Question 1   **20 marks**

In a pack of *hundreds & thousands*, for every thousand red *hundreds & thousands*, there are a hundred green *hundreds & thousands*. For every thousand green *hundreds & thousands*, there are a hundred blue *hundreds & thousands*. If I have ten thousand thousand red *hundreds & thousands*, how many hundred blue *hundreds & thousands* do I have?

(*Hundreds & thousands* are sprinkles that go on top of cakes.)

Question 1   **20 marks**

In a pack of *hundreds & thousands*, for every thousand red *hundreds & thousands*, there are a hundred green *hundreds & thousands*. For every thousand green *hundreds & thousands*, there are a hundred blue *hundreds & thousands*. If I have ten thousand thousand red *hundreds & thousands*, how many hundred blue *hundreds & thousands* do I have?

(*Hundreds & thousands* are sprinkles that go on top of cakes.)

Question 2 **20 marks**

A flower is formed by four circles of radius 1 centred at the points $(0, 1)$, $(1, 0)$, $(0, -1)$ and $(-1, 0)$. What is its perimeter?

Question 3   **20 marks**

Arrange the following numbers in increasing order:

$$\sqrt[2]{2} \qquad \sqrt[3]{3} \qquad \sqrt[5]{5}.$$

Question 4   **20 marks**

Kwok and Thara are running in a 100m race. Since Kwok is 50% faster than
Thara, he gives him a 4m head start, but still finishes 4 seconds faster. How
fast is Kwok in metres per second?

Question 5 **CHANGE RUNNER NOW** **20 marks**

A natural number x is said to be Fibbinary if, in its binary representation, the addition 2x + x has no carries. For example, 7 is not Fibbinary, since the addition

$$
\begin{array}{rr}
7 = & 1\,1\,1 \\
2 \times 7 = & 1\,1\,1\,0 \\
\hline
& 1\,0\,1\,0\,1
\end{array}
$$

requires carrying in the 2's, 4's and 8's columns. What is the 9th Fibbinary number?

Question 6   **30 marks**

Ray is playing darts. He always lands his dart in the 20-point section. He
has a 30% chance of hitting the 2x multiplier, and a 30% chance of hitting
the 3x multiplier. How many darts is Ray expected to throw to score at least
60?

Question 7   **30 marks**

How many ways are there of colouring a cube with 6 colours such that each face is a different colour? (Two colourings are considered the same if one is a rotation of the other.)

Question 8   **30 marks**

How many 3 digit numbers have digit sum of 10? (For example, 834 has digit sum of $8 + 3 + 4 = 15$.)

Question 9   **40 marks**

Find
$$1 \times \binom{10}{1} + 2 \times \binom{10}{2} + 3 \times \binom{10}{3} + \ldots + 10 \times \binom{10}{10}.$$

Question 10 **30 marks**

In a $3 \times 3 \times 3$ cubic lattice, how many lines pass through three lattice points?

Question 11   **CHANGE RUNNER NOW**   **30 marks**

A whole number is called a *brick-trick* if it can be expressed in the form $m^{n+1} - m^n - m^{n-1}$, where $m, n \in \mathbb{N}$. For example, $2009 = 7^4 - 7^3 - 7^2$ is a four digit brick-trick. What is the smallest positive n such that no four-digit *brick-tricks* exists in the form $m^{n+1} - m^n - m^{n-1}$?

Question 12   **40 marks**

Find all pairs of natural numbers (n, p) such that $p < n$ is a prime and $n! \times p$ is a square. (Give both p and n.)

Question 13   40 marks

What is the largest volume that can be constructed by cutting a sector out of a unit disc and bending to make a cone? (A sector is a part of a disc bounded by two radial lines and the arc between them.)

Question 14   **40 marks**

Find all solutions to the equation $x^2 + x + 29 = y^2$, where x and y are positive integers.

Question 15   **CHANGE RUNNER NOW**   **40 marks**

A sequence of pairs (a_n, b_n) is given by the recurrence $a_{n+1} = (a_n + b_n)/2$ and $b_{n+1} = 2a_n b_n/(a_n + b_n)$ for $n \geq 0$, with initial values $a_0 = 6$ and $b_0 = 24$. What is the limit $a_\infty + b_\infty$?

Question 16   **50 marks**

Given a parallelogram $ABCD$, let $BC = 1$, $\angle BAD = \frac{5\pi}{12}$ and let $\triangle ABD$ be a triangle where each angle of the triangle is acute. What is the maximum length possible for AB such that four unit radius circles centered at A, B, C and D will cover $ABCD$? (Your answer may contain trigonometric functions, but must be in simplest form.)

Question 17   **50 marks**

Evaluate

$$\sum_{j=0}^{\infty} j \binom{n}{j}^2$$

(Your answer may contain binomial coefficients, but must be in simplest form.)

Question 18   **50 marks**
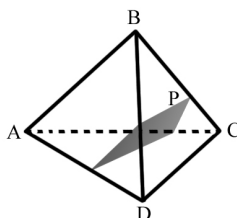
Find all degree 10 real polynomials f which satisfy

$$f(x)f(2x^2) = f(2x^3 + x).$$

Question 19   **50 marks**

Given a tetrahedron $ABCD$ with $AB = 15$ and $CD = 13$. Let P denote a plane parallel to both AB and CD so that the distance between AB and P is $3\sqrt{2}$ and the distance between CD and P is 3. Lastly, when AB and CD are projected on to P, the two projected lines intersect, with intersection angle $\frac{\pi}{5}$. What is the ratio of the volumes of the two pieces of the AB side of the tetrahedron compared to the CD half?
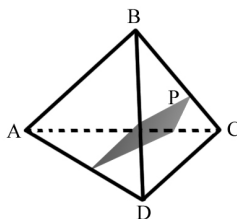
Question 20   **FINAL QUESTION!**   **50 marks**

Han and Yi are playing badminton. The first player to reach 11 points with at least a 2-point lead wins the match. Han and Yi are equally bad and have a 50% chance of winning each point. Assuming Han loses, what is his expected score?