Question 1 **20 marks**

Gil has a maths test coming up, which has 1,000,000 questions and goes for one hour. Without coffee, Gil takes one minute to solve each question, with 98% accuracy. With each successive cup of coffee, Gil's speed doubles, and his accuracy is reduced by 10% (e.g. 80% goes to 70%). What is the optimal whole number of cups of coffee for Gil to drink, in order to maximise the number of questions he answers correctly?

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Question 2 **20 marks**

What is the maximum score you can get in the 2011 University Maths Olympics?

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Question 3 **20 marks**

Richard and Jiaying have a square vegetarian pizza with side length 20 cm. Richard makes one straight cut of length at most 10 cm, and Jiaying takes the larger piece. What is the largest piece of pizza Richard can have?

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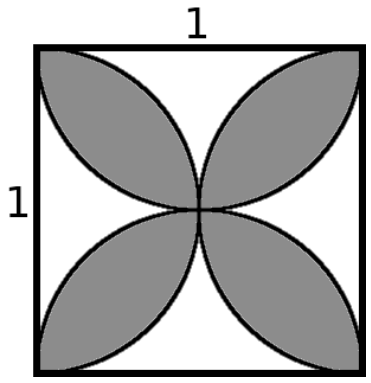
Question 4 **20 marks**

Four semicircles of diameter 1 are placed along each edge of a 1×1 square as shown below. What is the area of the shaded region?



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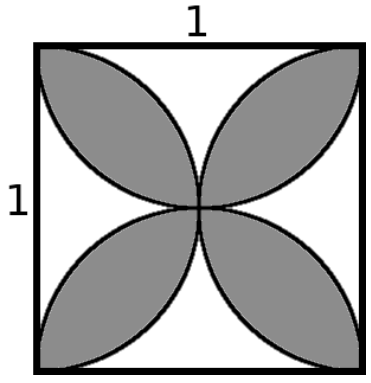

Question 5 **CHANGE RUNNER NOW** **20 marks**

Place the following in ascending order:

$$a = \frac{1001}{1002}, \qquad b = \frac{2001}{2003}, \qquad c = \frac{3002}{3005}$$

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Question 6 **30 marks**

Lawrence the lecturer only uses blackboard and chalk when teaching. With a full piece of chalk, Lawrence can write on the board for a full 20 minutes. However, some pieces of chalk have a hard tip that doesnt write easily, which Lawrence will snap off, after which he can only write for 15 minutes. Given that the chalk is faulty with probability 0.2, what is the probability that Lawrence can get through a full 50 minute lecture with just three pieces of chalk?

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Question 7 **30 marks**

Find the last digit of $(6^{66} + 66^6)^{666}$.

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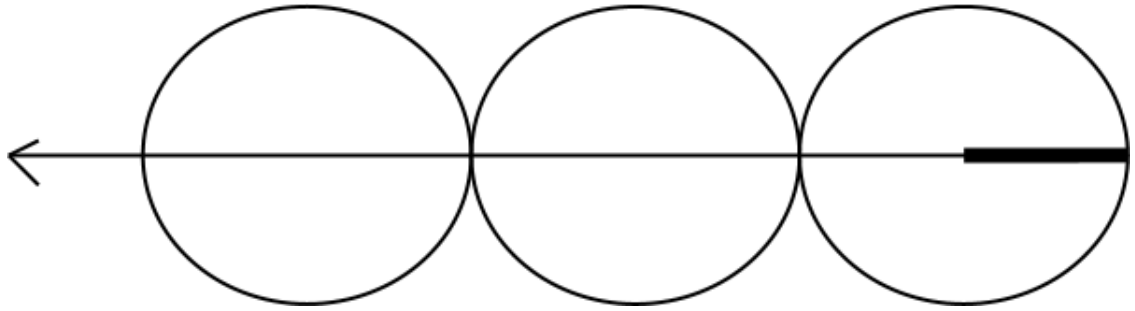
Question 8 **30 marks**

A table has an arrow drawn on it. On top of the table are three wheels, one of which is painted with a strip, as shown in the diagram below. After rolling the striped wheel over the other two so that it is now on the left, what angle will the strip make with the arrow?



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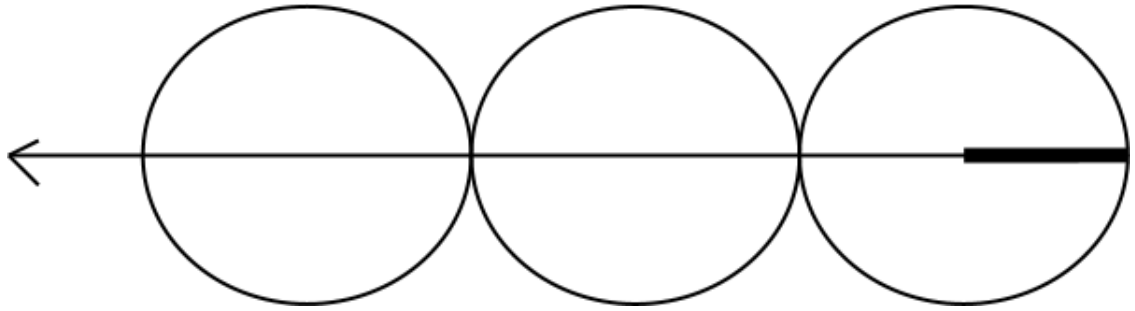

Question 9 **30 marks**

How many distinct words can be made by rearranging the letters in DANABANANA?

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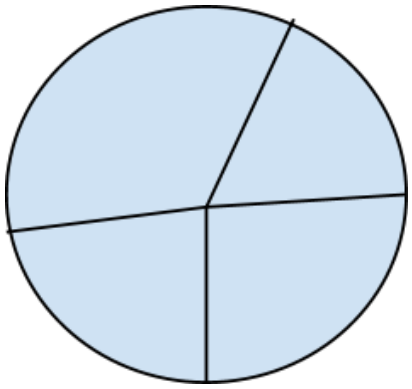
Question 10 **CHANGE RUNNER NOW** **30 marks**

Lu runs around a pie-shaped park. The park consists of a circular path, as well as straight paths to the centre which divide the circle into four arcs of lengths 13, 9, 20 and 11 furlongs. Lu can start and finish anywhere she likes, and insists on running along each path at least once. What is the shortest distance she can run?



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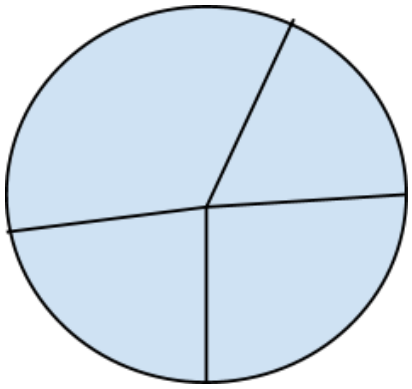

Question 11 **40 marks**

What is the largest multiple of 99 that can be made with the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, where each digit is used at most once?

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Question 12 **40 marks**

A wire of length $4 + 3\pi$ is to be made into a rectangle with a circle inside it so that the circle just touches the top and bottom edges. What is the maximum area that can be enclosed between the rectangle and the circle?

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Question 13 **40 marks**

How many non-negative integer solutions does the equation $x + 2y + 3z = 300$ have?

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Question 14 **40 marks**

Let d_n be the largest odd divisor of n. Find

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Question 15 **CHANGE RUNNER NOW** **40 marks**

A sequence $(a_0, a_1, a_2, \ldots) = (1, 7, 8, 49, 50, 56, 57, \ldots)$ is created by taking all the positive integers that are sums of distinct powers of 7. Similarly, $(b_0, b_1, b_2, \ldots) = (1, 6, 7, 36, 37, 42, 43, \ldots)$ is created by taking all the positive integers that are sums of distinct powers of 6. Find $a_{32} - b_{32}$.

Question 15 **CHANGE RUNNER NOW** **40 marks**

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Question 16 **50 marks**

Han has a machine that controls the rain in Park Orchards. Button A makes it rain with probability 0.6, while Button B makes it rain with probability 0.4. Han then plays the following game: Starting with Button A, he presses one button each day until it rains two days in a row. If he pressed Button A yesterday and it rained, he presses Button B today, otherwise he presses Button A. What is the expected number of days this will continue?

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Question 17 **50 marks**

Jeff owns a sheep who lives with him. One kilometre north of Jeff's sheep, is a castle made of ore, owned by Sam. Another kilometer east of Sam resides Han, in the palisade made of wood. Lastly, 1km south of Han is where the tricky Yi lives, in a house of bricks. In order to grow wheat, the four of them must fight for land. All four want a circular paddock (centred at their respective residences) with a radius of 1km. What is the area of land that is in dispute by all four of them?

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Question 18 **50 marks**

TriThang and Han play a game. Han plays first, announcing the number $X = 6$. After this, they take turns, starting with TriThang. On each player's turn, he rolls a fair dice, and multiplies the previous value of X by the number just rolled, to get a new value of X. The first player to reach $X \geq 42$ wins. What is the probability that TriThang wins?

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Question 19 **50 marks**

Five vertices are given, and Giles starts at one of them. Each time he's at a vertex, Giles draws an edge to another vertex, until he's drawn 10 edges. What is the probability that he now has a complete graph (a graph where every pair of vertices has an edge between them)?

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Question 20 **FINAL QUESTION** **50 marks**

Simplify:

$$\sum_{i=0}^{m}\sum_{j=0}^{n}\binom{m}{i}\binom{n}{j}\binom{m+n}{i+j}\prod_{k=0}^{m+n-1}\frac{1}{k+\frac{1}{2}}$$

Question 20 **FINAL QUESTION** **50 marks**

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