Question 1 **10 marks**

Ben, David, Emily, Julia and Patrick have some debts to settle. Ben owes each of David and Emily a chocolate bar, David owes each of Emily and Julia a chocolate bar, Emily owes each of Julia and Patrick a chocolate bar, Julia owes each of Patrick and Ben a chocolate bar and Patrick owes each of Ben and David a chocolate bar. What is the minimum total number of chocolate bars which needs to be bought to settle all of the debts?

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What is the smallest three digit number which is greater than the product of its digits?

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What is the size of the largest set of integers from 1 to 100 which contains no two consecutive numbers?

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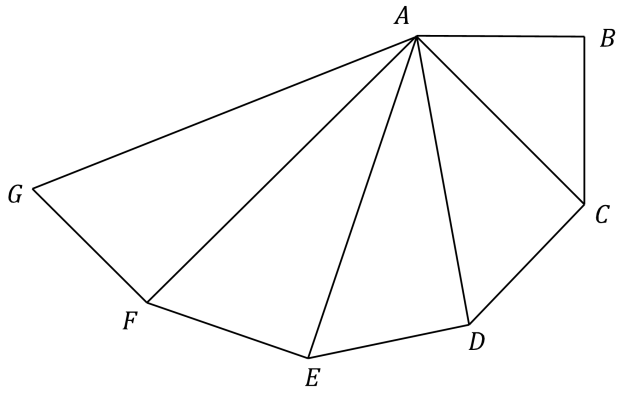
Question 4 **10 marks**

In the diagram, lengths $AB = BC = CD = DE = EF = FG = 1$ and angles $\angle ABC, \angle ACD, \angle ADE, \angle AEF, \angle AFG$ are all right angles. What is the length of segment AG?



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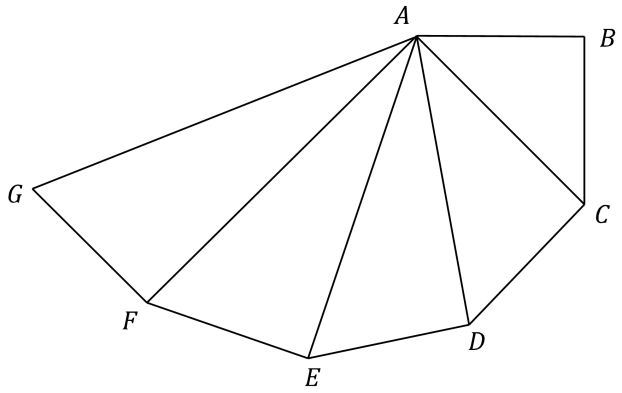

Question 5 **CHANGE RUNNER NOW** **10 marks**

Sam, Han and Andrew just finished eating a cake. Given that Han ate twice as much as Sam and Sam ate twice as much as Andrew, what proportion of the cake did Andrew eat?

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Question 6 **10 marks**

In the mathemagical square shown each square is filled with a positive number so that the product of the numbers in each row and column are the same. What is the value of '?'?

	9	
	6	
3		?

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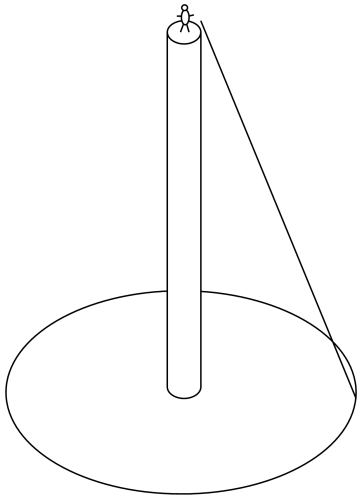
Question 7 **10 marks**

Franky is trapped on top of a tower that is surrounded by a moat. The moat has width x, and the tower has height y. To escape, Franky has a rope that is just long enough to reach the bank of the other side of the moat. If the rope is 30 metres long, and the tower is three times as tall as the moat is wide, what is the value of $x + y$?



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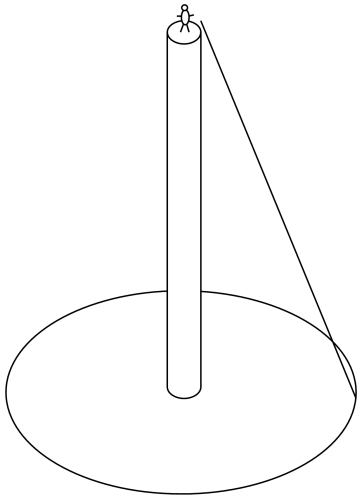
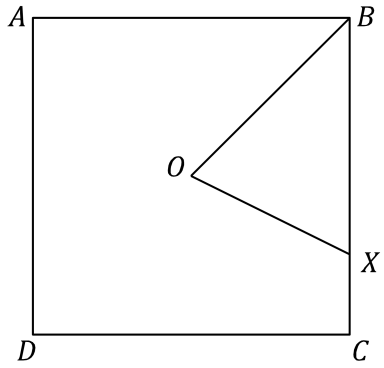


Question 8 **10 marks**

$ABCD$ is a square with side length 10 and centre O and X is a point on side BC. Given that the area of triangle OXB is one fifth of the area of the square, what is the length of segment BX?



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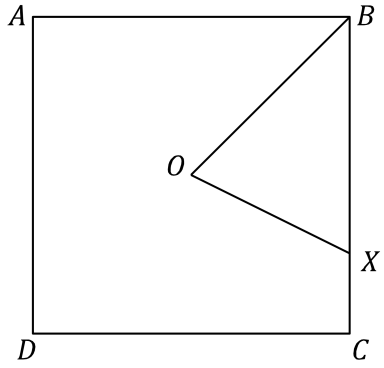

Question 9 **10 marks**

a, b, c and d are positive integers such that $a^8 < b^4 < c^2 < d$. What is the minimum possible value of d?

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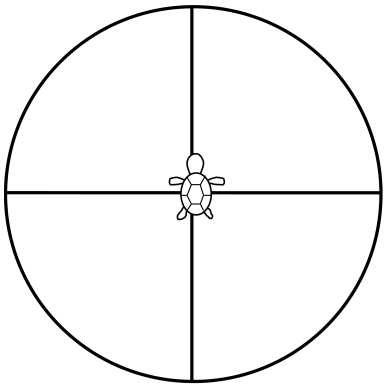
Question 10 **CHANGE RUNNER NOW** **10 marks**

In how many ways can the turtle travel along the lines shown, starting and ending in the middle, without retracing itself at any point? (Once the turtle gets back to the middle it stops immediately.)



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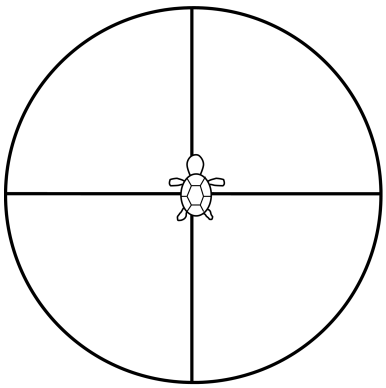
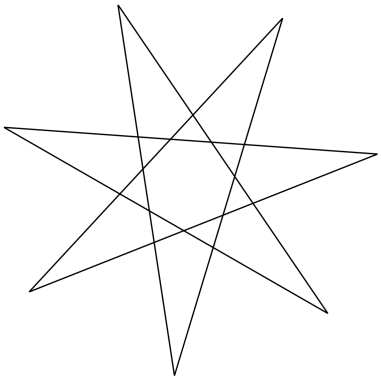


Question 11 **20 marks**

How many triangles are there in the following diagram?



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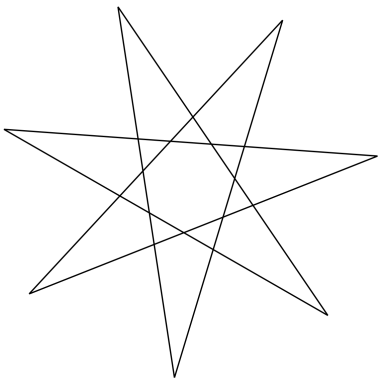

Question 12 **20 marks**

Yi has three boxes, one red, one blue and one green. He also has three identical balls of each colour. In how many ways can Yi distribute the nine balls amongst the boxes so that no ball is in a box of its own colour?

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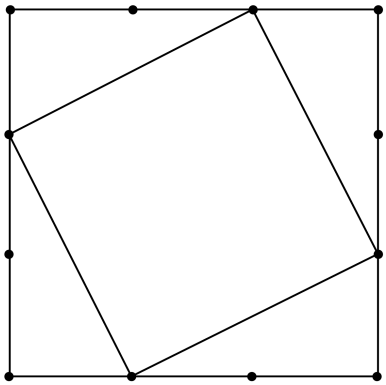
Question 13 **20 marks**

Each side of a square is broken into three equal parts by two points. Four of the eight points form a square. If the area of the new square is 1, what is the area of the original square?



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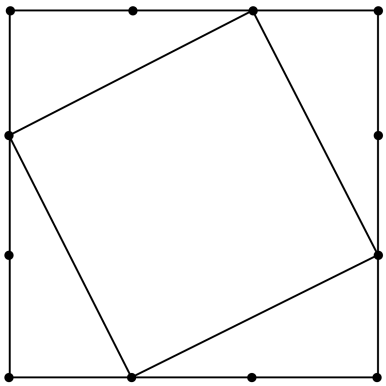

Question 14 **20 marks**

Aaron has a pile of blocks. When he groups them in sets of 3 there are 2 left over. Next he sorts them into groups of 5 and there are 3 left over. Aaron then sorts the blocks into groups of 7 and has 4 left over. Finally he separates them into groups of 9 and has 5 left over. Given that Aaron has fewer than 300 blocks, how many does he have?

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Question 15 **20 marks**

What is the size of the largest set of whole numbers, each of which lies between 0 and 2012 inclusive, such that no pair differs by a prime number?

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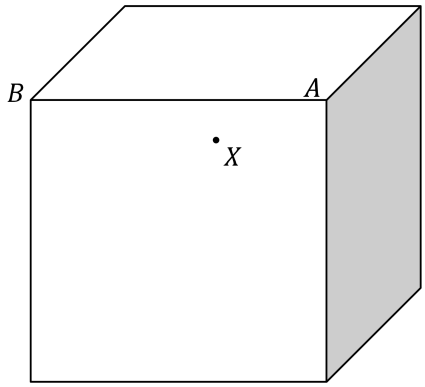
Question 16 **CHANGE RUNNER NOW** **20 marks**

A and B are adjacent vertices of a cube. A point X is chosen at random inside the cube. What is the probability that the closest vertex of the cube to X is A and the next closest vertex is B?



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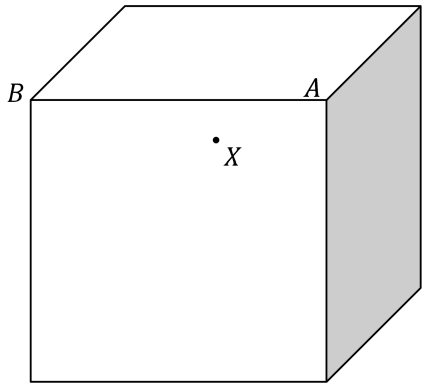

Question 17 **20 marks**

Dougal, Mel and Kristijan like archery. Each of them can shoot an arrow any distance up to 30 metres. For safety reasons they stand in a triangle such that each pair of them is exactly 30m apart. What is the area of the region which is within the range of both Mel and Dougal but not of Kristijan?

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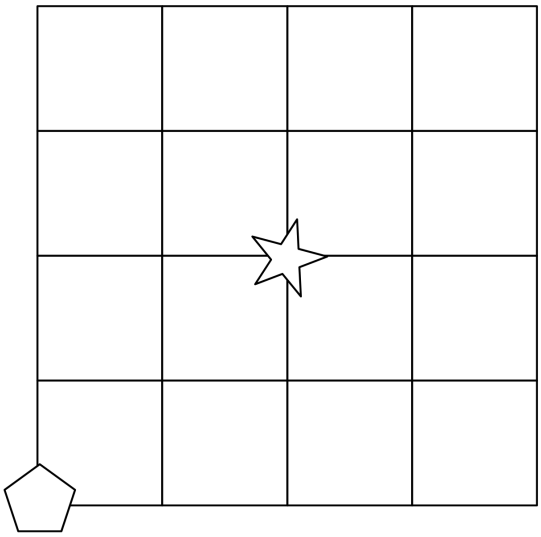
Question 18 **20 marks**

Jeff is standing at the Pentagon. In how many ways can Jeff reach the star if he only ever turns right, and cannot go back to a place he has been to before?



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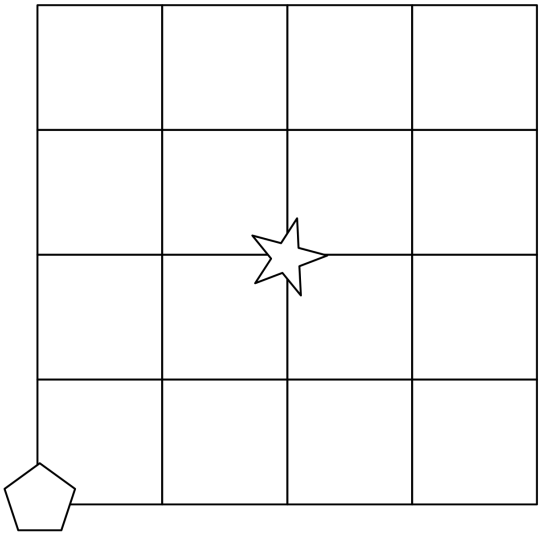

Question 19 **20 marks**

There is only one 3-digit number which is equal to 5 times the product of its digits. What is it?

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Question 20 **CHANGE RUNNER NOW** **20 marks**

A new operation $\ominus$ satisfies the following equations:

- $0 \ominus b = b + 1$
- $(a + 1) \ominus b = a \ominus (a \ominus b)$ for all $a \geq 0$

What is the value of $6 \ominus 7$?

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Question 21 **30 marks**

James has a stack of 6 sheets of paper. There are 3 white sheets, 2 green sheets and 1 yellow sheet, in a completely random order. James needs to sort out his sheets into 3 piles each of a different colour. An action is defined as when James takes a sheet from the 'random' pile and puts it into one of the sorted piles. If there are two or more sheets of the same colour on top of the 'random' pile, he takes all of them and sorts them in the one action, otherwise he can only take the one sheet per action. On average, how many actions does it take to sort his random pile?

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Question 22 **30 marks**

Gil and Giles decide to play a set of tennis with some unusual rules regarding serving. Instead of switching serve every game, they agree that the loser of each game gets to serve in the next game. Since they are lazy they decide to stop as soon as a player gets two games ahead. Given that Gil serves first and the probability of the server winning each game is $\frac{2}{3}$, what is the probability that Gil wins the set?

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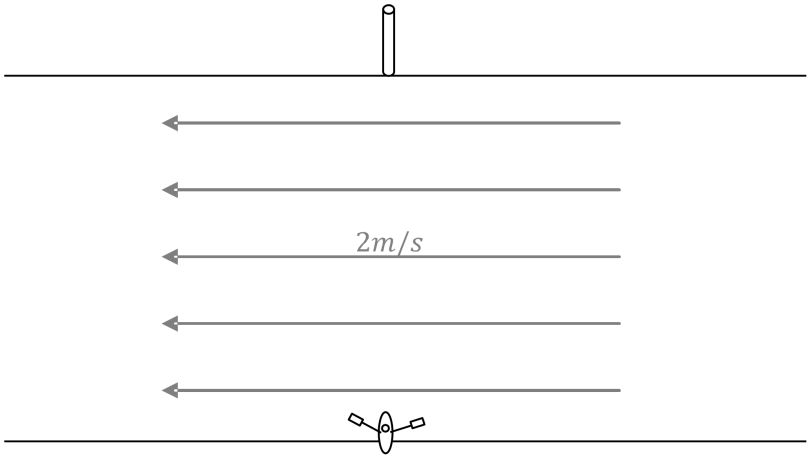
Question 23 **30 marks**

Nepa is at the south side of a 500m wide river. Directly across from Nepa is a post which he is trying to reach. Unfortunately the river is flowing at 2m/s and Nepa can only row at 1 m/s, so he can't possibly reach the post just by rowing. What is the minimum possible distance downstream of the post he can end up after rowing across the river?



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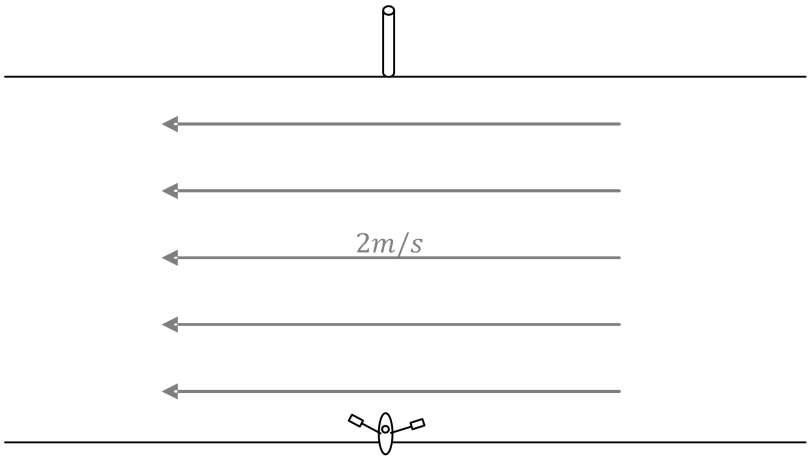

Question 24 **30 marks**

A sequence of positive integers satisfies $a_0 = 11$ and $a_{n-1} + a_{n+1} > 2a_n$ for all integers n. What is the minimum possible value of a_{2012}?

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Question 25 **30 marks**

Twelve people of ambiguous gender: Alex, Billie, Chris, Drew, Evan, Fran, Gay, Harley, Ivory, Jordan, Kim and Leslie want to cross a bridge. Unfortunately the bridge is very weak and will only support up to two people on it at the same time. Additionally the twelve people only have one torch, which they need to have with them while crossing the bridge. Alex, Billie, Chris, Drew, Evan, Fran, Gay, Harley, Ivory, Jordan, Kim and Leslie take 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 minutes to cross the bridge respectively and if two people cross at the same time they go at the pace of the slowest of the two of them. What is the minimum possible amount of time required for all ten to get across the bridge?

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