SMO 2007 Questions

Question 1

She sells seashells by the sea-shore,
A dollar a shell, not a cent more,
I purchase some seashells for my amore,
She cries, 'dear Sir, please buy twelve more,
I'll halve the whole price: only three dollars more!'
At thought, I acquiesced to what she implored.
How many shells bought I, for the one I adore?

Question 2

Each year MUMS holds a one day cooking extravaganza called the 'Cooking Frenzy of Win'. One of the items made in this year's cooking frenzy was a cylindrical pineapple pie of radius and height π. What is the volume of this pi-ly specified pie?

Question 3

7 consecutive positive integers sum to 91. What is the lowest of these?

Question 4

Out of the 365 days this year, what is the number of days with odd dates (the day of the month is odd) minus the number of days with even dates (the day of the month is even)?

Question 5

Han is sitting in a room with Joanna, Maurice and Ray. Han initially has a box of five chocolates. He takes a nap, and when he awakes he finds they have all been eaten.
Joanna says: Maurice ate an even number of chocolates, and Ray ate an odd number of chocolates.
Maurice says: Joanna ate an even number of chocolates, and Ray ate a square number of chocolates.
Ray says: Joanna ate a square number of chocolates, and Maurice ate a prime number of chocolates.
Han knows that two of them always tell the truth while one of them always lies. Who ate the most chocolates, and how many?

Question 6

4% tax is charged on an item, and the tax works out to be a whole number of cents. When the tax is added to the item, the total cost is a whole number of dollars. What is the cheapest the item can be, excluding tax?

Question 7

What is the lowest attainable value of the sequence of digits 32223, in that order, with one $\wedge$ (power of) and one $\times$ (multiplication) sign in between? (e.g. $32\times2\wedge23 = 32 \times (2^{23})$)

Question 8

Someone says to you 'Let's play a game. You tell me an integer. If I can find a real number so that the product of our numbers is equal to your number minus twice mine, then I win.'

After some thought, you find a number which defeats him. What is it?

Question 9

You are dealt 13 cards from a standard 52-card deck. What is the probability that you have at least four of the same suit?

Question 10

A 12×16 piece of paper is folded along the diagonal, and then flattened. What is the perimeter of the resulting shape?

Question 11

Sam has a shed containing 60 planks of wood, 40 pots and 50 theorems.
A woodchuck can chuck 15 planks of wood, push 2 pots and prove 1 theorem.
A pushpot can chuck 6 planks of wood, push 8 pots and prove 2 theorems.
A mathematician can chuck 2 planks of wood, push 2 pots and prove 7 theorems.
What is the smallest combined number of woodchucks, pushpots and mathematicians Sam needs to chuck all his wood, push all his pots and prove all his theorems?

Question 12

In a parking lot, 20% of the cars are red, 30% of the red cars are convertibles, and 25% of the convertibles are red. What percentage of cars in the parking lot are not red and not convertibles?

Question 13

In how many ways can 3 dice be rolled, one after the other, so that their topmost faces sum to 11?

Question 14

What number comes next in the series:

$$1.5, \quad 0.5, \quad 3.5, \quad 10.5, \quad 7.5, \quad 2.5, \quad 5.5, \quad 16.5, \quad 13.5$$

Question 15

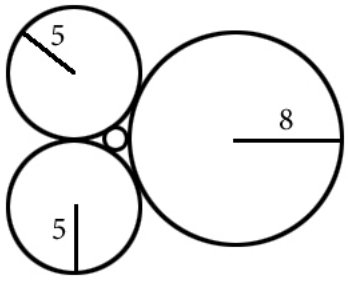
Twenty-seven $1\times1\times1$ cubes are put together to form a $3\times3\times3$ cube. If you remove some of the smaller cubes whilst leaving others intact (possibly suspended in mid-air), what is the maximum possible surface area of what is left?

Question 16

Given that the reciprocals of four distinct positive integers sums to 1, what is the largest value that any of these integers can take?

Question 17

Four circles are mutually externally tangent. If three of them have radii 5, 5 and 8, what is the radius of the fourth circle?



Question 18

The numbers $1, 2, \ldots, 100$ are written on a whiteboard. Kim picks two of the numbers, x and y say, erases them, and writes down the number $xy - x - y + 2$. He does this until there is only one number remaining on the whiteboard. What is this number?

Question 19

The following is a magic square. It has the property that the sum of each column, row and the two main diagonals are the same.

	19	96
1		

What is this sum?

Question 20

Kwok and James are playing rock, paper, scissors. They play games where the first to three points wins (one point for a win, none for a loss or draw). If they are always evenly matched, what is the average winning margin?

Question 21

You need to get from West Footscray to Camberwell by train which requires you to change trains at Flinders Street Station. Your train into the city is scheduled to arrive at Flinders Street at 9am and the next train to Camberwell leaves at 9:03am. However your friends says that in practice the 1st train arrives at some random time between 8:59am and 9:05am and the 2nd train never departs early but can be up to two minutes late. The probability of the arrival and departure times is spread evenly across the possible times. If it takes you 30 seconds to get between platforms at Flinders Street, what is the probability that you will miss the 2nd train?

Question 22

A tower is placed in each of the four corners of a square of area 1. Each tower is capable of detecting units within a radius of $1/\sqrt{3}$. What is the probability that a unit is detected if it randomly appears within the square?

Question 23

A guard dog is held near his post by a 5m chain. A sheep is tethered to a post 3m west of the dog's post by a 4m rope. A second sheep is tethered to a post 4m south of the dog's post by a 3m rope. The sheep are too scared to venture within the dog's range. What is the total grazing area of the two sheep?

Question 24

At 11.30am, there are 100 large cakes on a table, in preparation for a 1pm lunch. Spies inform us that one of them has been poisoned, and that the poison takes up to one hour to effect, and is then immediately lethal. In order to determine which one is poisoned, Alisa decides that some of us should sample small slices from the cakes (this doesn't take very long). How many of us need to risk our lives trying the cakes in order to ensure that lunch can go ahead at 1pm?

Question 25

A faulty calculator has all of its digit keys swapped around. No key is programmed to its stated digit, and no two keys are programmed to the same digit. The calculator is not faulty in any other way.

When "32×21" is typed, the calculator displays 672.
When "18×77" is typed, the calculator displays 1386.
When "99×43" is typed, the calculator displays 3300.

What does the calculator display when "46 + 30 + 95" is typed?