Find $\sum_{n=0}^{\infty} \frac{1}{2^n} \tan \frac{x}{2^n}$.

   Solution: note the identity $\tan x = \cot x - 2 \cot 2x$, so the sum telescopes.

   After cancelling most terms, we get

$$\sum = -2 \cot 2x + \lim_{n \to \infty} \cot(x/2^n)/2^n.$$

   Taking the limit, we get $1/x - 2 \cot 2x$ as the answer.

7. For positive integers $m$ and $n$, if $\frac{m}{n} < \sqrt{2}$, then $\frac{m}{n} < \sqrt{2}(1 - \frac{1}{4n^2})$.

   Solution: as $m^2 < 2n^2$, we then have $m^2 \leq 2n^2 - 1$. We complete the square on the RHS to obtain

$$m^2 \leq 2(n - \frac{1}{4n})^2 - \frac{1}{8n^2}.$$

   Hence $m^2 < 2(n - \frac{1}{4n})^2$. Taking the square root and simplifying produce the desired form. Note that this trick generalises to any $\sqrt{k}$.

8. Find all functions $f : \mathbf{Z}^+ \to \mathbf{Z}^+$ such that $f(m + f(n)) = f(m) + n$ for all $m, n$.

   Solution: (from Navin Ranasinghe) let $a = b + f(c)$, then $f(m + f(a)) = f(m) + a$, so:

$$f(m + f(b + f(c))) = f(m + f(b) + c) = f(m + c) + b = f(m) + b + f(c).$$

   So $f(m + c) = f(m) + f(c)$ for all $m$ and $c$. This is the famous Cauchy's equation and it has the unique solution of $f(n) = n$ in the natural numbers.

   To see the last assertion, we use induction to show that $f(n) = nf(1)$. Using this, the original equation reduces to $mf(1) + nf(1)^2 = mf(1) + n$, so $f(1) = 1, f(n) = n$.

Solution to puzzle: "If I were to ask you if there was a test today, what would you say?"

# Paradox **Problems**

Below are some problems for which cash prizes are awarded. Bear in mind that anyone who submits a clear and elegant solution may claim the indicated amount (unless two solutions are the same, in which case only the first submission will be rewarded). Either email the solution to the editor (see inside front cover for address) or drop a hard copy into the MUMS room (G24) in the Richard Berry Building; please include your name.

1. (\$2) Find the sum of squares of all entries in the $n$th row of Pascal's triangle. What about the sum of the product of all adjacent pairs of entries?

2. (\$4) Using a standard deck,

   (a) (infamous SMO problem) find the probability of getting at least four cards of the same suit in a hand of 13 cards,

   (b) find the probability of getting at least five cards of the same suit (i.e. a flush) in a hand of 13 cards.

3. (\$3) In an equilateral triangle $ABC$, $P$ is on $AB$ so that $AP = AB/3$, $Q$ is on $BC$ so that $BQ = BC/3$, and $R$ is on $CA$ so that $CR = CA/3$. The lines $CP, AQ, BR$ enclose a triangle. Find the ratio of the area of this triangle to that of $ABC$.

4. (\$5) Find $\int_0^1 \frac{\log x}{x^2-1}\mathrm{d}x$.

5. (\$4) In a triangle $ABC$, let $a, b, c$ be the sides opposite angles $A, B, C$ respectively. Show that

$$\frac{\cos(B/2)\sin(B/2+C)}{\cos(C/2)\sin(C/2+B)} = \frac{a+c}{a+b}.$$

Paradox would like to thank Sam Chow, David Halprin, Kate Mulcahy, Han Liang Gan, Yi Huang, Rick Tankard and Adib Surani for their contributions to this issue.