Question 1 **10 marks**

As the saying goes: "an apple a day keeps the doctor away". Your particular doctor takes this adage a little too seriously, and vows to pay you a visit if you don't eat an apple on any given day. You have 2008 apples, and beginning today, plan to eat one per day, doubling the daily dosage with each passing week in anticipation of acquiring an apple-associated-addiction. On which day of the month is today?

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Question 2 **10 marks**

When expressed in Roman numerals, which positive integer below 1000 takes up the most number of characters?

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Question 3 **10 marks**

What is the largest two digit number ab with the property that in base 7, $ba_7 = ab_{10}$?
(a and b are digits, and the subscript denotes the base.)

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Question 4 **10 marks**

A square floor is tiled with identical square tiles. The tiles on the two main diagonals are black, whilst the rest are white. If there are 281 black tiles, how many white tiles are there?

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Question 5 **CHANGE RUNNER NOW** **10 marks**

Arrange the numbers a, b, c and d in increasing order, where:

$$a = 2^{2^{2^3}}, \quad b = 2^{2^{3^2}}, \quad c = 2^{3^{2^2}}, \quad \text{and} \quad d = 3^{2^{2^2}}.$$

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Question 6 **15 marks**

A polynomial $P(x) = x^3 + ax^2 + bx + c$ satisfies $P(1) = 1^1$, $P(2) = 2^2$ and $P(3) = 3^3$. What is $P(4)$?

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Question 7 **15 marks**

If 11 mathematicians, 17 physicists and 13 chemists produce 36 papers per year, and 23 mathematicians, 20 physicists and 10 chemists produce 45 papers per year, how many papers do 5 mathematicians, 6 physicists and 4 chemists produce in one year?

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Question 8 **10 marks**

As of the date of the 2008 Maths Olympics, the largest known prime number is $2^{32,582,657}-1$. What is its remainder when divided by 9?

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Question 9 **15 marks**

A circle both circumscribes a regular hexagon and is inscribed in a different regular hexagon. What is the ratio of the area of the outer hexagon to that of the inner hexagon?

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Question 10 **CHANGE RUNNER NOW** **15 marks**

Consider two cogs locked onto each other. The big cog has 20 teeth, while the small one has 12. The big one stays fixed, while the small one rolls around the big one until it gets back to its original position. How many revolutions does the small cog make?

Question 10 **CHANGE RUNNER NOW** **15 marks**

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Question 11 **20 marks**

Having finally moved away from the prying eyes of his parents, the evil mastermind Dr. James Zhao commences his nefarious plot of global domination. Unfortunately, he quickly blows his Stanford scholarship money on a state-of-the-art freeze-ray, and decides to ask his parents for more money. He sends the following note:

$$\begin{array}{r} S\ E\ N\ D \\ +\ M\ O\ R\ E \\ \hline M\ O\ N\ E\ Y \end{array}$$

Each letter stands for a distinct digit. Find out how much $MONEY$ James wants.

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Question 12 **20 marks**

Book publishers sometimes choose to omit the page number on the first page of each chapter. Following this printing scheme, and assuming that each chapter is at most twice as long as any other, what is the difference between the minimum and maximum numbers of page number digits printed in a 200-page book with 12 chapters?

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Question 13 **20 marks**

For each positive integer n, the *n-driver* is the smallest positive integer with exactly n divisors (including 1 and n). An n-driver is said to *overtake* an m-driver if $n > m$ but the n-driver is smaller than the m-driver. This overtaking is called *fair* if n and m have the same number of divisors. What is the smallest n-driver that overtakes fairly?

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Question 14 **20 marks**

Given $\{a_i\}$, a sequence of positive integers such that:

$$(a_n - a_{n-1})(a_{n-1} - a_{n-2}) \cdots (a_2 - a_1)(a_1 - a_0) = \sqrt{a_n^n},$$

for all $n \geq 1$. What is a_{2008}?

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Question 15 **CHANGE RUNNER NOW** **20 marks**

There are two triples (a, b, c) of positive integers satisfying $a^3 + b^3 + c^3 = 2008$, ignoring permutations. One such triple is $(2, 10, 10)$. What is the other one?

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Question 16 **25 marks**

Yi has an equilateral triangular sandbox of side length $16\sqrt{3}$cm, in the middle of which is a vertical stick of diameter 1cm. He wants to play horseshoes with a metal ring of inner diameter 7cm and outer diameter 8cm, but he has very poor aim and can only throw this projectile anywhere entirely inside the sandbox with uniform probability. What is the probability that he succeeds in getting the ring around the stick? (Assume the ring falls vertically, and bounces off if it touches the stick.)

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Question 17 **25 marks**

The planet of Cubonia consists of a perfect cube of rock of side length 1 megametre, and oceans that form a spherical shell which entirely covers the planet except at the vertices, two of which are the poles of the planet. Cubonians live in underwater cities. The Cubonian Navigator-General has defined latitude at any point on the ocean floor as that of the surface circle of latitude which is coplanar to that point, and latitude at any other underwater point as that of the nearest point on the ocean floor. What volume of water lies between the latitudes specified by the non-pole vertices?

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Question 18 **25 marks**

Find the largest integer k satisfying the following conditions:

- k^2 is the difference of two consecutive cubes; and
- $2k + 79$ is a perfect square.

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Question 19 **25 marks**

Given nine points in space, no three of which are collinear, find the smallest integer n such that for any colouring with red or blue of n line segments between these nine points, there always exists a triangle having all edges of the same colour.

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Question 20 **END OF MATHS OLYMPICS** **25 marks**

The bunyip is a humanoid species with strange breeding patterns. Male bunyips only reproduce between 3 and 11 years of age, while female bunyips reproduce between 4 and 7 years of age (inclusive). A particular family of bunyips consists of six members:

- Fiona is Alice's mother and grandmother;
- Alice is Eddie's grandmother and half-sister;
- David is Cathy's father and great-grandfather; and
- Brian is Eddie's father and great-grandfather.

If Eddie is 8 years old, how old is Fiona?

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