Question 1 **30 marks**

Giles has brought 11 boxes of cookies to the MUMS Trivia Night and Sam has brought some packets of lollies. To ensure that there is room on each of the 12 tables' equally-sized plates for lollies, Giles divides all of the cookies evenly amongst the plates so that they are all three-quarters full. Over the course of the Trivia Night, each table eats two-thirds of their cookies. What is the smallest number of cookie boxes that Giles can use to take all of the remaining cookies home?

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Question 2 **30 marks**

Samoil tells Emma "I have three times as many sisters as brothers and one sibling who is neither male nor female," to which his little sister adds "I have two times as many sisters as brothers and one sibling who is neither male nor female." How many children are there in Samoil's family?

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Question 3 **30 marks**

Last year, at the University Maths Olympics, all but three teams wore yellow t-shirts, all but three teams wore green t-shirts, all but three teams wore white t-shirts, and all but three wore orange t-shirts. How many teams participated in the UMO?

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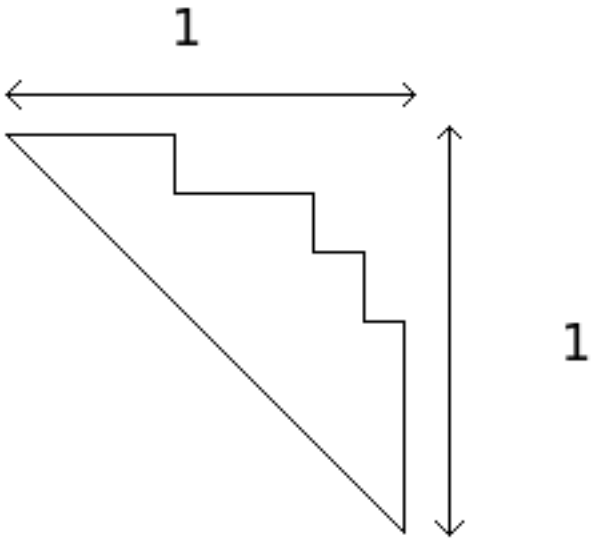
What is the perimeter of the following shape?



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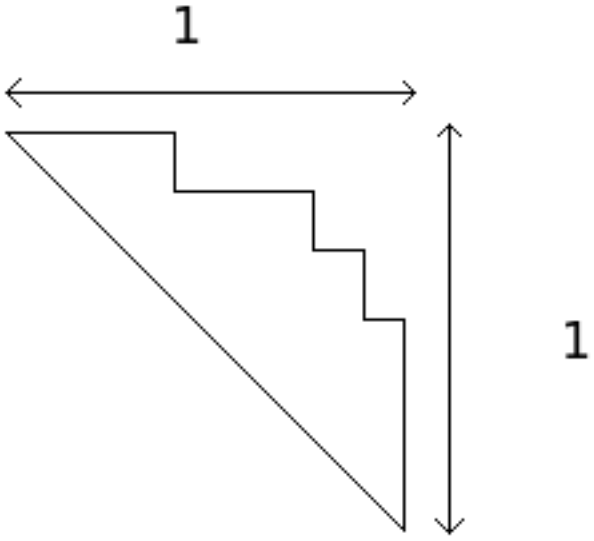

Question 5 **CHANGE WALKER NOW** **30 marks**

David, Song, and Ruwan are splitting up a garlic chicken pizza. David can finish the entire pizza in 20 minutes, Ruwan can eat it all in 30 minutes, while Song, who likes to savour the flavours, takes an hour to devour a whole pizza. How many minutes do they take to finish the pizza?

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Question 6 **40 marks**

Emma is making herself some cordial. She pours in the concentrate and then adds water, so that there is 200mL of cordial in the glass, of which 50% is water. This looks too strong for her taste, so she adds some more water, so that the cordial is now 75% water. How much cordial is now in her glass?

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Question 7 **40 marks**

David is playing a carnival game where he hopes to win the rare Master Ball. The carnie has three boxes: two of the boxes contain two pokeballs; the other contains one pokeball and one master ball. David believes in the heart of the cards and chooses a box at random, but to his bitter disappointment pulls from it one pokeball. What is the probability that the other ball in the box he chose is the elusive master ball?

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Question 8 **40 marks**

Sam and Adib are taking the supplies from the MUMS Room for the End of Semester BBQ. They have two tubs to carry: the tub of food and the tub of drinks. The tub of food weighs 2/3 as much as the tub of drinks, and the tub of drinks weighs 1kg more than the tub of food. Adib is stronger, so he carries the heavier tub. How many kilograms will Sam carry?

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The clocks in the MUMS Room and the main corridor of the maths building are rather old. The MUMS Room clock gains 10 minutes every hour and the clock in the corridor loses 10 minutes every hour. Consequently, even though they were set at the same time earlier today, the clock in the MUMS Room reads 6pm, while the clock in the main corridor says that it is only 3pm. What time were the two clocks set?

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Question 10 **CHANGE WALKER NOW** **40 marks**

Giles is challenged to a game of chance by his sinister supervisor Ian. The game proceeds by a fair[a] coin being tossed until the two most recent coin tosses show a certain sequence. If the last two flips were HEADS TAILS then Ian wins; if the flips were TAILS TAILS then Giles wins. If the last two flips are neither of these sequences then the coin is flipped again, and this continues until one of the two sequences appears.
What is the probability that Giles wins?

[a]Fair: the probability of a heads is equal to the probability of a tails and all flips are independent of each other.

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Question 11 **50 marks**

For a positive integer n, let $f(n)$ be the smallest positive integer with exactly n factors (including itself). So $f(1) = 1$, $f(10) = 48$ and so on. What is $f(f(f(f(2))))$?

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In a particularly unusual first-year mathematics class, there are 2017 students all of which weigh the same except for one. Adib has a set of scales, intent on determining which student is the odd one out. The scales allow him to put two groups of students either side (of any number, they're big scales) and they will dip in the direction of the heavier group of students. Luckily, Adib knows a weighing strategy that will guarantee that the maximum number of weighings he needs to perform is as small as possible. What is this minimum maximum number?

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If $1 + \frac{1}{2} + \frac{1}{2}^2 + \frac{1}{2}^3 + \ldots + \frac{1}{2}^{2017} = a - \frac{1}{2}^b$ what is $a + b$?

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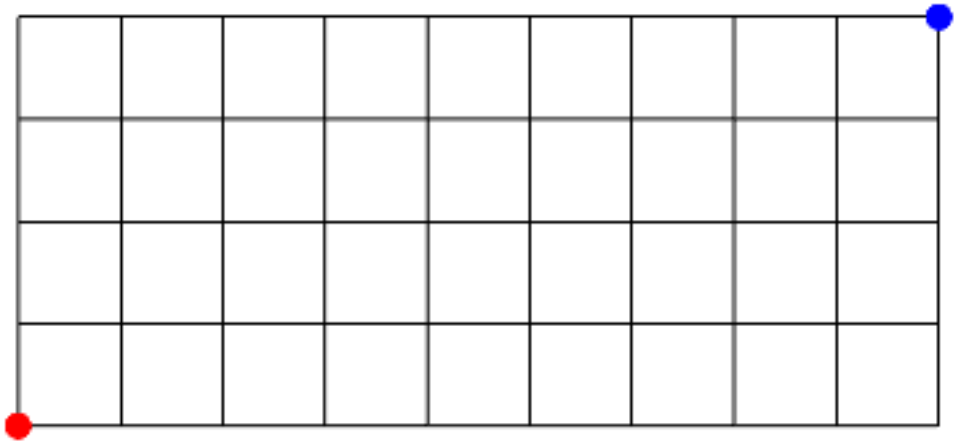
Samoil is traversing the following 5×10 grid from the bottom-left corner to the top-right corner taking only up and right steps. How many paths can he follow if he cannot take two consecutive vertical steps?



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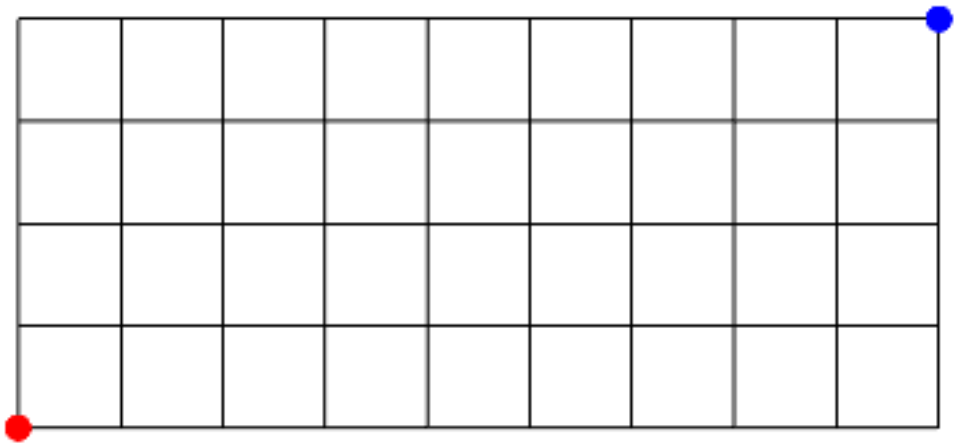

Question 16 **60 marks**

Buses arrive at a stop following an unusual schedule. First, one arrives after 20 minutes, then the next one arrives after 30 minutes. This repeats forever so that two buses arrive over the course of any 50 minute period. Song arrives at the bus stop at a random time. On average, how many minutes will she have to wait till the next bus?

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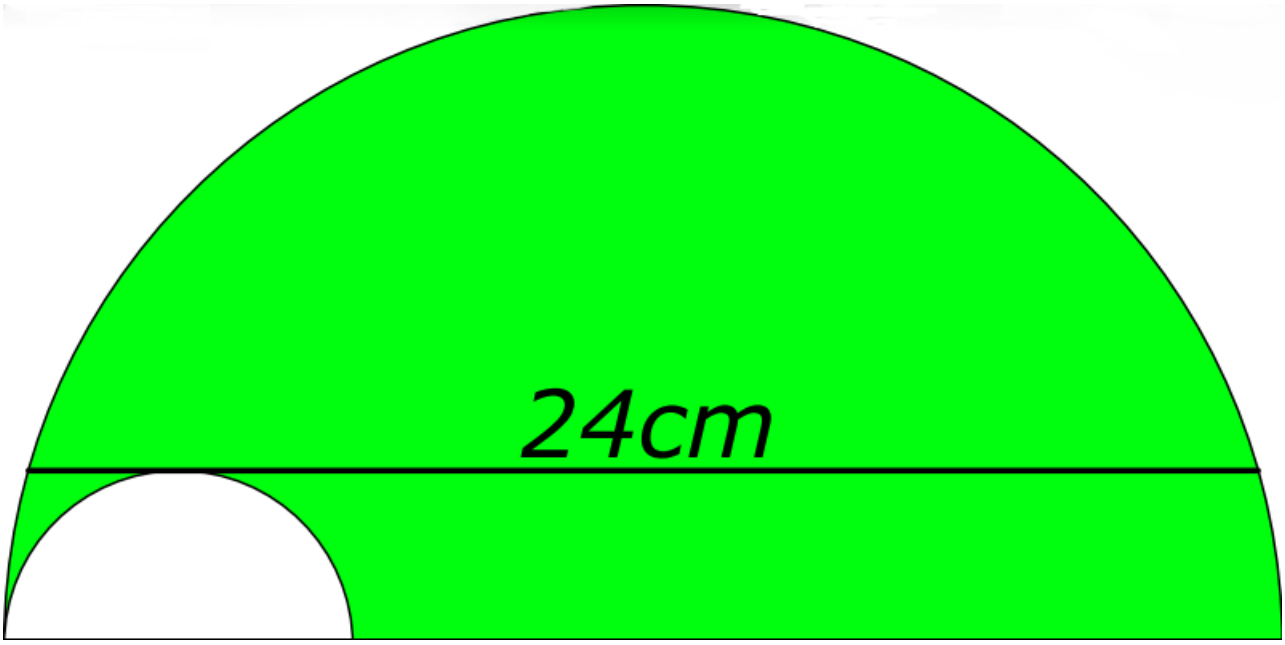
What is the area of the shaded region in cm^2?



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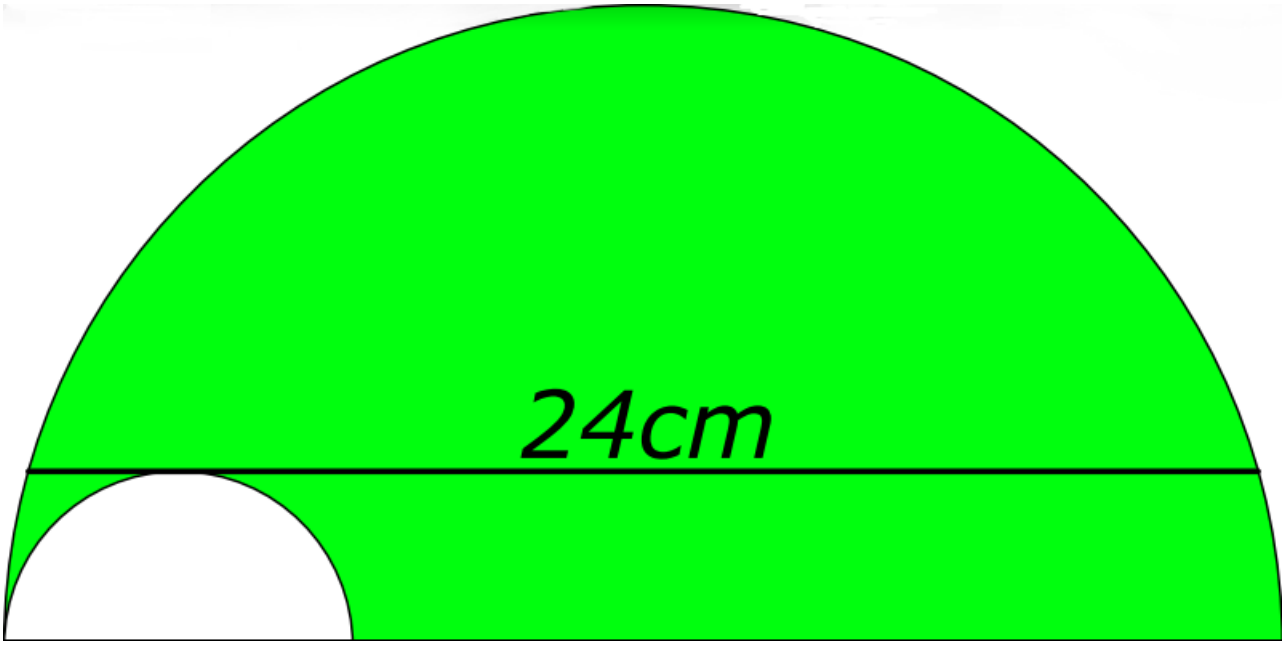

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