Question 1 **10 marks**

Han the chicken farmer has 150 chickens each starting with two legs. Han is extremely hungry one morning so he eats one leg of each of 30 of his chickens. Now that he's had a satisfactory breakfast Han gets to work putting a bunch of his chickens into a pen. When he has finished this he notices that the chickens in the pen have a total of 54 legs between them. Assuming that the proportion of one-legged chickens to two-legged chickens in the pen is the same as that of the total population, how many one legged chickens are in the pen?

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Question 2 **10 marks**

Find two (not necessarily distinct) positive integers such that their sum is equal to their product.

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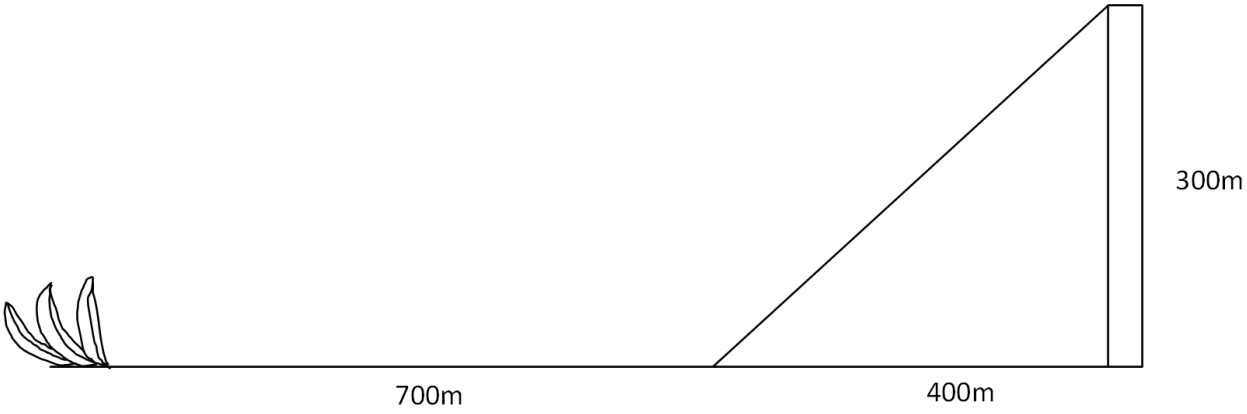
Question 3 **10 marks**

Frederick the Monkey is sliding down a zip line. One end of the zip line is attached to the top of a 300 metre high tree. The other end is attached to the ground 400 metres away from the base of the tree. When he reaches the bottom he needs to run 700 metres to get to the bunch of bananas. If Frederick travels along the zip line at 5m/s and he runs at 14m/s how long will it take him to get from the top of the tree to the bananas?



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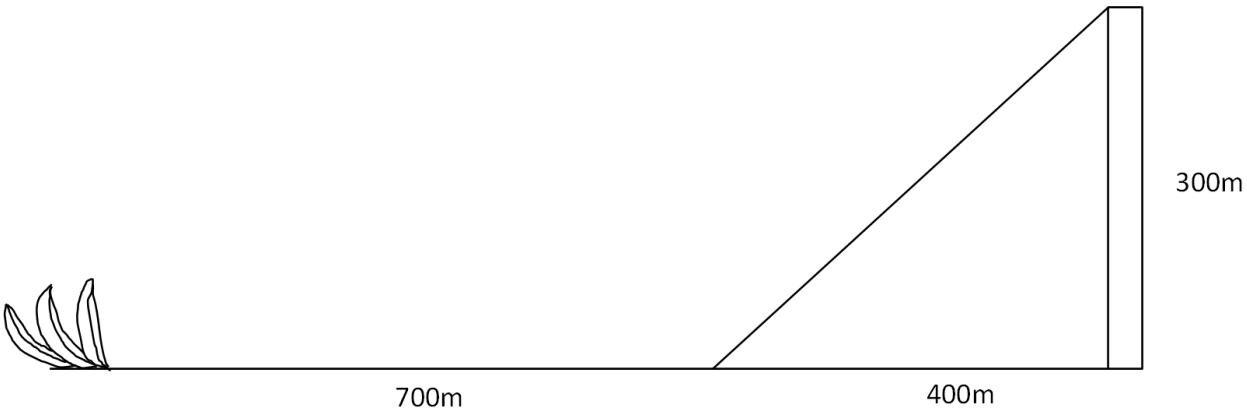

Question 4 **10 marks**

What is the last digit of 9^9?

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Question 5 **CHANGE RUNNER NOW** **10 marks**

The first term in a sequence is 1. Each other term is the sum of all of the previous terms. What is the 10th term?

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Question 6 **10 marks**

What is the sum of the squares of the values of x for which $x^3 - 7x + 6 = 0$?

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Question 7 **10 marks**

Han takes 6 minutes to eat a cake, while Jiaying takes 12 minutes. How many minutes does it take them to eat the cake together?

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Question 8 **10 marks**

In how many ways can \$11 be made out of \$1 coins, \$2 coins and \$5 notes?

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Question 9 **10 marks**

What is the largest number that can be subtracted from the denominator of $\frac{3}{4}$ and the numerator of $\frac{4}{3}$ so that the resultant fractions are equal?

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Question 10 **CHANGE RUNNER NOW** **10 marks**

Kristijan, Mel, Dougal, Adib and Stephen each have a number. Kristijan, Mel and Stephen's numbers are all positive integers. Dougal's number is equal to Kristijan's number plus the reciprocal of Mel's number whereas Adib's number is equal to Mel's number plus the reciprocal of Kristijan's number. It turns out that the product of Adib's number and Dougal's number is one twelfth plus Stephen's number. What is Stephen's number? (The reciprocal of a number is one divided by that number.)

Question 10 **CHANGE RUNNER NOW** **10 marks**

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Question 11 **20 marks**

Sam, Han and Yi have ordered a chicken pizza and a vegetarian pizza for lunch but the pizza chef forgot to cut them. Sam sees this as an opportunity - he can finally have exactly one third of each pizza like he has always wanted. All Yi wants is to have the same number of pieces of each pizza. Han the chicken lover requests that he have (strictly) more slices of the chicken pizza than anyone else. To make up for it he agrees to have (strictly) fewer slices of the vegetarian pizza than anyone else as long as he can have at least one slice. The three of them agree that each pizza should be cut into a whole number of equally sized pieces but the two pizzas may have differently sized slices from each other. Given that each person gets a whole number of slices of each pizza, with none being wasted, and everyone gets what they want, what is the minimum total number of slices of pizza possible?

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Question 12 **20 marks**

Find the largest possible fraction $\frac{m}{n}$ where $0 < m < n < 100$ such that m and n are each the product of two primes.

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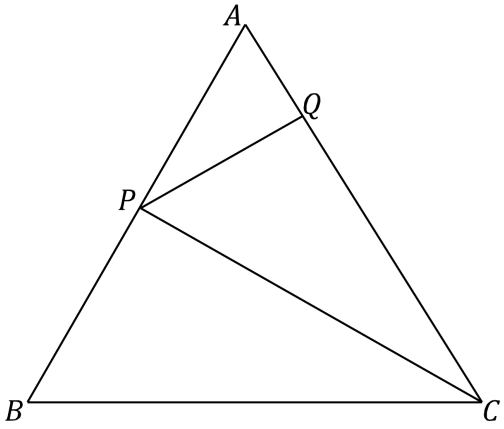
Question 13 **20 marks**

Let ABC be an equilateral triangle with side length 12. P and Q are points on sides AB and AC such that CP is perpendicular to AB and PQ is perpendicular to AC. Find the length of line segment QC.



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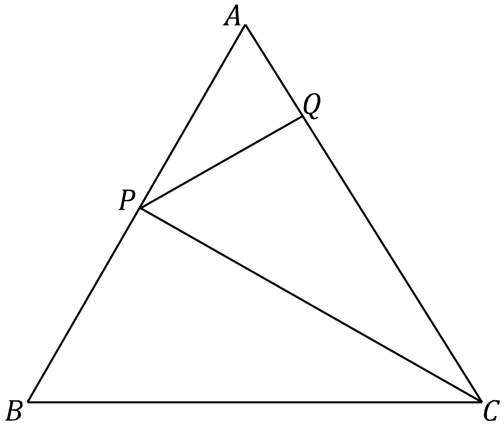

Question 14 **20 marks**

Jeff the chef is cooking up a storm. Jeff puts up to 10 ingredients into a dish that will sell for $500. Each ingredient costs $50 and Jeff only has to pay this if the dish sells. If the probability that the dish will sell is equal to a tenth of the number of ingredients, how many ingredients should he put into the dish to maximise his expected profits?

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Question 15 **CHANGE RUNNER NOW** **20 marks**

A sequence is given by

- $a_1 = 100$
- $a_{n+1} = \frac{a_n}{2}$ if a_n is even
- $a_{n+1} = a_n + a_{n-1} + 2$ if a_n is odd

Find a_{2011}.

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Find a_{2011}.

Question 16 **20 marks**

Gil and Giles each have an analogue 12-hour clock. Giles'clock gains 4 seconds each hour and Gil's clock loses 1 second each hour. Given that they both start off showing 12:00, how many days will it be until they next show the same time as each other?

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Question 17 **20 marks**

What is the sum of the digits of every integer between 1 and 999 (inclusive)?

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Question 18 **20 marks**

At a party there are a number of boys and a number of girls, some of whom have brought dogs with them. Each boy knows 5 girls and 2 dogs at the party. Each girl knows 4 boys and 3 dogs at the party. There are 46 dogs in total and each knows exactly 5 humans. Assuming that knowing is mutual, how many girls are at the party?

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Question 19 **20 marks**

Heckyl and Jeckyl share the same birthday. Heckyl notices that his age is a factor of Jeckyl's age. Jeckyl points out that this will only happen one more time and that will be in 8 years. How old is Jeckyl now?

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Question 20 **CHANGE RUNNER NOW** **20 marks**

Lu has a four digit number . It turns out that the sum of the digits of n form the last two digits of n and the product of the digits of n form the first two digits of n. Find n.

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Question 21 **30 marks**

Jack the ant likes to walk along the edges of a cube. Whenever Jack reaches a vertex (corner) of the cube he immediately chooses an edge connected to that vertex with equal probability (possibly the one he came from) and walks along that until he reaches another vertex. He stops this when he reaches the grain of sugar which is at one of the vertices. If he starts at the vertex opposite the grain of sugar and he takes one minute to walk along each edge of the cube how long is he expected to take to reach the grain of sugar?

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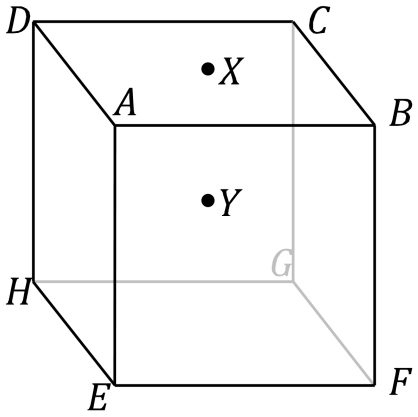
Question 22 **30 marks**

Let the vertices of a cube be labelled A, B, C, D, E, F, G, H as shown. Let X be the centre of face $ABCD$ and let Y be the centre of the cube. Consider the tetrahedron (triangular pyramid) formed by A, B, X and Y. What is the sum of the six angles defined by taking pairs of the faces of this tetrahedron?



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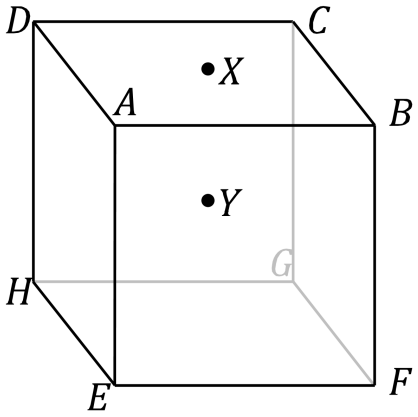

Question 23 **30 marks**

3 of each of 4 different species of penguins (Emperor, Fairy, Macaroni and Galapagos) are having a meeting. In how many ways can they be seated around a circular iceberg such that no two Emperor penguins are next to each other? (Rotations are considered to be different arrangements and penguins of the same species are identical.)

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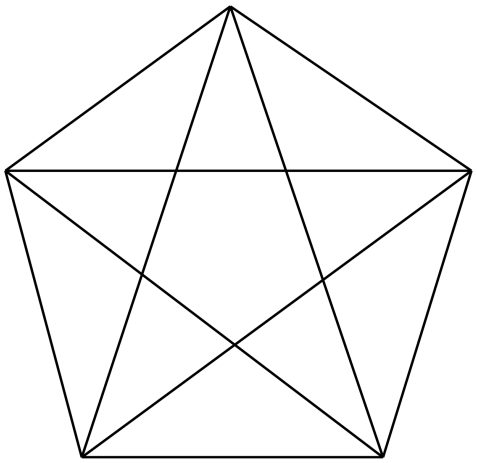
Question 24 **30 marks**

What's the ratio of the area of the big regular pentagon to the area of the small one in the diagram? Express your answer as $\frac{a+b\sqrt{c}}{d}$ where a, b, c and d are positive integers.



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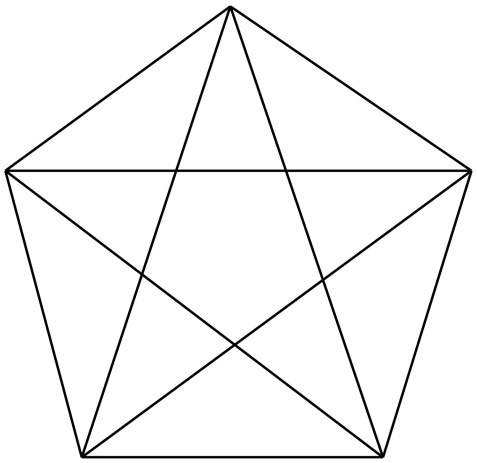
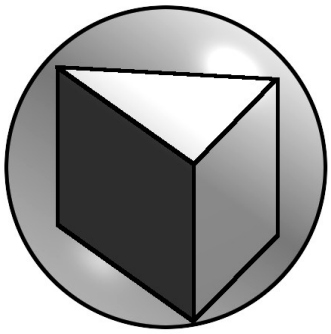


Question 25 **30 marks**

What is the largest possible volume of a triangular prism which is contained inside a sphere of radius 1?



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