1. Cinderella is given one 3 Litre and one 5 Litre bucket by her stepmother, who tells her to bring exactly 4 Litres of water back into the house in one trip. With no other available measuring tools given, what is the minimum number of times that she must fill the 3 Litre bucket up to the brim?

2. Thara and Yi went down to the local oval to do some running. They both started at the same point on the 400m oval, but ran in opposite directions. Thara is a great runner and can easily manage 9m/s. On the other hand, Yi wears trousers and thus can only manage 7m/s. If they both ran until they met up again, how much distance did Thara cover?

3. Yi, Alisa and Han all share the same birthday. Alisa is one year older than Yi and Han is two years older than Alisa. This year, the sum of their ages is 118. How old is Alisa?

4. 'At least one of us is lying,' said Andrew.

   'Only one of us is lying,' said Bertas.

   'Squeak (two of us are lying),' said the Chipmunk.

   'Either three or four of us are lying.' said Daisy.

   'Elmo thinks that everybody's lying!' said Elmo.

   How many liars are there altogether?

5. Newspapers are created by laying sheets of paper on top of each other, and then folding down the center line, creating a 'book'. A 100-sheet newspaper has its pages numbered from 1 to 400. A sheet falls out of the newspaper and you notice that one of the pages is numbered 67. What is the sum of the numbers of the other 3 pages on the same sheet?

6. A 4L cylindrical tank has a hole drilled half way up its side, thus allowing water to leak out at 2L/min. We can fill up the tank in $2\sqrt{3}$ minutes if we pump water into the empty tank at a steady rate of v L/min, what is the value of v?

7. Han has made \$100 by selling mini-Mars bars. His coin jar holds exactly 100 coins, and contains only 50c, \$1, and \$2 coins. If we assume that he has at least 1 coin of each type, how many combinations of the one hundred coins are possible?

8. Given that for any x,

$$ax^3 + bx^2 + cx + d = (x^2 + x - 2)(x + 4) - (x - 2)(x^2 - 5x + 4)$$

   What is the value of $a + b + c + d$?

9. A rectangular sheet of paper, $ABCD$, has sides $AD = 1$ and $AB = \frac{5}{4}$. The paper is folded along a line through A so that the side AD falls onto the side AB. Without unfolding, the paper is folded again along a line through B so that the side CB also lies on AB. A region of the resulting triangle is four sheets thick. What is the area of this region?

10. In an arithmetic sequence $a_1, a_2, a_3, \ldots, a_{46}, a_{47}$, the sum of the odd numbered terms is 1272. What is the sum of all 47 terms in the sequence?

11. Stephen erased two of the digits in the following 9 digit number.

$$347\_47\_64$$

What is the probability that the original number was divisible by 36?

12. What is the biggest area that two 5 metre sticks, and one 8 metre stick can be used to enclose?

13. Consider a regular hexagon H. How many distinct equilateral triangles exist such that at least two of the vertices of such a triangle are also vertices of H?

14. What is the smallest positive integer that has twenty positive factors, including 1 and itself?

15. A Sydney-sider comes down to Melbourne and checks out a game of Australian Rules footy. He doesn't understand the scoring system (6 points for a goal, and 1 for a behind) and thinks that the game is scored by multiplying the number of goals by the number of behinds. Using his erroneous scoring system, how many combinations of goals and behinds are there that allow him to calculate the correct final score?

16. A quadrilateral, $DEFG$, circumscribes a circle of radius 2. DE is parallel to GF and $\angle DEF = \angle EFG = 90°$. If $DE = 3$, what is the area of the quadrilateral?

17. Speed-skating champion Daniel Yeow draws m infinitely long, straight lines on the MUMS whiteboard. He notices that there are 2006 intersections, and that no three lines meet at one point. If the first k lines drawn are all parallel to the first one drawn (with $k < 10$) and none of the rest are parallel to any of the others, what are the values of m and k?

18. A function f is defined over the natural numbers, such that $f(n + 1) = n - f(n)$. Evaluate $f(2006) + f(1)$.

19. A certain base-8 three digit number has its digits reversed when multiplied by 2. What is it in base-10?

20. What is the value of
$$\frac{1}{2^0} + \frac{2}{2^1} + \frac{3}{2^2} + \ldots + \frac{n}{2^{n-1}} + \ldots$$

21. The average class sizes at school X and school Y are 32 and 18 students respectively. Next year, the number of students at each school is expected to increase twice as much for school X compared to school Y. This will mean that the average class size at X will be double that of Y. If the number of classes at each school (both prime numbers greater than 12) will stay the same, what is the difference between the number of classes at each school?

22. An equilateral triangle of side length 2006 is divided up into small equilateral triangles each of side length 1. The large triangle is positioned so that one of the three sides lies at the bottom. What is the number of paths from the small triangle in the top row to the bottom right triangle so that adjacent triangles in the path share a common edge and the path never travels from a lower row to a higher row or revisits a triangle? (A sample path is shown in the diagram below.)
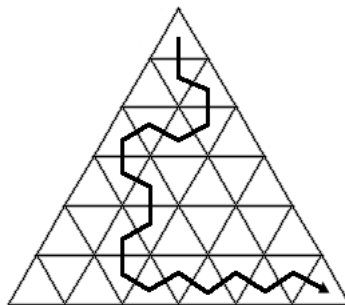
23. A certain biased coin is known to turn up tails twices as often as heads. On average, how many times do you need to flip this coin to get two successive tails?

24. Joshua lives in an apartment with three friends: Eric, Raymond and Ryan. The apartment has nine square rooms arranged in a 3 by 3 grid, with doors between adjacent rooms. Each of the housemates live in one of the corner rooms and Joshua lives in the room diagonally opposite to Eric's.

    Whilst on a study break, Joshua decides to see what his housemates are up to. He walks from one room to another until he goes into one of his housemate's rooms. Each time he decides to leave a room, he will choose any of the doors (including the one that he came through) with equal likelihood. What is the chance that he will visit Eric's room before the others?

25. Let n be a positive integer. There is at least one set of positive integers $\{a_0, a_1, \ldots, a_n\}$ such that
$$\frac{99}{100} = \frac{a_0}{a_1} + \frac{a_1}{a_2} + \ldots + \frac{a_{n-1}}{a_n}$$
where $a_0 = 1$ and $(a_{k+1} - 1) a_{k-1} \geq a_k^2 (a_k - 1)$ for $k = 1, 2, \ldots, n - 1$.

For all such sets $\{a_0, a_1, \ldots, a_n\}$, what is the largest possible value of a_n?