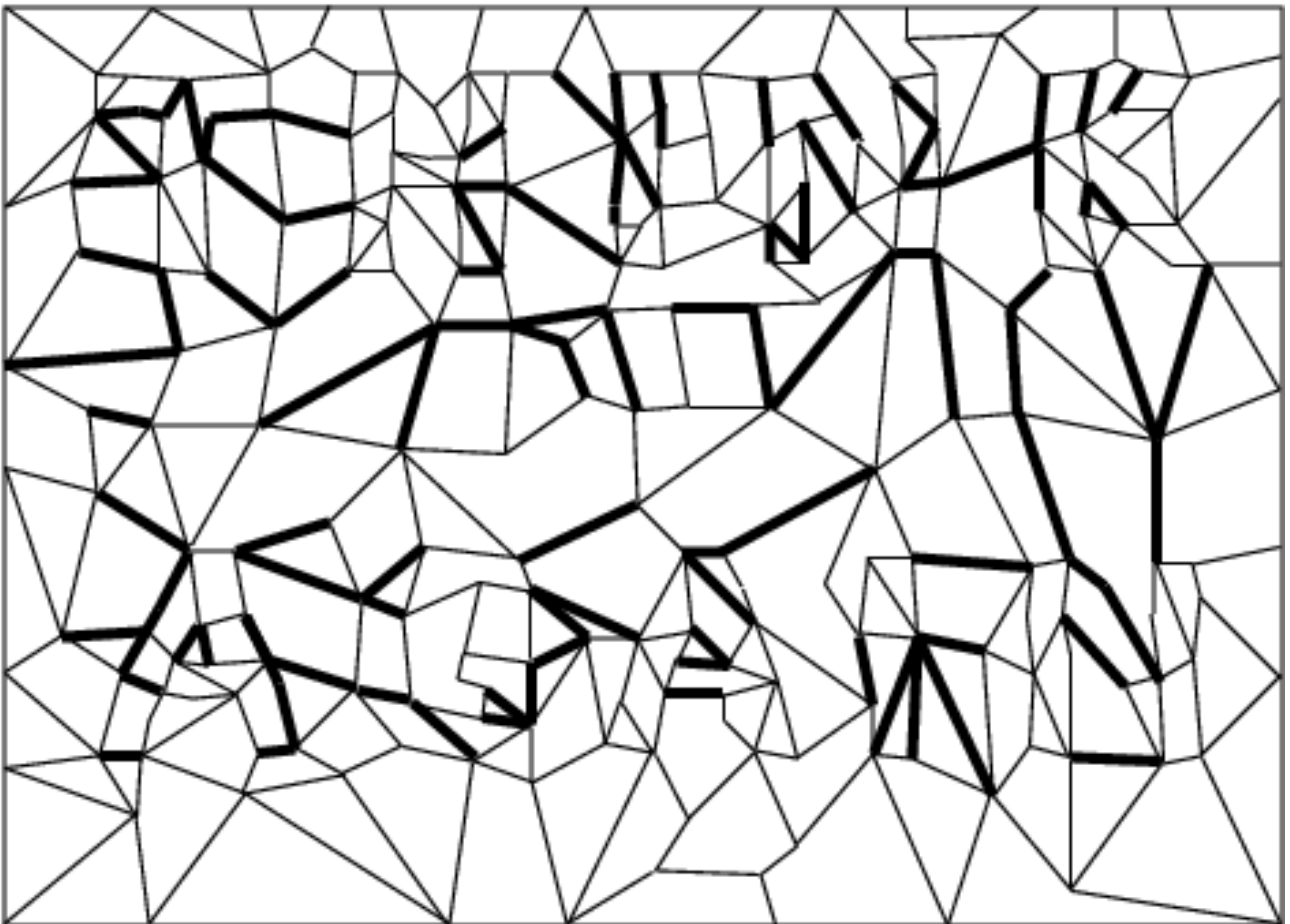

Paradox

Issue 1, 2005

THE MAGAZINE OF THE MELBOURNE UNIVERSITY MATHEMATICS AND STATISTICS SOCIETY



MUMS

PRESIDENT: Damjan Vukcevic
damjan@ms.unimelb.edu.au

VICE-PRESIDENT: Julian Assange
j.assange@ugrad.unimelb.edu.au

TREASURER: Andrew Kwok
a.kwok2@ugrad.unimelb.edu.au

SECRETARY: Joanna Cheng
j.cheng1@ugrad.unimelb.edu.au

EDUCATION OFFICER: Maurice Chiodo
m.chiodo@ugrad.unimelb.edu.au

PUBLICITY OFFICER: James Saunderson
j.saunderson@ugrad.unimelb.edu.au

EDITOR OF Paradox: Nick Sheridan
n.sheridan@ugrad.unimelb.edu.au

1ST YEAR: Bobbi Ramchen
maple_leaf9@hotmail.com

2ND YEAR: Michael de Graaf
m.degraaf@ugrad.unimelb.edu.au

3RD YEAR: Daniel Yeow
d.yeow1@ugrad.unimelb.edu.au

HONOURS: Chris Goddard
c.goddard@ugrad.unimelb.edu.au

POSTGRADUATE: Norm Do
norm@ms.unimelb.edu.au

WEB PAGE: <http://www.ms.unimelb.edu.au/~mums>

MUMS EMAIL: mums@ms.unimelb.edu.au

PHONE: (03) 8344 3385

Paradox

EDITOR: Nick Sheridan

LAYOUT: Stephen Farrar

PRINTER: Daniel Yeow

WEB PAGE: <http://www.ms.unimelb.edu.au/~paradox>

E-MAIL: paradox@ms.unimelb.edu.au

PRINTED: March, 2005

Words From the Editor

Once again, we welcome every variety of maths-lover, from the pure maths gurus to the dilettantes, to the latest edition of *Paradox*. *Paradox*, for those of you who don't read the social glossies, is the prestigious magazine of the Melbourne University Mathematics and Statistics Society, better known as MUMS, and we strive to offer mathematical delicacies of all description.

In this edition, you will find reproduced an amusing exchange of papers between the mathematicians B. Mandelbrot and H.A. Simon, demonstrating that even mathematicians get hot under the collar sometimes. We have an article about the vacation scholarship program from Stefan Rampertshammer, as well as a description from James Zhao of how to calculate square roots by hand, an art that has come close to extinction. As always, *Paradox* is interspersed with mathematical jokes and morsels of all description, including the Pizza Theorems and an abridged list of all even primes, which appears on every page for your convenience. The article called 'The Probability of Getting Wet', which looks at a question from the entrance exam for Cambridge, was contributed by Willie Yong and Jim Boyd of Singapore, giving this edition a somewhat international flavour. Willie edits a journal called the Mathematics and Informatics Quarterly, which has a website at olympiads.win.tue.nl/ioi/misc/miq.html. Bruce Craven has composed an ode to the bitter-sweet experience that is learning topology.

The crowning glory of any edition of *Paradox* is its mathematical cartoon. This time, our cartoon was contributed by Chris Tuffley of the University of California, and our noble genus-two hero is Sammy the Graduate Student. I think you will agree that Sammy surpasses all previous mathematical cartoons on the scales of punnery and nerdery. More Sammy cartoons (including the only known proof of the h-Cobordism theorem by cartoon) are available at www.math.ucdavis.edu/~tuffley/sammy/.

It has become customary for the *Paradox* editorial to finish with an exhortation to students to write articles for us so that we can publish them. This has not, too date, proven to be particularly successful, and most articles still arise out of the time-honoured method of threats and/or emotional blackmail of the editor's acquaintances. We nevertheless persist in this convention. It is beneficial and fun to write an article for *Paradox* because you will research an area of maths which you otherwise would not get to know much about, or you might decide to interview and get to know someone you otherwise wouldn't. You will also

get your work published, which is fun. Even if you don't have any idea what you would write an article about, we would very much like to hear from anyone who is interested. We are very committed to publishing articles by students, and to encouraging first-time article writers (two of this edition's articles were contributed by students who had not written for *Paradox* before). You can email us at either of the addresses below.

—Nick Sheridan

`paradox@ms.unimelb.edu.au`

`n.sheridan@ugrad.unimelb.edu.au`

President's Words

I would like to welcome you all, especially those of you in first year, to a new year of MUMS. Once again, we will be running our traditional activities: seminars, trivia nights, competitions and, of course, publishing *Paradox*. The fortnightly seminars are the 'bread and butter' of the MUMS calendar, where we invite everyone from current students to distinguished professors to give a talk on their favourite weird and quirky topic.

The idea is to see something that would not be seen in lectures, but is nonetheless highly interesting, or at least entertaining. The seminars are also a good place to meet other like-minded people, especially since we serve refreshments afterwards.

A special event to look out for this semester is the Puzzle Hunt, back again by popular demand. After the amount of interest this event generated last year, it would be a crime not to run it again. Those of you who entered a team in last year's Puzzle Hunt will have experienced the adrenaline rush of competitive puzzling. To the newcomers: beware, it is very addictive!

—Damjan Vukcevic

<p>The First Pizza Theorem: If a circular pizza is divided into 8, 12, 16... slices by making cuts at equal angles from an arbitrary point inside the pizza, then the sums of the areas of alternate slices are equal.</p>

Mandelbrot and Simon's Debate

- HA Simon (1955), "On a class of skew distribution functions", *Biometrika* <<http://www.jstor.org/journals/00063444.html>>, 42: 425 – 440. [PDF <<http://linkage.rockefeller.edu/wli/zipf/simon55.pdf>>]
- BB Mandelbrot, "A note on a class of skew distribution function. Analysis and critique of a paper by H.A. Simon", *Information and Control*, 2, 90 – 99 (1959). [ABSTRACT: This note is a discussion of H.A. Simon's model (1955) concerning the class of frequency distributions generally associated with the name of G.K. Zipf. The main purpose is to show that Simon's model is analytically circular in the case of the linguistic laws of Estoup-Zipf and Willis-Yule. Insofar as the economic law of Pareto is concerned, Simon has himself noted that his model is a particular case of that of Champernowne; this is correct, with some reservation. A simplified version of Simon's model is included.]
- HA Simon, "Some further notes on a class of skew distribution functions", *Information and Control*, 3, 80 – 88 (1960). [ABSTRACT: This note takes issue with a recent criticism by Dr. B. Mandelbrot of a certain stochastic model to explain word-frequency data. Dr. Mandelbrot's principal empirical and mathematical objections to the model are shown to be unfounded. A central question is whether the basic parameter of the distributions is larger or smaller than unity. The empirical data show it is almost always very close to unity, sometimes slightly larger, sometimes smaller. Simple stochastic models can be constructed for either case, and give a special status, as a limiting case, to instances where the parameter is unity. More generally, the empirical data can be explained by two types of stochastic models as well as by models assuming efficient information coding. The three types of models are briefly characterized and compared.]
- BB Mandelbrot, "Final note on a class of skew distribution functions: analysis and critique of a model due to H.A. Simon", *Information and Control*, 4, 198 – 216 (1961). [ABSTRACT: We shall restate in detail our 1959 objections to Simon's 1955 model for the Pareto-Yule-Zipf distribution. Our objections are valid quite irrespectively of the sign of $p - 1$, so that most of Simon's (1960) reply was irrelevant. We shall also analyze the other points brought up in that reply.]
- HA Simon, "Reply to 'final note' by Benoit Mandelbrot", *Information and Control*, 4, 217 – 223 (1961). [ABSTRACT: Dr. Mandelbrot's original

objection (1959) to using the Yule process to explain the phenomena of word frequencies were refuted in Simon (1960), and are now mostly abandoned. The present “reply” refutes the almost entirely new arguments introduced by Dr. Mandelbrot in his “final note”, and demonstrates again the adequacy of the models in (1955).]

- BB Mandelbrot, “Post scriptum to ‘final note’”, *Information and Control*, 4, 300 – 304 (1961). [ABSTRACT: My criticism has not changed since I first had the privilege of commenting upon a draft of Simon (1955).]
- HA Simon, “Reply to Dr. Mandelbrot’s post scriptum”, *Information and Control*, 4, 305 – 308 (1961). [ABSTRACT: Dr. Mandelbrot has proposed a new set of objections to my 1955 models of the Yule distribution. Like his earlier objections, these are invalid.]

Editorial note: Dr. Mandelbrot feels that no further comment is needed and this debate terminates herewith.

The Second Pizza Theorem: A circular pizza can be equally divided among n people using $n - 1$ vertical cuts. In this sense, “equally” refers to the area of each slice. But different people may like different toppings, and therefore value some parts of the pizza more than others (technically speaking, the people may use different *measures* in determining the area of their slice of pizza as a proportion of the whole). In fact, even if there are a variety of different toppings, it is still possible to divide the pizza so that each person believes they have received $\frac{1}{n}$ of the ‘tasty bits’ of the pizza. If there is a piece of pizza, the relative tastiness of which two people disagree on, then it is possible to divide the pizza so that each person believes they have received *more* than $\frac{1}{n}$ of the net tastiness of the pizza!

$$3 \int (\text{ice})^2 d(\text{ice})$$

Science is a differential equation. Religion is a boundary condition.
—Alan Turing

Vacation Projects

If, like me, you've ever had the fleeting thought that the mathematics building might be an interesting place to live, then the Summer Vacation Scholarship Program is for you.

The mere thought of doing maths for the summer, as opposed to customer service, was enough to get me hooked, but upon arrival I found that, amid the plethora of forms, was a key to my very own office (well not just mine but we'll get to that) as well as a swipe card for access to the building whenever I should desire. Discovering that my office was the now-infamous room 106 was a pleasant surprise, as was discovering Southy the fridge. Very early on I decided that the whiteboard should, as quickly as possible, be filled with unintelligible symbols and scribble. Unfortunately my office mates had reached the same conclusion and had beaten me to it. What was really great was the three of us explaining our projects to the others, and watching the excitement on their faces as they'd just worked something fairly major out, and not having the heart to tell them that you didn't understand a thing of what they were talking about.

I suppose at some stage I should mention my project. I was doing a statistics project with Associate Professor Ray Watson as my supervisor and there were two distinct parts to it; the first was an exploration of a variation on the standard epidemic model, whereby people are afflicted with a disease, infect others and then are removed from the system (death, quarantine). In that I looked at the distribution of people unaffected by the illness when it had affected a certain percentage of the population. Like most things, it turned out that, to a reasonable squint of the eye, the distribution was normal.

The second part was to do with a model for breast cancer treatment. In this one the goals were not so well-defined, I wanted to produce output that would match real data and I also wanted to be able to see, from the output, what occurred in each case; whether the cancer was detected before it became too big, how many screenings it took before detection. In order to match the real data I had to play around with some of the parameters and once I could see the effect of changing the parameters on the output distribution I set upon the task of estimating these parameters. In doing this I delved into all manner of different distributions. I think that was what really appealed to me about research; in attempting to solve one problem, you come across a dozen more and each of those is interesting in its own right. I believe that this project will

somewhere along the line be used to model the effects of Hormone Replacement Therapy on the cancer.

The most wonderful part of the program was being forced to learn new skills. I hadn't done any computer programming before and suddenly I was to write a program to simulate an epidemic. It was a little daunting but I've discovered that, besides the stray segmentation fault, computers are quite obedient. When I was to write up a report I learned how to use the basic features on LaTeX and I discovered why many of the handouts we receive in our subjects are all formatted the same way.

Throughout the program there were a series of seminars, that is to say each vacation scholar was dragged up and forced to give a little talk on their project to the other research students. It was really interesting to hear what all of the other people were up to, from ideas as abstract as knotted graphs to things that were "useful", such as finding the formula to calculate the date of Easter Sunday (one of the students got up and said that they like 'useful' maths as opposed to pure maths). Also a weekly occurrence was the lunches we were strongly encouraged to attend on a Friday afternoon. As everyone knows, we maths students are extremely lacking of the social skills, and with this in mind we were dragged off to various corners of the earth within walking distance of uni to have pizza, noodles and even a picnic or two.

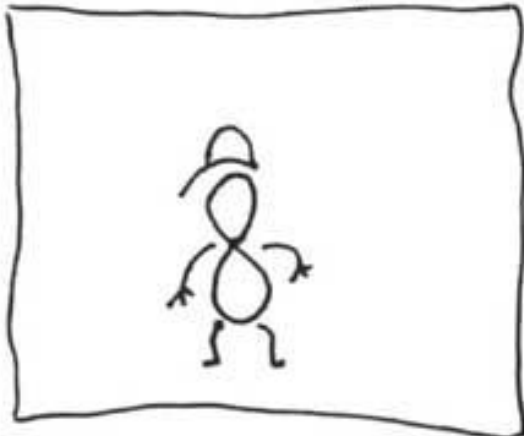
I'd like to take this opportunity to thank everyone who helped throughout the project; Ray for putting up with my erratic visits to his office asking inane and obscure questions; Nick, James, Joanna and Damian for patiently explaining all things computer to me; Olivia and Michael for telling me "its ok" when my program wasn't working as I unleashed a torrent of abuse at the computer; and iTunes. I'd also like to thank Peter Forrester and all of the other staff involved in the organization of such a fantastic program. Thank you all.

— Stefan Rampertshammer

The Third Pizza Theorem: Two arbitrarily-shaped pizzas can always be simultaneously bisected by a single cut (otherwise known as the Pancake Theorem).

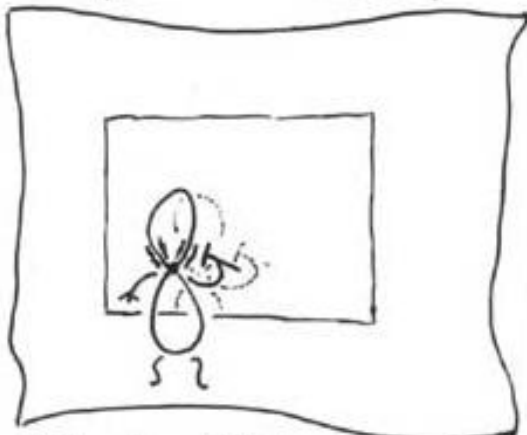
"I ate my tenderloin with a fork; I nine my elevenderloin with a fivek. "
—Victor Borge

This is Sammy the Graduate Student. Maybe you've seen him around the department.



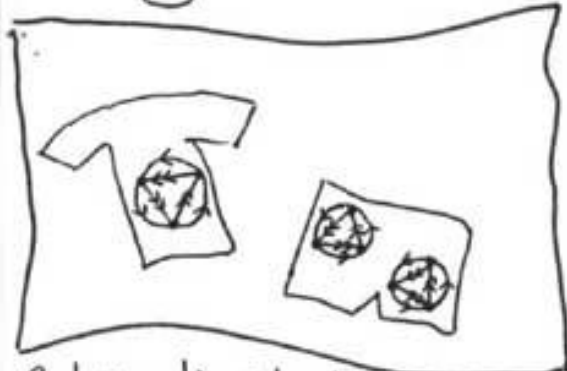
Each morning Sammy gets up and dresses, because it doesn't do to go out in public uncovered.

Then he shaves himself carefully, until his cheeks are smooth. Really smooth. Obviously smooth.

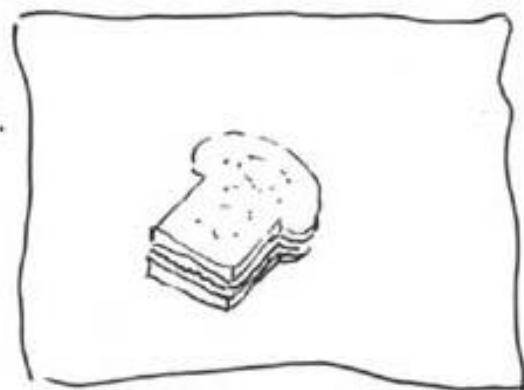


After breakfast Sammy makes his lunch. He likes to use bread with plenty of fibre.

Today he's wearing normal clothing...

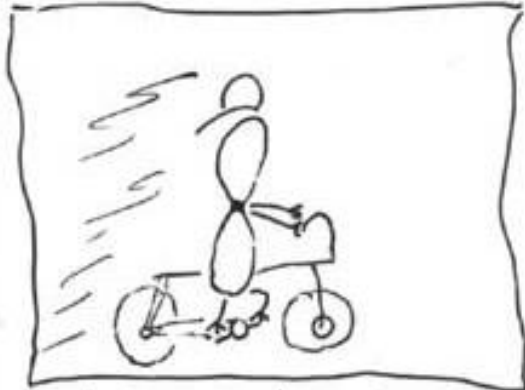


But sometimes he likes to wear something a bit more outlandish



Today he is having ham sandwiches. He cuts them carefully in half, then bundles them up and goes to school.

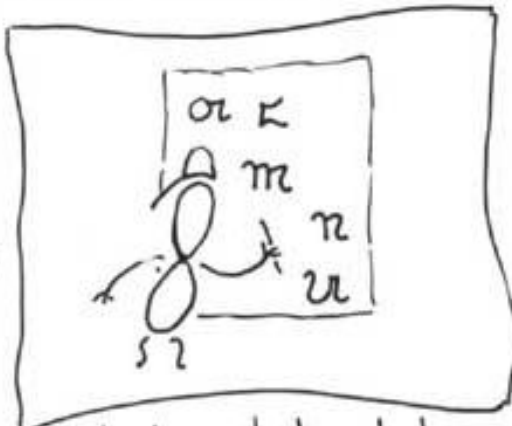
He always commutes by cycle



and dismounts when he reaches the campus boundary.

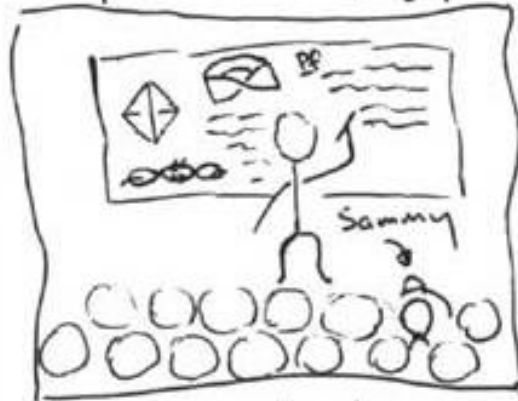
A day in the life of Sammy the Graduate Student

His next class is calligraphy - he's learning to write Gothic letters. Even though he thinks using them is less than ideal.



Luckily he only has to learn some Gothic letters. *w* is one of them though. And *w* is complicated. Much more complicated than *v*.

This is Sammy in class. Today he has topology.



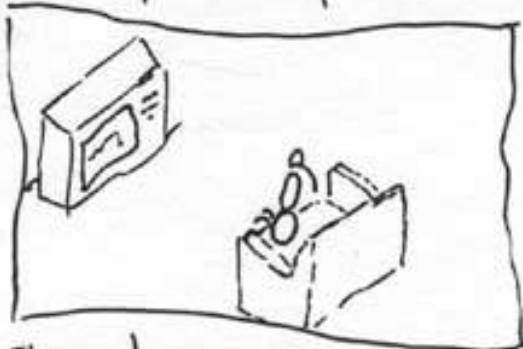
Sammy enjoys topology. Much of it seems very familiar, but at times he has difficulty getting a handle on it.



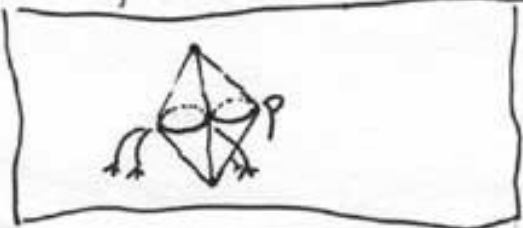
Some of the other students have portable phones, but Sammy doesn't. This gives him a cellular complex.

Complicated. Much more complicated than v.

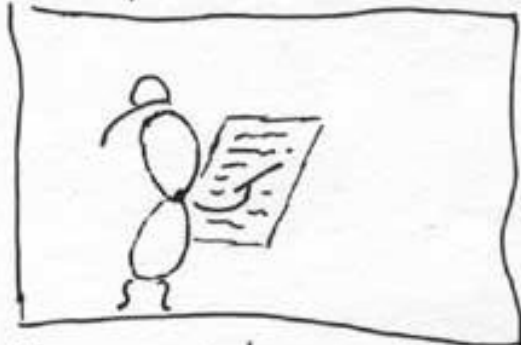
Sammy's favourite programme is on today: Berkeley Hills 94720



They always end on a cliffhanger that leaves him in suspense



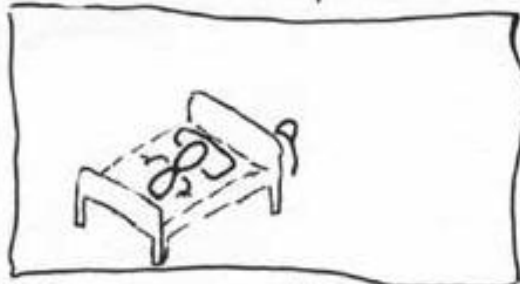
Having disentangled himself, Sammy does some studying.



Sammy works very hard and goes through sheaves of paper.



When he's finished he brushes his teeth then hops into bed.

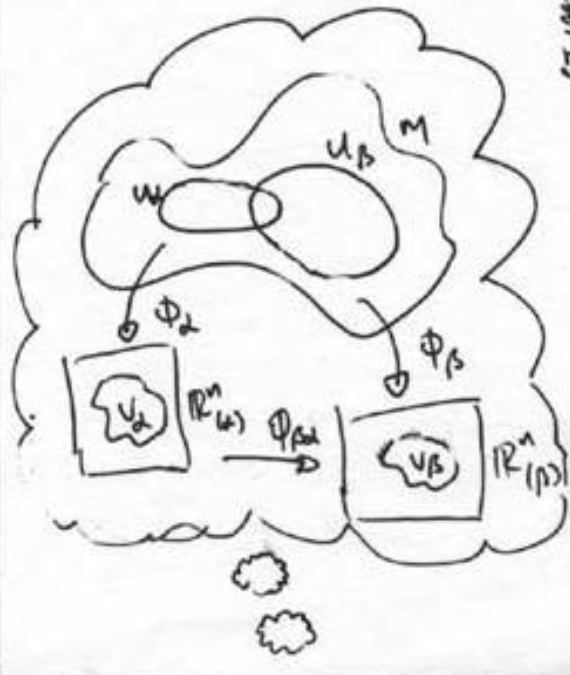


Before turning off the light he reads for a while. The book is yellow.



All of Sammy's books are yellow.

Sweet dreams Sammy.



A Recipe for Handmade Square Roots

Our arithmetic lives take many twists and turns, beginning with times tables, progressing through BODMAS towards the holy electronic calculator, then passing by the immortal graphics calculator before finally reaching the divine computer, which can perform calculations faster than one can say “a million factorial”. We space out our progress quite evenly, mastering the four basic operations in primary school, discovering some new, more exotic functions in high school and learning how to use sequences and series to approximate them in university.

But along the way, something seems to have been lost. What of the humble square root, so simple yet important enough to have its own calculator button? Many will dismiss its calculation as merely a special case of the binomial expansion, while others will leap to tried-and-true approaches like Newton’s method, giving up after a few tedious iterations to consult some logarithmic tables. Despite the huge repertoire of approaches available to us today, few would look back into the reign of the abacus and rediscover an elementary algorithm for evaluating square roots by hand, not unlike the long division which often plagues the early years of those less mathematically-minded.

So here’s how you do it. Take the number whose square root is to be calculated, and chop it into pairs of digits, working outwards starting from the decimal point. For example, to calculate the square root of 152.2756, we’d write it as 1 52. 27 56. Counting the lone 1 as a pair, each pair of digits will give one significant figure in the result; extra pairs of zeroes can be added to the end for extra precision.

Divide your page into two columns, and write your number on the right. Rule a horizontal line, bring down the first pair of digits (in this case the lone 1), place a zero on the left and you’re ready to start!

$$\begin{array}{r}
 \\
 \\
 \hline
 0 \\
 x
 \end{array}$$

Step A - filling in two numbers without a horizontal line

Put for x the largest integer such that x plus the number on the left, all multiplied by x , is less than the number on the right. Here, we want the largest x such that $x(x + 0) \leq 1$, so $x = 1$.

For w we put the value of this product, so here, $w = x(x + 0) = 1$.

	1	52.	27	56
0	1			
1	1			
y	z			

Step B - filling in two numbers with a horizontal line

Set y to be the number two spots above the line plus twice the number directly above it, all multiplied by 10. Here, $y = 10(0 + 2 \times 1) = 20$.

For z , we do as we do in standard long division, perform a subtraction then pull the next *pair* of numbers down.

	1	52.	27	56
0	1			
1	1			
20	0	52.		
x	w .			

Step A - no horizontal line again

Maximise x for $x(x + 20) \leq 52$ gives $x = 2$

$$w = x(x + 20) = 44$$

	1	1		
20	0	52.		
2		44.		
y	z .			

Step B - horizontal line returns

$$y = 10(20 + 2 \times 2) = 240$$

$z = 52 - 44 = 8$, pull down the next pair for 827

	2	44.		
240	8.	27		
x		w		
y	z			

Steps A & B - taking a shortcut

Maximise x for $x(x + 240) \leq 827$ gives $x = 3$

$$w = x(x + 240) = 729$$

$$y = 10(240 + 2 \times x) = 2460$$

$z = 827 - w = 98$, then pull down the final 56 from above for 9856

3	7.	29	
2460		98	56
x		w	
y		z	

Steps A & B - one last time

Maximise x for $x(x + 2460) \leq 9856$ gives $x = 4$

$$w = x(x + 2460) = 9856$$

$$y = 10(2460 + 2 \times x) = 24680$$

$z = 9856 - w = 0$, no more to pull down so we're done!

	1	52.	27	56
0	1			
1	1			
20	0	52.		
2		44.		
240		8.	27	
3		7.	29	
2460			98	56
4			98	56
24680				0

Voilà! The table is completed. Quite conveniently, the final difference is zero, indicating that the number you started with is a perfect square, a coincidence that won't normally occur. Though perhaps at this point, one might ask, "Dude, where's my square root?" But never fear, simply take all the x values you worked out, 1, 2, 3 and 4, put the decimal point in the right spot (it appears in the calculation during the bits with $x = 2$ and $x = 3$, so put it between the 2 and the 3), and so, the square root of 152.2756 is 12.34.

This seems quite long and tedious, but speeds up drastically with some practice. You might notice that the final y is unused, so why bother calculating it, or that after a while, $x \ll y$, so one can use $x_{next} \approx z/y$ to get x quickly. Whether you use it to save electricity, in your 620-202 exam or to entertain your little cousin, this is certainly an elegant and much underrated way of calculating square roots, useful to have in anyone's mathematical arsenal.

The Probability of Getting Wet

Author's Note: This problem is based upon a problem which appeared on the 1983 Entrance Examination for Cambridge University. The original problem had to do with conditional probabilities of life and death before a firing squad. We have retained the hard mathematics but softened the scenario.

Dr. Goat of the Institute for the Advancement of Elegance in Euclidean Geometry was serious about maintaining high standards of elegance in proof. Consequently, he was quite upset by the introduction of analytic methods into a series of lectures being given by his colleague Professor Bear. He was so upset that he felt that Professor Bear deserved to be doused by the water balloon which he intended to drop from a third-floor dormitory window overlooking the sidewalk which leads from the lecture hall to the Institute's reception centre.

Dr. Goat knew the Professor Bear would leave the lecture hall after he had finished his last presentation and responded to his audience's comments and questions. He would then head for the reception to be held in his honour and would walk beneath Dr. Goat's window at some time between 8 and 9 o'clock that evening. It was not possible to know in advance the exact time at which Professor Bear would pass by. So, in effect, Dr. Goat would have to drop his water balloon at some instant randomly chosen between 8 and 9.

Since sunset occurred at 8:30, the one hour period from 8 until 9 p.m. began in full light but it was quite dark by 9 o'clock. The visibility of Dr. Goat's target affected his aim in such a way that at 8 his aim was perfectly true but at 9 he was sure to miss. He figured that the probability of his dropping the water balloon on target decreased linearly from 1 to 0 as the time progressed from 8 to 9.

Also, the balloon should not be too full of water; otherwise, it might burst as Dr. Goat transported it from the bathroom tap to the vantage point at his window. Thus the charged and loaded balloon was not at all tightly stretched with the result that the probability that it would burst when it struck an object below was reduced to 0.5.

Dr. Goat was in place with his balloon at the ready by 7:45. Having nothing to do but wait for the inelegant and analytic Professor Bear, Dr. Goat decided to compute probabilities. Here is one of Dr. Goat's problems:

Problem: What is the probability that the balloon does burst if Professor Bear does not get wet?

Solution: A suitable probability space for the scenario described may be partitioned into three mutually exclusive events:

E1: The balloon bursts and Professor Bear stays dry.

E2: The balloon bursts and Professor Bear gets wet.

E3: The balloon does not burst and Professor Bear stays dry.

The event E3 might just as well have been defined by “The balloon does not burst” and the probabilities of the three events may be denoted by $p(E1)$, $p(E2)$ and $p(E3)$.

Dr. Goat’s problem is to compute the conditional probability $p(\text{The balloon does burst if Professor Bear stays dry})$, which is equal to:

$$\frac{p(E1)}{p(E1|E3)} = \frac{p(E1)}{p(E1) + p(E3)}$$

Recall that ‘ $E1|E3$ ’ means ‘ $E1$ OR $E3$ ’. The probability that Dr. Goat’s aim will be true when Professor Bear walks beneath his window is clearly 0.5. However, for anyone who wished this claim to be argued, Dr. Goat would have been happy to provide the following calculation.

The probability that Professor Bear will walk into the target zone during the interval of time dt at time $t \in [0, 1]$ where $[0, 1]$ represents hour between 8 and 9 p.m. is $\frac{dt}{1}$. The probability that Dr. Goat’s aim will be true at time t is $(1 - t)$. Integration yields the probability $p(\text{Dr. Goat’s aim is true when Professor Bear walks by}) = p(T) = \int_0^1 (1 - t)dt = 0.5$. It is obvious that $p(\text{Dr. Goat’s aim is not true when Professor Bear walks by}) = 0.5$ as well.

Now $p(E1) = p(F) \times p(\text{The balloon bursts}) = 0.5 \times 0.5 = 0.25$ and $p(E3) = 0.5$ since Dr. Goat had decided that $p(\text{The balloon bursts}) = p(\text{The balloon does not burst}) = 0.5$.

Therefore, the desired conditional probability is $p(\text{The balloon does burst if Professor Bear stays dry}) = \frac{0.25}{0.25+0.5} = \frac{1}{3}$. In the event, none of Dr. Goat’s computations were relevant. They were based upon a model whose assumptions were invalidated when Professor Bear appeared at 8:30 carrying an umbrella. He had been warned.

TOPOLOGY LIMITED

The student of Topology
Finds Surfaces on every hand.
Sometimes, by serendipity,
He even starts to understand.
To make a start to classify
Would certainly be fine and grand.
But troubled thoughts come, by and by –
Oh say, should Möbius be banned?

Approach combinatorially
Your torus, plane and twisted band;
Forget about Homotopy –
Just now it isn't in demand.
Equivalence is all our cry,
By transformations, nicely planned.
But still I wonder, with a sigh,
Oh say, should Möbius be banned?

Like pilgrims bound for Samarkand,
We trudge on, Categorically
Determined, one day we will land
Some theorem, new and fine to see.
And surely we should use, or try
Some techniques not of standard brand –
Quaternion Homology?
Oh say, should Möbius be banned?

Prof., in your boundless armoury
Of Groups and Mappings to command,
Where do you draw the Boundary?
Oh say, should Möbius be banned?

—Bruce Craven

The Fourth Pizza Theorem: The volume of a circular pizza of thickness a and radius z is, of course, *pizza*.

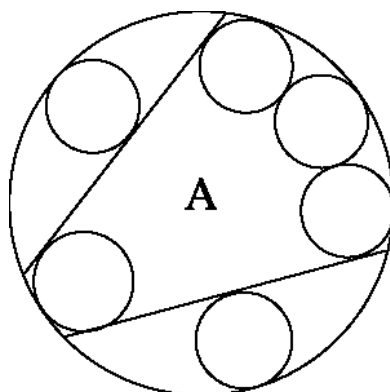
Paradox Problems

The following are some maths problems for which prize money is offered. The person who submits the best (clearest and most elegant) solution to each problem will be awarded the sum of money indicated beside the problem number. Solutions may be emailed to

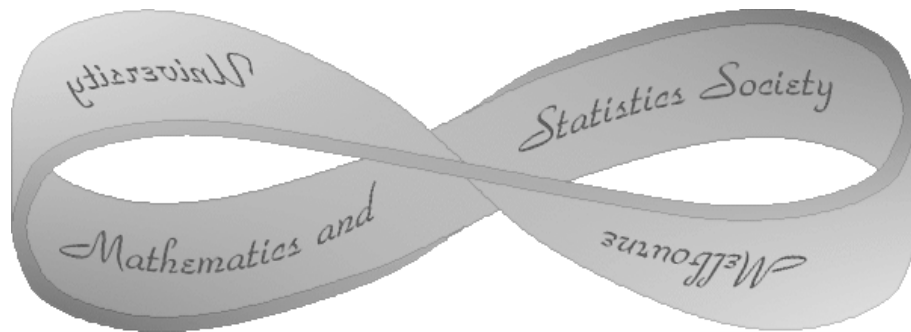
`paradox@ms.unimelb.edu.au`

or you can drop a hard copy of your solution into the MUMS pigeonhole near the Maths and Stats Office in the Richard Berry Building. Congratulations to Derek Weeks, Shaun Gladman and Sally Zhao, who submitted correct solutions to Question 1 from the last edition of *Paradox*, and also to Olivia Madill, who solved Question 3. Unfortunately for Sally, Derek and Shaun got there first on Question 1, so they get the prize. Derek, Shaun and Olivia can come by the MUMS room to pick up their prizes whenever they feel like it. There is no prize for the puzzle on the front cover (by the way, the answer is not ‘think again’).

1. (\$5) Show that a number is rational if and only if its decimal representation is eventually periodic.
2. (\$5) In the diagram below, the small circles have radius 1. Show that the area A in the diagram is equal to $8 - 2\pi$.



3. (\$10) Find all subsets X of the reals such that for each ‘stretch’ S , where $S(x) = ax$, there exists a translation T , where $T(x) = x + t$, such that $S(X) = T(X)$.



Melbourne University Mathematics and Statistics Society

Mystery and Mayhem

Melbourne Metropolis Mutilated!

How? Why? Who? When? What?

Find out in the

2005 Melbourne Uni Puzzle Hunt

For more information:

- visit <http://www.ms.unimelb.edu.au/~mums/puzzlehunt>
- keep an eye out for posters in the Richard Berry Building
- subscribe to our mailing list at:
<http://www.ms.unimelb.edu.au/~mums/mlist>