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$$\begin{array}{r} X \quad Y \quad Y \\ +X \quad Y \quad Y \\ \hline Y \quad Y \quad Z. \end{array}$$

What are X, Y and Z?

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Today's date (6/10/2015) exhibits two properties:

1. The sum of the last two digits of the year is the date ($1 + 5 = 6$).
2. The first two digits of the year, treated as one number, is exactly double the month ($2 \times 10 = 20$).

Not including today, how many dates with these properties have occurred since 1/1/1900?

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Question 5 **CHANGE RUNNER NOW** **20 marks**

Find the larger number in 222^{333} and 333^{222}.

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Tien is thinking of the smallest possible positive number such that when it's doubled, the sum of the digits of the new number is equal to half of the original. What number is Tien thinking of?

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Question 8 **30 marks**

How many zeroes are at the end of 2015! ?

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Question 9 **30 marks**

Consider the following three 6-sided dice:

- Dice A has sides $2, 2, 4, 4, 9, 9$;
- Dice B has sides $1, 1, 6, 6, 8, 8$;
- Dice C has sides $3, 3, 5, 5, 7, 7$.

These set of dice have the curious property that the probability that Dice A rolls a number higher than Dice B is the same the probability that Dice B rolls a number higher than Dice C and is the same as the probability that Dice C rolls a number higher than Dice A. What is this probability?

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Question 10 **CHANGE RUNNER NOW** **30 marks**

Paul the alcoholic tries to go home after a long night of drinking. With each step, he either goes forward or backwards with equal probability. After 8 steps, what is the chance that he has gotten strictly more than 4 steps from his starting point?

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Question 11 **40 marks**

How many ways are there of (possibly unevenly) dealing 8 different cards to 3 people, so that everyone gets at least 1 card?

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Question 12 **40 marks**

The lengths of the three sides of a triangle are given by $x-2, x$ and $x+2$. If its area is 6, what is its perimeter?

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Question 13 **40 marks**

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Find all positive integer solutions $a, b, c \in \mathbb{Z}_+$ to the equations

$$a^3 - b^3 - c^3 = 3abc \text{ and } a^2 = 2(b + c).$$

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Question 15 **CHANGE RUNNER NOW** **20 marks**

Given a triangle $\triangle ABC$, let M denote the midpoint of B and C, and O denote its orthocentre, and let P denote its circumcentre. If B, O, P, C lie on the same circle, what is $\frac{\overline{BC}}{\overline{PM}}$?

(Hint: The orthocentre O is where the three altitudes of the triangle meet, and the circumcentre P is the centre of the smallest circle containing a triangle.)

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Question 16 **50 marks**

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$$\prod_{k=1}^{7} \cos\left(\frac{k\pi}{15}\right).$$

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Question 17 **50 marks**

Adib, Tien, Damian, Ben and Aaron are standing in a pentagon passing a pair of balls around. Every five seconds, any person who has a ball passes it to the person immedately to their left with probability $\frac{1}{2}$ and to the person to their right with probability $\frac{1}{2}$.

If Adib and Tien, who are standing next to each other start with the balls, what is the expected number of passes before the first time that someone is holding both balls?

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Question 18 **50 marks**

What is the area of the largest square that you can fit inside a regular hexagon with length 1 edges? Express your answer in the form $a + b\sqrt{c}$, where a, b, c are rational numbers.

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Question 19 **50 marks**

Define a function $Fu : \mathbb{Z} \to \mathbb{Z}$, where

$$Fu(n) := \begin{cases} n + 1820, & \text{if } n < 2015 \\ n - 2210, & \text{if } n \geq 2015. \end{cases}$$

What is the smallest m so that $Fu^m(0) = 0$?

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Question 20 **50 marks**

Line segments are distributed on an infinite two-dimensional integer lattice in the following way: for each pair of (non-diagonally) adjacent lattice points, there is a segment between the points with probability $\frac{1}{2}$.

What is the probability that there is an unbroken path of segments connecting the points $(0, 0)$ and $(0, 1)$?

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