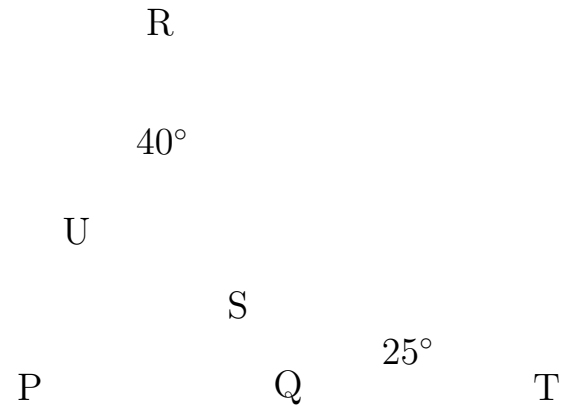
2001 MUMS Schools Maths Olympics Questions

Question 1. What is the sum of the first five square numbers ending in a 1?

Question 2. In the diagram, $PR = QR$, $\angle PRQ = 40°$, and $\angle PTU = 25°$. What is $\angle RST$, in degrees?



Question 3. Simplify

$$\frac{16\sqrt{7}}{\sqrt{7} + \frac{1}{\sqrt{7}}}$$

Question 4. Mary has sat for 10 tests this term and has an average score of 68. What mark must she gain in the next one to raise her average to 70?

Question 5. The digits 3, 1, 4 and 1 can be arranged to form how many different 4-digit numbers?

Question 6. What is the surface area, in square centimetres, of a cube having volume 343 cm^3?

Question 7. What is the area of the smallest circle in which you can fit six equilateral triangles of area 3 without overlap? (Express your answer *exactly*)

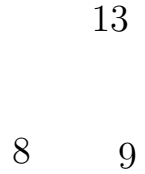
Question 8. Luke is a fugitive from justice. He steals a car in Melbourne at 8:00 am and aims to drive it to his freedom. Unfortunately the Magna he stole is a real bomb and he can only travel at a constant speed of 80 km/h. The police are notified of the theft and commence a chase at 8:30 am from the location of the theft. They follow the trail of oil which has been dripping out of the Magna's engine and so follow Luke's route exactly, at a constant speed of 100 km/h. At what time will Luke be arrested? Remember to specify in your answer whether the time is am or pm.

Question 9. Norm and Geordie each roll a die. What is the probability that the product of the two numbers rolled is less than 6? (Express your answer as a fraction in simplest form.)

Question 10. Three jolly professors, Tim, Swarup and Kris, are gambling by a billabong. They start with sums of money in the ratio $7 : 6 : 5$ and finish with sums of money in the ratio $6 : 5 : 4$, in the same order of wealth. One of the professors won \$12. How many dollars did he start with?

Question 11. Many years ago Melbourne University had under 5000 students enrolled. A third of the students were in first year, two-sevenths were in second year, one-fifth were in third year and the rest were postgraduate students. The mathematics department offered a popular course in which were registered a fortieth of all the first-year students, a sixteenth of all the second-year students, and a ninth of all the third-year students, while the remaining third of the maths class were all postgraduates. How many students were there in the maths class?

Question 12. In the diagram, the lengths of the sides of the triangle are 8, 9, 13 centimetres . The centres of the circles are at the vertices of the triangle, and the circles just touch. What are the radii of the three circles, in increasing order?



Question 13. Lines from the vertices of a square to the midoints of the sides are drawn, as shown. If the area of the large square is 1, what is the area of the smaller square in the middle?

Question 14. The diagram shows a 5 by 5 table. The top row contains the symbols A,S,I,D,E. The fourth row contains the symbols A,S,I at the centre. The remaining squares can be filled with A's, S's, I's, D's and E's such that no row, column or diagonal contains

the same symbol more than once. What are the symbols in the bottom row, from left to right?

A S I D E

A S I

Question 15. Let $f(n)$ be the number of letters used when writing out the digits of the base ten representation of n. For example, $f(27) = 8$ since "two seven" has eight letters. What is the value of $f(f(f(\cdots f(2^{16}))))$ where f is applied 10 times?

Question 16. How many points with positive integer coordinates are there strictly inside the area bounded by the lines $x = 0$, $y = 0$ and the graph $y = 10/x$?

Question 17. Solve the following equation for the integer n:

$$\sqrt[3]{n+\sqrt{n^2+8}}+\sqrt[3]{n-\sqrt{n^2+8}}=8$$

Question 18. On the planet Dankuhn lives a being which can be one of three sex types: male, female and emale. Any two different sexes may breed and the offspring from such a union is of the third sex. How many of an emale's great great great great grandparents were emales?

Question 19. Find the number of three digit numbers whose digit sum is ten.

Question 20. "Baker's Dozen" doughnuts are sold only in boxes of 7, 13 or 25. To buy 14 doughnuts you must order two boxes of 7, but you cannot buy exactly 15 since no combination of boxes contains 15 doughnuts. What is the largest number of doughnuts that cannot be ordered using combinations of these boxes?