Question 1 **10 marks**

Mel is making lasagne. He has 2010 layers' worth of pasta and an unlimited amount of meat and cheese. He starts by making a layer of meat for the base followed by a layer of pasta and then a layer of cheese. After this he uses the following rules to decide what to add:

- If the top layer is meat he adds a layer of pasta
- When the top layer is cheese, he adds a layer of whatever layer is below the cheese
- Above a layer of pasta Mel adds a layer of the same type as the layer directly below the previous layer of pasta.

Mel continues this until he is unable to continue to due to lack of pasta. How many layers of meat are there in Mel's lasagne?

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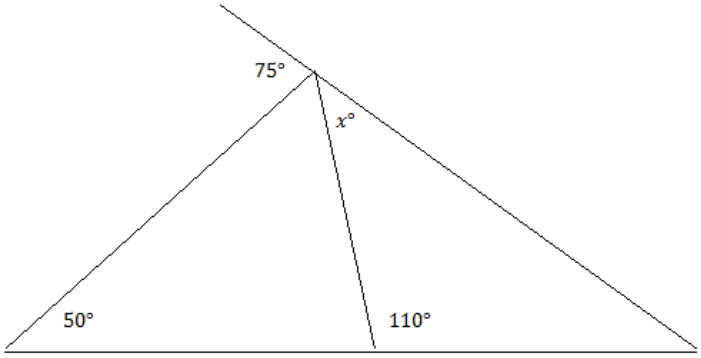
Question 5 **CHANGE RUNNER NOW** **10 marks**

In the diagram, what does x equal?



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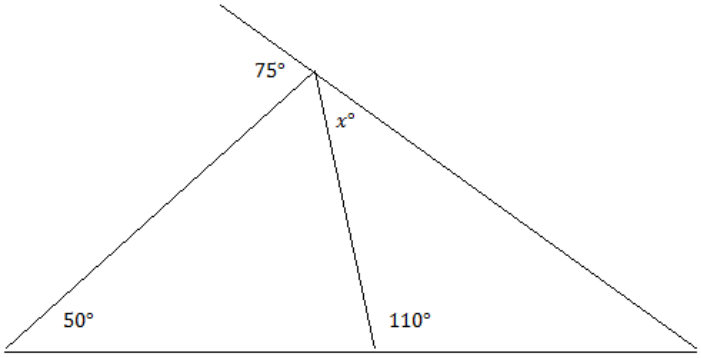

Question 6 **10 marks**

A group of old ladies meet for an afternoon tea party. They bring all their cats. In all, there are 22 heads and 72 feet. How many old ladies and how many cats are in the room?

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Question 7 **10 marks**

The SMO has been held each year starting in 1999, with 25 teams on average. Some students participate in multiple years, with the average being 1.5 times (not including future participation). Given these assumptions, as well as the assumption that every team is full, how many students have participated in the SMO (including this year)?

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A Dudeney number is a natural number (positive integer) that equals the cube of its sum of digits. For instance, $4913 = (4 + 9 + 1 + 3)^3$. Find the next Dudeney number after 1.

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Question 9 **10 marks**

While Julia always tells the truth, four of her friends - Yi, Stephen, Sam, and Jiaying - tell the truth randomly only in one out of three instances. Yi makes a statement. Stephen tells Sam that Yi's statement is the truth. Sam tells Jiaying that Stephen is right, and Jiaying says to Julia that Sam is telling the truth and Julia agrees with her. What is the probability that Yi's statement is the truth?

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Question 10 **CHANGE RUNNER NOW** **10 marks**

In the Maths and Stats Society there are 84 members, with half as many women as there are men. The next month, 8 women joined (none left) and some men left (none joined). If the ratio of women to men is now 3 to 4, how many men left?

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Question 11 **20 marks**

How many four-digit even numbers use only the digits $1, 2, 3, 4, 5$?

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Three faces of a rectangular prism have area 12, 25 and 27. Find its volume.

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Question 13 **20 marks**

Paul the Octopus correctly predicted the results of 8 out of 8 matches in the 2010 FIFA World Cup. We divide these into two types: 3 of these were group matches (where a draw is possible), and the other 5 had to have a winner. Paul predicts that one country or the other wins, and if a group match is drawn, Paul is deemed to be incorrect. As a fraction in lowest terms, what was the probability of Paul's guesses all being correct, given:

- he tries for 3 group matches and 5 later-stage matches
- the probability of any group match being drawn is 25%
- he always has a 50% chance of picking either country?

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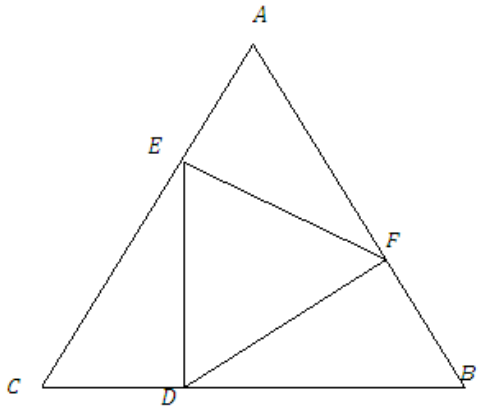
Question 14 **20 marks**

In the diagram $AF = BD = CE = 2EA = 2FB = 2DC$. If the area of triangle DEF is 4, what is the area of triangle ABC?



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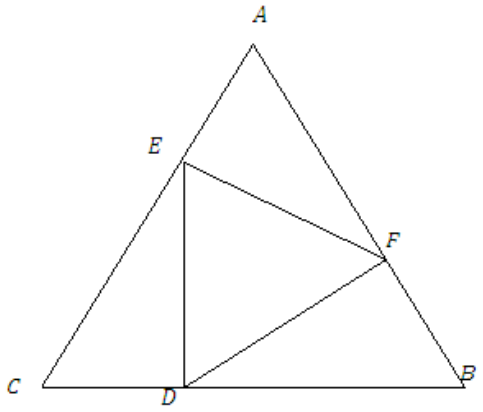
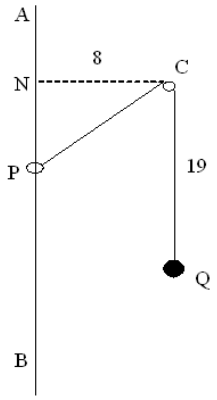


Question 15 **CHANGE RUNNER NOW** **20 marks**

AB is a fixed vertical rod and C is a fixed peg 8 metres from AB. PCQ is a rope 36 metres long supporting the solid ball Q. The other end of the rope is attached to a ring P which can slide on the rod AB. Initially, Q is 19 metres below C. If Q is pulled down a further 7 metres, find by how much P rises.



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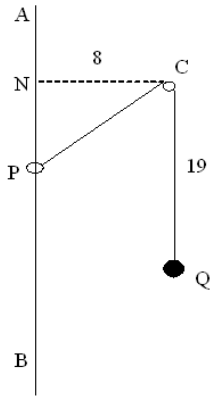

Question 16 **20 marks**

Grandma sent Johnny some money for his birthday. Johnny spent all of it in five stores. In each store, he spent $1 more than half of what he had when he came in. How much money did he get from grandma?

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A fair coin is flipped three times. Given that at least one coin landed tails, what is the probability of observing two consecutive heads?

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If n is a three-digit number, and $s(n)$ is the sum of its digits, determine the smallest possible value of

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In the good old days, McDonalds sold chicken nuggets in packets of 6, 9 and 20. What was the greatest number of chicken nuggets you could ask for that they could not provide?

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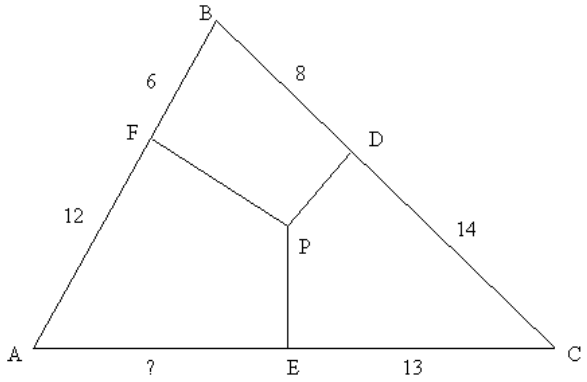
From point P inside triangle ABC, perpendiculars are drawn to the sides meeting BC, CA, AB at points D, E, F, respectively. If $AF = 12$, $FB = 6$, $DC = 14$, $CE = 13$, determine the length of AE.



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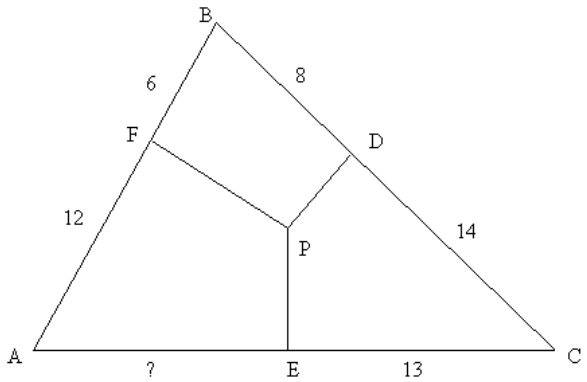

Question 21 **30 marks**

Find a four-digit number n such that the last four digits of n^2 in fact form the number n.

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Question 22 **30 marks**

On a maths test there are 6 problems. For each one, a score of 0, 1, 2 or 3 may be awarded. In how many ways can a score of exactly 15 be achieved on this test?

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Question 23 **30 marks**

In Jeff-land, the Business Section is sold by itself, and is 10 pages long. Daily sales are equal to 10000 times the square of the number of articles per page. Each page can hold 9 articles, and any number of articles can be replaced by 2 advertisements each. The cost of an article is $400, while the revenue from advertisements is $0.01 per advertisement per Business Section sold.

Assuming no other revenue or expenditure, what is the maximum daily profit that can be made, in dollars?

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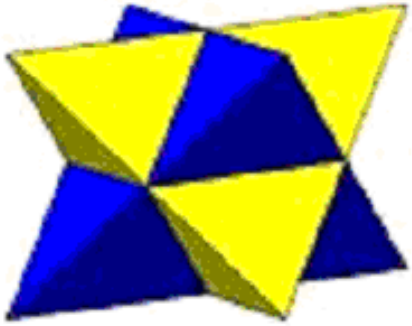
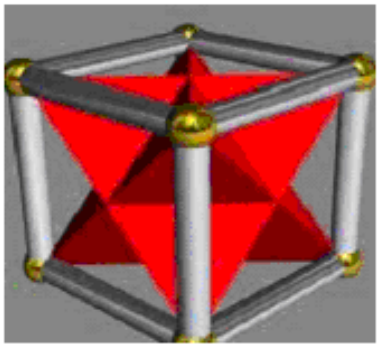
Question 24 **30 marks**

Han, a professional cake baker, baked a cake for a client that was a cube, but because that shape was boring he decided to make some cuts in order to make a stella octangula. Han makes the largest possible stella octangula. What proportion of cake (by volume) was wasted i.e. left over for Han to eat in his capacity as professional cake eater?





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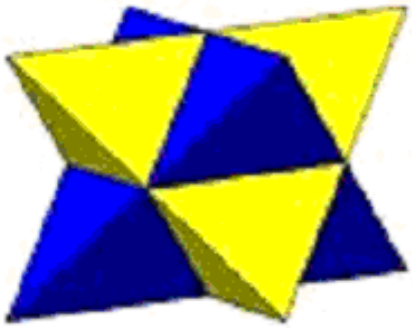

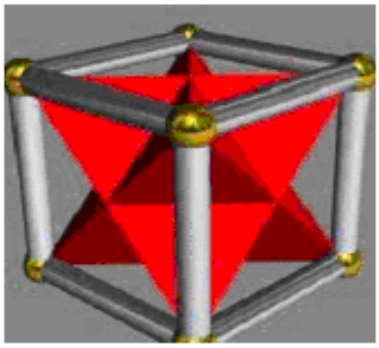

Question 25 **30 marks**

I have two children, at least one of whom is a boy born on Tuesday. Assuming that children are equally likely to be born on any day of the week, and that any birth has a 50% chance of producing a boy, what is the probability that I have two boys?

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