SMO 2022 questions

Questions

1. (20 points) The university plans to recycle paper cups. It is given that 9 used cups can make 1 new cup. If they have 404 paper cups, how many new cups can be made from recycling in total?

   REMINDER: All of the questions can be answered with a number of form $a + b\sqrt{c}$ where a and b are simplified rational numbers, and c is an integer. Any of a, b, c may be 0, or whole numbers.

   Source: MS Outreach

2. (20 points) I choose a 4-digit number A, and swap any two of the digits to create another 4-digit number B. For example, if I chose $A = 3021$, I can swap the 3 and the 1 to create another 4-digit number $B = 1023$

   What is the largest value of $A - B$ I can create?

   REMINDER: All of the questions can be answered with a number of form $a + b\sqrt{c}$ where a and b are simplified rational numbers, and c is an integer. Any of a, b, c may be 0, or whole numbers.

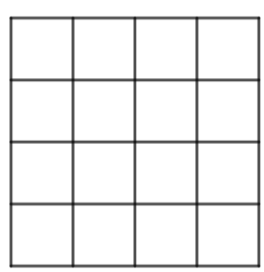
   Source: Quang Original

3. (20 points) What is the largest number consisting only of the digit 1 (in base ten) which is a perfect square?

   Source: Oxford admissions interview question

4. (20 points) Find the maximum number of bishops you can place on a 4×4 chessboard such that no two attack each other.

   Two bishops attack iff they are placed on the same diagonal.



Source: Quang Original

5. (20 points) If we randomly pick 4 of the vertices from a cube, what is the probability that the chosen vertices share a same plane? Give your answer as a simplified fraction.

Source: Gaokao 2022

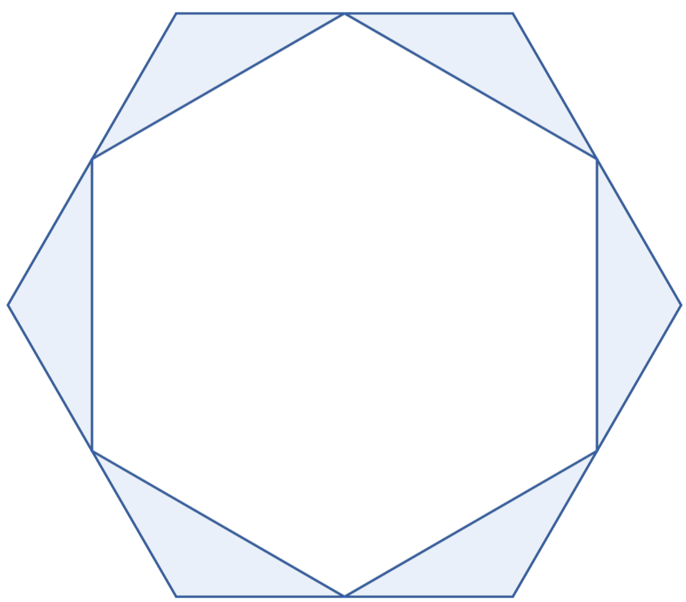
6. (20 points) In a given week, Lewis and Marnie each randomly choose days to visit the Stardrop Saloon. Lewis will uniformly randomly choose 3 different days, and Marnie will choose 2 days. What is the probability that they will end up meeting in the Stardrop Saloon at least once this week?

Source: Yadan Original

7. (20 points) You have 2 points A, B on the 2d plane at coordinates $(0, 3), (5, 9)$ respectively. Find the length of the shortest path from A, to some point on the x axis, then to B.

Source: Folklore (Heron's Problem)

8. (25 points) What is the area in the region between a regular hexagon of area 1 and the hexagon made by connecting the midpoints of each side?



Source: Bon Original

9. (25 points) I want to run a Beat Saber tournament among my 8 friends. Not only do I want to find who is objectively the best at Beat Saber, I also want to find who is objectively the worst. Each friend has a distinct skill level, and whenever I schedule a match between two players, the person with the higher skill level always wins.

   What is the minimum number of matches I need to schedule, in order to determine the best, and worst among the 8 players? Note I can look at tfitshe results of the previous matches before deciding who plays in the next match.

   Source: Quang Original

10. (25 points) The local farmer Quang from Stardew Valley is going to visit Pierre's store for some sunflower seeds before the store is closed. He walks from home to the store at 3.5 km/hr in the first hour, but realizes he will be an hour late if he continues at this speed. He drinks some coffee instantly and increases his speed by 1.5 km/hr for the rest of the way to the store and arrives 30 mins early. How many kilometres is the store from his home?

    Source: 2014 AMC 10A Problems/Problem 15 (Modified)

11. (25 points) I want to run another beat saber tournament among my 8 friends. This time, I want to find who is objectively the best at beat saber. Each friend has a distinct integer skill level, and whenever I schedule a match between two players, the person with the higher skill level always wins.

    What is the minimum number of matches I need to schedule, in order to determine the best among the 8 players? This time, I must schedule all my matches ahead of time without seeing the results of any rounds, and I must always be able to unambiguously determine who is the best, no matter how their skill levels are ordered.

    Source: Quang Original

12. (25 points) We choose nine points on a circle and draw chords to connect every pair of points. No three chords intersect in a single point inside the circle. How many triangles with all three vertices in the interior of the circle are created?

    Source: 2010 AMC 10A Problems/Problem 22

13. (25 points) The 25 integers from 1 to 25 can be arranged to form a 5 by 5 square in which the sum of the numbers in each row, the sum of the numbers in each column, and the sum of numbers along each of the main diagonals are all the same. What is the value of this common sum?

    Source: 2020 AMC 10A Problems/Problem 7

14. (25 points) A particular 12-hour digital clock displays the hour and minute of a day (starting from 12:00 to 11:59). Unfortunately, whenever it is supposed to display a 1, it mistakenly displays a 9. For example, when it is 1:16 PM the clock incorrectly shows 9:96 PM. What fraction of the day will the clock show the correct time?

Source: 2009 AMC 12B Problems/Problem 10

15. (25 points) Find the length of the shortest string constructed by 1,2,3 which contains every permutations of 1,2,3 as a substring.

Source: Superpermutations

16. (30 points) Squareman lives in a 2D world. He wishes to pack his prized possessions, 5 unit squares into a square suitcase of length L. The 5 unit squares must fit entirely within his suitcase, and may not overlap with each other.

    What is the length L of the smallest possible suitcase for which this is possible?

Source: https://erich-friedman.github.io/packing/squinsqu/

17. (30 points) Little Don Don is playing with her toy train set. Being the spoiled child she is, she has an unlimited number of wagons of length 1, and wagons of length 3. She can connect wagons together to form a whole train, with length equal to the sum of the length of each wagon! Don Don is fascinated by all the combinations she can make, and would like to know how many different kinds of trains she can make of length 10.

    Two trains are different if the sequence of wagon lengths differ at at least one place. For example, 1-3, and 3-1 are both different kinds of trains of length 4.

Source: Quang Original

18. (30 points) Evaluate $\sqrt[\log_3 5]{\log_2 18 + \log_{\sqrt{3}} 18 - (\log_2 5)(\log_5 9) + \frac{\log_{\frac{1}{25}} 16}{\log_5 3}}$

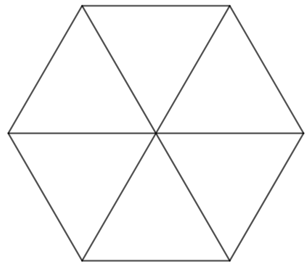
Source: Andres Original (Modified)

19. (30 points) If a positive integer is unable to be expressed as the difference of two perfect squares, it is called a “quirky” number. What is the 2022nd “quirky” number?

    Note that 0 is considered a perfect square.

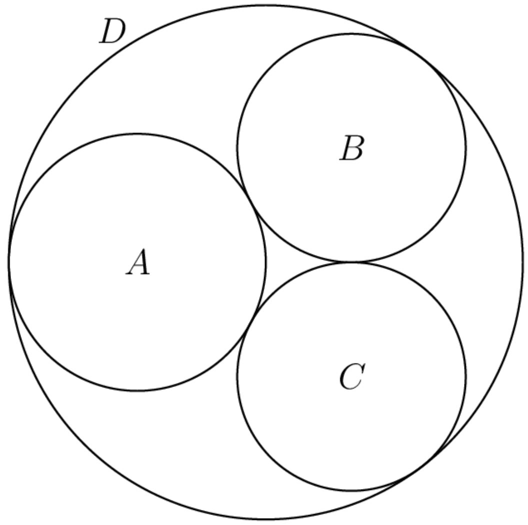
Source: Ravon Original

20. (30 points) What is the number of ways to colour the triangles in this diagram using at most 4 colours such that no two adjacent triangles have the same color.



Source: Harry from notes

21. (30 points) Circles A, B and C are externally tangent to each other, and internally tangent to circle D. Circles B and C are congruent. Circle A has radius 1 and passes through the center of D. What is the radius of circle B?



Source: 2004 AMC 12A Problems/Problem 19

22. (30 points) Let $ABCD$ be a square with side length 2, and let E be the midpoint of CD. What is the radius of the largest circle that fits in quadrilateral $ABCE$?

Source: Bon Original