

Question 1

20 marks

Five positive integers have a mean of 2016, a median of 2016, and a unique mode of 2014. What is the largest of these integers?

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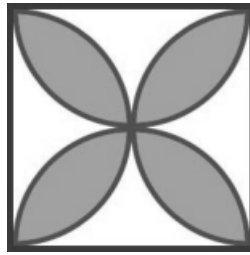
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Question 2

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A square of side length 2 units has semicircles drawn on each side.

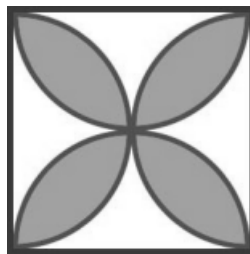


What is the area of the resulting four petals?

Question 2

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Question 3**20 marks**

If $x^2 - x^{-2} = 4$, then what is the value of $x^6 - x^{-6}$?

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Question 4**20 marks**

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Question 5**CHANGE RUNNER NOW****20 marks**

Tasmania Jones has come out of retirement to hunt for more treasure. He's entered the temple, but as soon as he grabs some, he sees his rivals, who will even kill him to get the treasure. To escape, Jones runs back to his boat, along a straight path. Anticipating this, his rivals also run on a straight path to the boat, so that their path and Jones' path intersect at a 30-degree angle. When Jones first saw his rivals, his path to the boat was perpendicular to the line between him and his rivals. If Jones' rivals run at 2 metres per second, and the treasure in Jones' hands weighs 5 kilograms, what is the speed s metres per second such that if Jones runs at any speed $s' > s$, he will beat his rivals to the boat?

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Question 6

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David is thinking of a number equal to four times the sum of its digits. What is the largest number that he could be thinking of?

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A helix structure wraps around a cylinder exactly 4 times. The circumference of the cylinder is 40 units and its height is 120 units.



What is the length of the helix?

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Question 8**30 marks**

Emma has raised \$2016 for the University Maths Olympics by selling 100 fundraising tickets. Some tickets were sold at full price (whole dollar amount), and the rest at a 10% discount. How much was raised from the discounted tickets?

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Question 9**30 marks**

If Adib and Cassy work together to build a house, it would take them 10 days. If Adib and Michael build a house, it takes them 12 days. If Cassy and Michael build a house, it would take them 15 days. If all three of them worked together, how many days would it take them to build a house?

Question 9**30 marks**

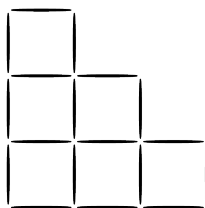
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Question 10

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Ruwan enjoys constructing toothpick staircases. For example, he can produce a staircase with six squares using just 18 toothpicks, as in the following diagram.



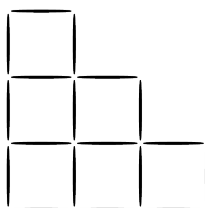
How many toothpicks would he need to construct a staircase with 2016 squares?

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Question 11**40 marks**

James is once again running late for his statistics lecture. He sits down and does some calculations. If he runs at an average speed of 10km/h he will be there 10 minutes late, if he runs at a constant speed of 5km/h he will be there 12 minutes late. How far away in metres is James from the lecture theatre?

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Question 12**40 marks**

Two non-congruent isosceles triangles have the same area and perimeter. If one of them has side lengths 5, 5 and 6, what is the shortest side length of the other triangle?

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Question 13**40 marks**

How many 5-digit numbers satisfy all of the following conditions:

1. The number begins with a 5;
2. The number is divisible by 5; and
3. The sum of the number's digits is divisible by 5?

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Question 14**40 marks**

Sam is generating numbers with 10 balls. He has written the numbers 1–10 on the balls, and placed them in a bag (so that each number occurs exactly once in the bag). He draws one ball out, records its number, and puts it back in the bag. He does this another nine times. Disregarding order, what is the total number of possible combinations of numbers which can be obtained with this procedure?

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Question 15

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Song constructs a cuboid of dimensions $8 \times 9 \times 10$ by gluing 720 unit cubes together. She then pokes a long, infinitely thin rod through one corner of the cuboid so that it exits at the opposite corner. How many unit cubes does this rod go through?

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Question 16**50 marks**

Let $f(n)$ be the largest power of the largest prime factor dividing n , e.g. $f(144) = 9$. What is the largest integer m such that 2016^m divides $f(2)f(3)f(4) \cdots f(2016)$?

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Question 17**50 marks**

If x and y are real values, determine the minimum possible value of

$$\sqrt{x^2 + y^2} + \sqrt{(x - 1)^2 + y^2} + \sqrt{x^2 + (y - 2)^2} + \sqrt{(x - 3)^2 + (y - 4)^2}$$

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Question 20**50 marks**

Let $ABCDE$ be a regular pentagon inscribed in a unit circle centered at O . Consider the line tangent to this circle at A , and pick a point P on this tangent so that AOP is isosceles. What is $AP \cdot BP \cdot CP \cdot DP \cdot EP$?

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