Question 1 **10 marks**

I wish I was a pirate, on a farm,
Where the clouds are sweet, where I'll be no harm.
Six swishing sticks is all I need,
To mill the seed,
Pile the feed,
Expound my greed.
Now, once more, you must read,
This story again,
And tell me then,
How many letters did I not pen,
That the alphabet does contain?

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Question 2 **10 marks**

To convert from degrees Celsius to Fahrenheit, you multiply by $\frac{9}{5}$ and then add 32. If N degrees Celsius is the same temperature as N Fahrenheit, then what is the value of N?

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Question 3 **10 marks**

Nick has a treasure map with the following instructions:

> Walk 100 metres north.
> Turn left 90° and walk 200 metres.
> Turn right 90° and walk 300 metres.
> Turn right 90° and walk 50 metres.

In his excitement, Nick becomes disorientated and turns left whenever he should have turned right and vice versa. How far, in metres, from the treasure will Nick eventually be?

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Question 4 **10 marks**

If it takes 12 mathematicians 12 minutes to prove 1800 theorems, how many minutes does it take 4 mathematicians to prove 700 theorems?

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Question 5 **CHANGE RUNNER NOW!** **10 marks**

Jennifer, Sally and Dave were rounded up by MUMS yesterday, because one of them was suspected of having stolen Knotman's 'Utility Belt'. The three medical students made the following statements under very intensive interrogation:

> Sally: "I'm innocent!"
> Jennifer: "I'm innocent!"
> Dave: "Jennifer is the guilty one."

If only one of these statements is true, who took Knotman's prized possession?

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Question 6 **15 marks**

What are the last four digits of the largest multiple of 8 which does not have a repeated digit (when written in decimal representation)?

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Question 7 **15 marks**

A *cross* on a chessboard consists of a square together with its neighbours above, below, to the right, and to the left. Find the largest number of crosses that can fit without intersection on a regular 8×8 chessboard.

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Question 8 **15 marks**

Chris walks from A to B. If Chris had walked 500 metres per hour faster, he would have taken only $\frac{4}{5}$ of the time. If he had walked 500 metres per hour slower, it would have taken $2\frac{1}{2}$ hours longer. What is the distance from A to B in metres?

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Question 9 **15 marks**

In how many ways can one colour two squares of a 3×3 checkerboard such that no coloured square is adjacent to another coloured square? Two squares are adjacent if they share a common edge.

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Question 10 **CHANGE RUNNER NOW!** **15 marks**

A group of 25 physicists, 20 lawyers, 18 mathematicians and 12 doctors were at a chocolate festival. The whole group spent a total of 133 dollars on chocolates (which they all consumed immediately with invigorated passion). It is known that 5 physicists bought as much as 4 lawyers, that 12 lawyers bought as much as 9 mathematicians, and that 6 mathematicians bought as much as 8 doctors. How much did the most chocoholic group (the group with the highest total spending) spend in dollars?

Question 10 **CHANGE RUNNER NOW!** **15 marks**

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Question 11 **20 marks**

If $ABCD$ is a square, how many distinct points O are there such that OAB, OBC, OCD, ODA are all isosceles triangles?

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Question 12 **20 marks**

What are the last two digits of the number:

$$1 \times 1! + 2 \times 2! + \cdots + 2004 \times 2004! \ ?$$

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A *nice* number equals the product of its positive divisors excluding 1 and the number itself. Find the sum of the first 10 nice numbers.

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Question 14 **20 marks**

A sequence of positive integers includes the number 68 and has average 56. When 68 is removed the average of the remaining numbers is 55. What is the largest number that can occur in the sequence?

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Question 15 **CHANGE RUNNER NOW!** **20 marks**

Which four-digit number has the property that when its digits are reversed the resulting number is six less than twice the original number?

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Question 16 **20 marks**

Bobbi:	Think of a square number.
Zhihong:	Ok.
Bobbi:	Now, add 2004.
Zhihong:	Done.
Bobbi:	Is your result a perfect square?
Zhihong:	Yes!
Bobbi:	Hmm. . . is your number big?
Zhihong:	How big is 'big'?!
Bobbi:	Say, bigger than 100,000?
Zhihong:	No.
Bobbi:	Aha, your original number is. . .

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Find the coefficient of x^2 in the polynomial:

$$(1-x)(1+2x)(1-3x)\cdots(1-15x)$$

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Question 18 **20 marks**

Hyam says to Maurice, "Today is the birthday of my two children, see if you can calculate their ages. They're under 10. I'll give you a clue. I won't tell you the product of their ages, because that won't help you. However the quotient of their ages is mutter, mumble, mumble..." (Maurice understands the mumbling, we don't!)

Maurice says, "I still can't tell their ages."

Hyam says, "You're right. But if I had told you the difference between their ages instead of the quotient, you would have known."

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Question 19 **20 marks**

Let $ABCD$ be a convex quadrilateral. Let M be a point on AB with $AM = \frac{1}{3}AB$, N be on BC with $CN = \frac{1}{3}BC$, R be on CD with $CR = \frac{1}{3}CD$ and S be on DA such that $AS = \frac{1}{3}AD$. Find the ratio of the area of $MNRS$ to $ABCD$, and express it as a fraction in lowest terms.

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Question 20 **CHANGE RUNNER NOW!** **20 marks**

Let $S_1 = 1$, $S_2 = 2 + 3$, $S_3 = 4 + 5 + 6$,...
Find S_{21}.

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Question 21 **30 marks**

A regular dodecahedron is a solid with 12 regular pentagons for faces. If each face is to be coloured by one of 12 colours and each face has a different colour, how many distinct colourings are there? Two dodecahedra with the same colouring but in different orientation are not considered distinct.

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Question 22 **30 marks**

Two regular tetrahedra A and B are contained in a cube of volume 1. The vertices of A coincide with the vertices at the ends of a diagonal in the bottom side of the cube and a diagonal of the top side of the cube. The other vertices of the cube are the vertices of B. What is the volume of $A \cup B$?

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Question 23 **30 marks**

Find the value of:

$$\sum_{i=0}^{1002} (-3)^i \binom{2004}{2i}$$

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Question 24 **30 marks**

Andrew typed up the following equation at 346 wpm:

$$(abc)(bca)(cab) = 234235286$$

where $a > b > c$. Unfortunately, in doing so, Andrew compromised his accuracy — he got all the first eight digits of the nine-digit number on the RHS in the wrong position — that is to say, only the last digit 6 is in the correct position. Find the number that Andrew should have typed.

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Question 25 **FINAL QUESTION!** **30 marks**

A 10-digit number is said to be *interesting* if its digits are all distinct and it is a multiple of 11111. How many interesting integers are there?

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