Question 1 **20 marks**

The first two numbers in a sequence are 5 and 6. Each successive number is the sum of the digits of the product of the two numbers before it. What is the 2013th number in this sequence?
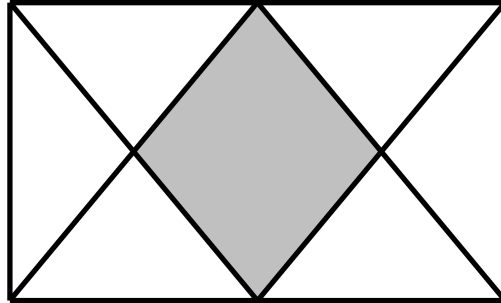
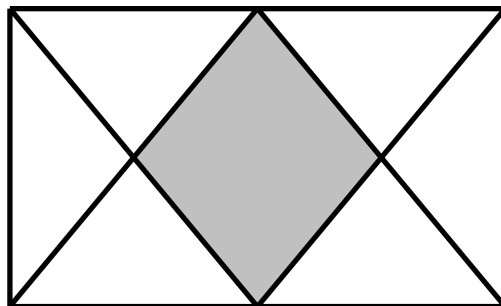**Question 2**   **20 marks**

The rectangle shown has base length 10 and height 6. If the points on the top and bottom of the rectangle are midpoints, what is the area of the shaded region?

Question 3   **20 marks**

Dougal, Jinghan, Matt and Jenny wish to finish a cake together. If Matt has twice as many pieces as Jinghan, who has 3 times as many pieces as Dougal, who has half as much as Jenny. What is the minimum number of pieces the cake could have been cut into?
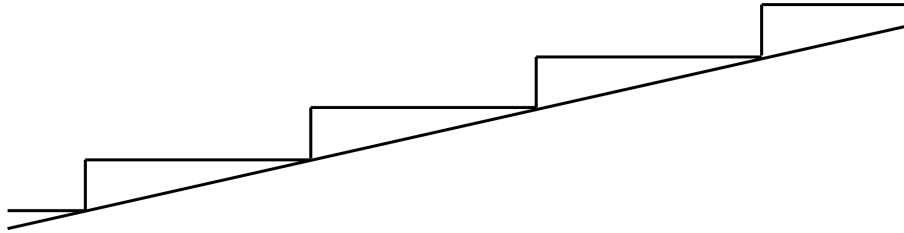
Question 4   **20 marks**
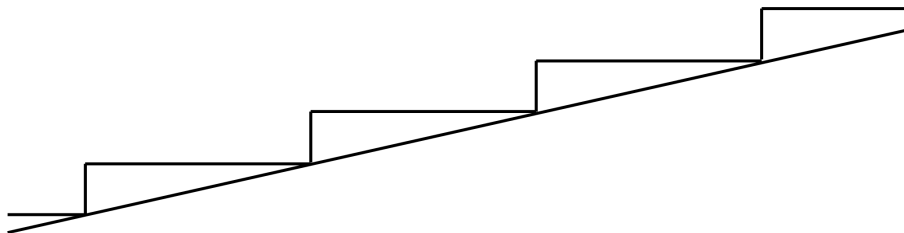
There are a set of steps, each one 40 cm long and 9 cm high. Next to it is a ramp of constant slope which touches the bottom of each step. Two ants start at the top of the steps, one goes down via the steps and the other the ramp. If one ant travels 192 cm more than the other to get to the bottom, how many steps are there?

Question 5   **CHANGE RUNNER NOW**   **20 marks**

Order $3^{-555}, 4^{-444}, 5^{-333}$ in increasing order.

Question 6 **30 marks**

Garry the cockroach is on one corner of a cube and wants to walk to the opposite corner. In how many ways can he do this by only walking along three edges of the cube?

Question 7   **30 marks**

How many real solutions are there to the equations

$$\frac{a-b}{c} = \frac{b-c}{a} = \frac{c-a}{b} = 1$$

Question 8   **30 marks**

Suppose a and b are positive integers satisfying

$$\log_2(\log_{2^a}(\log_{2^b}(2^{256}))) = 0.$$

What is $a + \log_2(b)$?

Question 9   **30 marks**

MUMS has decided to run a new competition called the EMO (Extreme Maths Olympics).
In the EMO, there are 25 questions. Questions 1-5 each worth 10 points, questions 6-10
each worth 20 points, questions 11-15 each worth 30 points, questions 16-20 each worth 40
points and Questions 21-25 each worth 50 points.
How many different possible scores are there in the EMO?

Question 10   **CHANGE RUNNER NOW**   **30 marks**

Han wishes to tile a large equilateral triangle region of edge length 2013 with small equilateral triangles of edge length 1. He has only black or white tiles, and intends to place 3, 10 and 13 rows of black tiles immediately within the three sides of this region, tiling everything-else with white tiles. How many black tiles will Han need?

Question 11   **40 marks**

How many ways are there to put the numbers from 1 to 8 on an octahedral (8-sided) die so that no two adjacent numbers share a common prime factor? (Consider two numberings equivalent if they can be rotated into each other.)
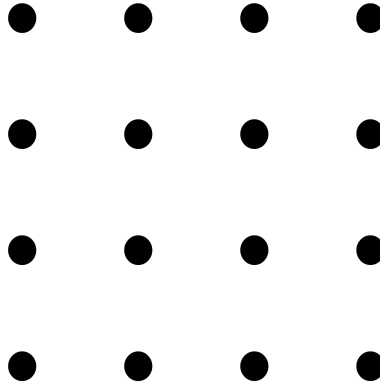
Question 12   **40 marks**
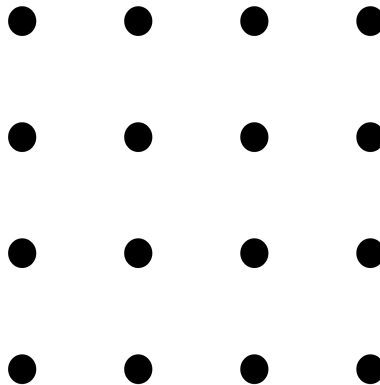
Three circles of equal radius are drawn so that every dot in the 4×4 array of dots lies on one of the circles. What is the diameter of each circle?

Question 13   **40 marks**

A right-angled triangle has perimeter 28 and area 21, what is the length of its longest side?

Question 14   **40 marks**
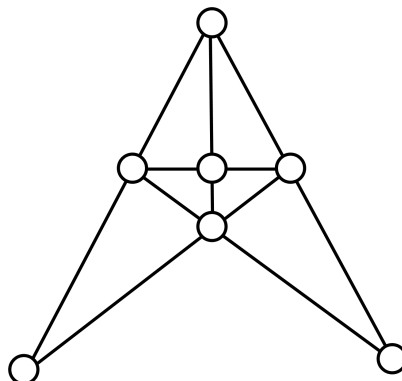
Simplify $\sqrt[3]{3\sqrt{3}+5} - \sqrt[3]{3\sqrt{3}-5}$.
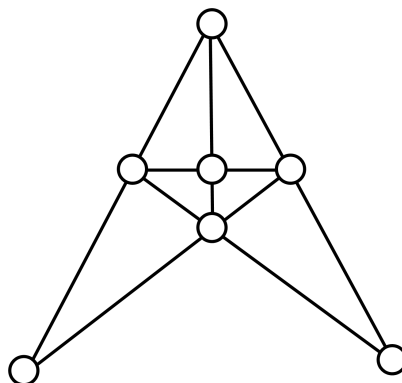
Question 15   **CHANGE RUNNER NOW**   **40 marks**

How many ways are there of filling in the circles with positive integers so that the sum of the three numbers on each line is 2013?

Question 16   **50 marks**

Jinghan places 5 positive integers around a circle so that every two adjacent numbers have a common factor greater than one, but no two non-adjacent numbers have a common factor greater than 1. What is the smallest possible sum of these five numbers?

Question 17 50 marks

Define a sequence by $a_1 = 2$ and $a_{n+1} = \sqrt{2 - 2\sqrt{1 - (a_n/2)^2}}$ for $n \geq 1$
What is $\lim_{n\to\infty} 2^n a_n$?

Question 18   **50 marks**

Three of the four faces of a tetrahedron with integer edge lengths are similar triangles, but no two of them are congruent. What is the minimum possible perimeter of the fourth triangle?

Question 19   **50 marks**

The number 1 is written on one of the whiteboards in the MUMS room. If Yi comes into the room, he replaces the number on the board with that number plus 1. And if Trithang comes in, he replaces the number with that number cubed. If the number is changed at most 12 times by the end of the day, how many possible values can it take?

Mel has 8 litres of water distributed amongst 3 enormous buckets placed in a row. There are 6 litres in the left-most one and 1 litre in each of the other two. There are two types of moves which Mel can do:

- Choose a positive real number x and remove x litres of water from one of the buckets (with at least x litres in it), then multiply the number of litres in the bucket directly to the right of that by x

- Remove one bucket completely (and all the water in it) and change the order of the other buckets

What is the largest possible amount of water Mel can end up with in a single bucket?