Question 1 **20 marks**

It takes Damian 1 minute to walk up an escalator, and Ben 2 minutes without walking. If the escalator moves at 5 km/hr, how quickly does Damian walk?

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Question 2 **20 marks**

Damian is trying to find a new house, but is quite picky. He would like to live in a house whose number is equal to six times the sum of its own digits (in the usual decimal expansion). What is the lowest house number that he can live in?

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Paul flips 10 (fair) coins. What is the probability that all 10 coins come up tails?

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Question 5 **CHANGE RUNNER NOW** **20 marks**

Sarah has sent a proposal to the General Conference on Weights and Measures that the standard unit of length be changed to the General Earth Unit (geu). The geu has the interesting property that the (volume of the Earth in geu)^3 is equal to the (surface area of the Earth in geu)^2. What is the radius of the Earth in geu?

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Question 6 **30 marks**

Jenny is standing is a very long corridor with thousands of numbered closed doors leading into it. First, Jenny goes through the corridor opening every door. Then she returns to the first door and does another walk of the corridor. This time, she stops at every second door and opens it if it is closed and closes it if it is open. She continues to do these walks, changing the status of every third door, fourth door and so on. When Jenny is finished, what is the number of the 100th open door?

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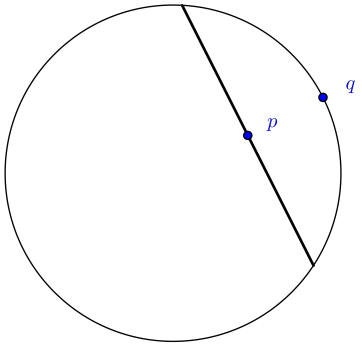
Question 7 **30 marks**

Declan picks a point p uniformly at random from the interior of the unit circle C. He then picks the point q which is the point on the circumference of C closest to p. Lastly, he draws a chord through p that is parallel to the tangent of C at q. What is the probability that this chord has length greater than 1?



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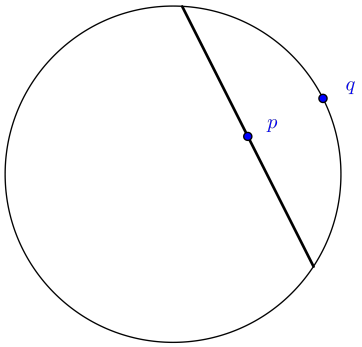

Question 8 **30 marks**

Yi has a pair of loaded dice. The first die lands on a value n with probability $1/2$ and all other values with uniform probability. The second die lands on a value m with probability $1/3$ and all other values with uniform probability.
If Yi throws his dice, their sum has an expected value of $83/10$. What is the product $m \times n$?

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Question 9 **30 marks**

Ben Hague is being vague about the plague. He has 100 lab rats, each numbered with a serial number from 1–100. A few of his lab rats have died from the plague. If he says that the serial number of the dead rats satisfy 1 mod 3, 2 mod 4, and 4 mod 5, what are the serial numbers of the dead rats?

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Question 10 **CHANGE RUNNER NOW** **30 marks**

Consider two spheres of radius 1/4, whose centres are at antipodal points of the unit sphere in $\mathbb{R}^3$. Now take two antipodal points on those two spheres so that you have four points in total. How many choices of four points (ignore reordering) are there so that the four points have the same first two coordinates? (i.e. $(x, y, z)(a, b, c)(e, f, g)(h, i, j)$ should satisfy, $x = a = e = h$ and $y = b = f = i$.)

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Question 11 **40 marks**

TriThang permutes the numbers 1–10 at random. He first reads the first number in the permuted list. From then on, having read a number n, he next reads the number in position n of the permuted list. What is the probability that TriThang will read all the numbers 1–10?

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Question 12 **40 marks**

How many ways are there to place 3 rooks on a 3×3 chessboard so that each square is either occupied or threatened by at least one rook?

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Question 13 **40 marks**

Mel looks at a clock and sees that the time is 4:15. How many minutes will Mel have to wait before the minute hand crosses the hour hand? (give an exact, unrounded answer)

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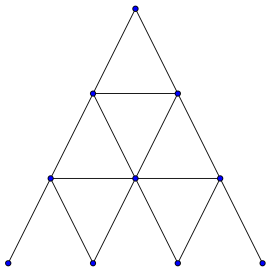
Question 14 **40 marks**

TriThang wants to build a very tall card house, so he buys 10 whole packs of 52 cards. How many stories does the highest card house that he can build have?



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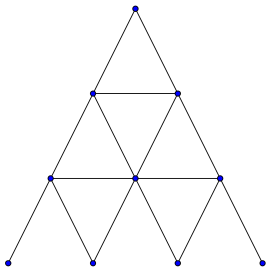
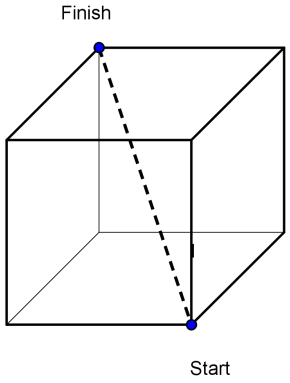


Question 15 **CHANGE RUNNER NOW** **20 marks**

A termite and a black ant are on one vertex of a cube. Black ants love to eat termites, so the termite decides to escape by burrowing through the centre of the cube through to the opposite vertex at a constant speed. The ant sets off running along the surface of the cube to the opposite vertex and manages to catch the termite the instant it emerges from the cube. If the ant took the shortest path connecting the vartices, find the ratio of the ant's speed to the termite's speed.



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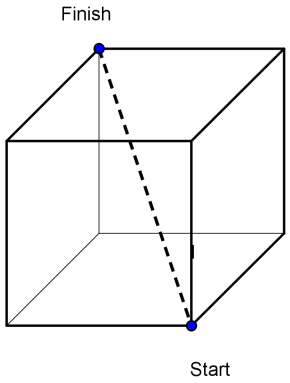

Question 16 **50 marks**

Yi is packing himself a lunch consisting of three oranges of radius 3 cm each. He arranges them in a triangle shape and wraps them in glad wrap. What is the area of the glad wrap that Yi uses?

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Question 17 **50 marks**

Four cannonballs of radius 10 are stacked in a pyramid. What is the radius of the largest ball that can fit in the space between the cannonballs.

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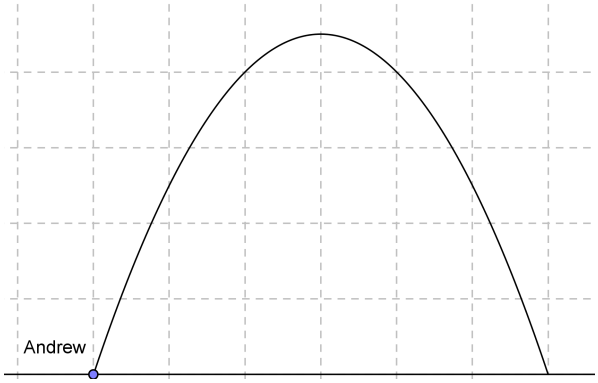
Question 18 **50 marks**

Andrew is shooting an arrow in an alternate reality where things fall at 10 m/s^2. Suppose he is shooting the arrow East at an angle of 60° from the horizontal, and at a speed of 10 m/s. If we place a grid with a spacing of 1 m, the x-axis pointing East, and the y-axis pointing up, what is the value of $\frac{d^2y}{dx^2}$ for the trajectory of the arrow.



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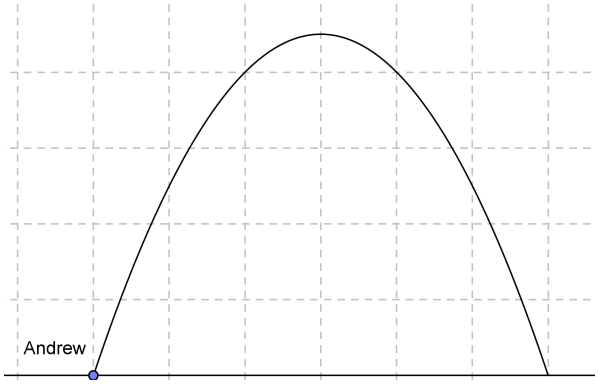
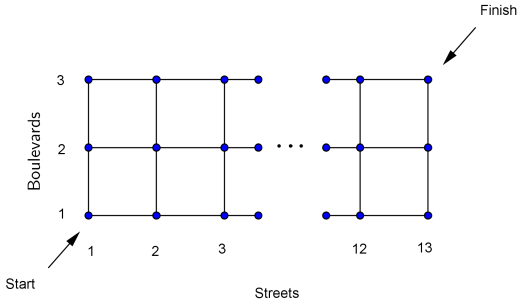


Question 19 **50 marks**

MUMSbourne is a city built according to a grid. Specifically, it consists of 13 streets running North-South and 3 boulevards running East-West so that each North-South road crosses each East-West Boulevard exactly once, and at right-angles. Ben wants to visit all of MUMSbourne's intersections, but unfortunately his car can only turn left. Starting from the corner of 1st st and 1st blvd and ending on the corner of 13th st and 3rd blvd, how many different tours (i.e. routes visiting all the intersections) of MUMSbourne can Ben make without driving over the same section of road twice?



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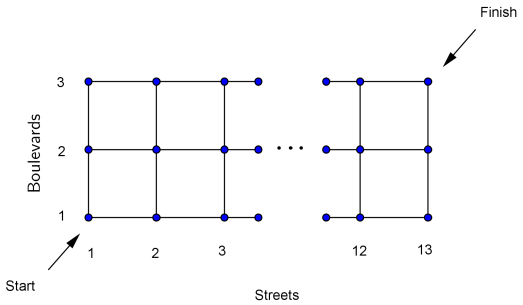

Question 20 **50 marks**

Mel is standing at a point A. He walks 1 km to another point B. Next, he walks to a point C that is equidistant from A and B. Next, he walks to a point D that is equidistant from A, B, C. Finally, he returns from point D to point A. If Mel wants to minimise his walking distance, how far should he walk from B to C?

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