Issue 1, 1998

Paradox

The Magazine of the Melbourne University Mathematics and Statistics Society

Animated Still Life, Salvador Dali, 1956. Oil on canvas.

Paradox

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Printed:	April, 1998

Greetings from the Editor

Hello, and welcome to another year of *Paradox*. A year in which we are planning to bring you several exciting and innovative editions of this magazine, to complement the entertaining schedule of mathematics and statistics related talks, seminars, Olympics, BBQs and more organised by the Melbourne University Mathematics and Statistics Society (MUMS). This year, we hope to include articles by interested maths and stats students of all levels, and in general encourage participation by everyone who wants to see 1998 be a fun and stimulating year (or just wants to pad their C.V. and see their name in print). So anyone who wants to submit any maths or stats jokes, puzzles, articles, pictures, recipes or anything at all — just email it to us at `paradox@ms.unimelb.edu.au`.

— Jeremy Glick, *Paradox* Editor

The *Paradox* team would like to thank the following people: Chaitanya Rao, Vanessa Teague, Jon Faulkner, Tony Wirth, Andrew Oppenheim, Lawrence Ip, Koula Courtot, and everyone else who assisted us in some way but who we have forgotten to name.

El Presidente says: ¡Viva la revolución!

You may not know it, but you're probably a member of MUMS already! Anyone who is enrolled in a maths or stats subject is automatically a member, and anyone else with an interest is most welcome to join. There's no need to fill out any membership forms, just turn up to our events.

So what do we do?

On the academic side, on Friday afternoons MUMS holds seminars on *interesting* topics in maths and stats (not the stuff you get in lectures!) with refreshments (drinks and nibbles) served afterwards. One of the more interesting speakers we have lined up is Michael Barnsley, the mathematician who invented fractal image compression and who has gone on to make millions from it.

Last year we didn't have many social functions, but with the new fresh energetic committee, things have changed! We've already had a pizza lunch, and a trivia afternoon is coming up after Easter. Get together your teams now.

Of course the big event of the year is the annual Maths Olympics, a team contest featuring quick thinking, lots of running and the chance to find out what happens when you mix rugby with maths! Look out for the posters early in 2nd semester.

For more information on MUMS and our events, check out our new web page (`http://baasil.stats.mu.oz.au/Mums`).

If you have suggestions on things we should hold or would like to be placed on our mailing list for announcements, just send email to me.

— Lawrence Ip, MUMS President
`lip@ms.unimelb.edu.au`

Seminars

Professor Michael Barnsley (1998 Miegunyah fellow) will be presenting the *Miegunyah Lecture* entitled *Mathematics: Vision, Education and Money* on Tuesday 28th April at 6:30pm in Theatre A, Richard Berry building. For more information, see the notices posted in the Richard Berry building.

In addition, MUMS will be continuing its Friday afternoon seminars. Check the notice boards around the Richard Berry building and *This Day* regularly to be sure you don't miss one!

Paradox on the Web

Did you have to step over your best friend in order to grab the last remaining copy of *Paradox*? Well no longer! *Paradox* is now available on-line at

`http://baasil.stats.mu.oz.au/Mums/Paradox`.

You will be able to read new and old editions of *Paradox* using your favourite web browser, or print out hard-copies to have and to hold for ever.

Mathematical humour

To many, "mathematical humour" is an oxymoron; to a quizzical few, it is a tautology. Regardless, here are some examples …

An engineer, a mathematician, and a statistician went to the races one Saturday, and laid their money down. Commiserating in the bar after the race, the engineer said, 'I don't understand why I lost all my money. I measured all the horses and calculated their strength and mechanical advantage and figured out how fast they could run … '

The statistician interrupted her: '… but you didn't take individual variations into account. I did a statistical analysis of their previous performances and bet on the horses with the highest probability of winning … '

'… so if you're so hot, why are you broke?' asked the engineer. But before the argument could escalate, the mathematician took out his pipe and they got a glimpse of his well-fattened wallet. Obviously, here was a man who knew something about horses. They both demanded to know his secret.

'Well,' he said, between puffs on his pipe, 'first I assumed that all the horses were identical and spherical … '

Said the Dean to the physics department: 'Why do I always have to give you so much money for laboratories and expensive equipment and stuff? Why couldn't you be like the maths department? — all they need is money for pencils, paper and waste-paper baskets. Or even better, like the philosophy department — all *they* need is pencils and paper.'

An apology

The *Paradox* team would like to apologise for the poor quality of the jokes. However, they were the best we could find. If you believe you can do better, we are willing to offer *cash.* Whoever submits the funniest maths jokes (printable in *Paradox* without exposing us to defamation litigation) will win \$10. ("Funniest" will be decided by the *Paradox* team.)

Two boxes of cash

You and a friend are each presented with a closed opaque box which contains a certain sum of money. You are told that one box contains twice as much money as the other. You open your box and find to your dismay that there is only \$10 in your box. (You are not averse to looking a gift horse in the mouth.) As a saving grace, you are given the option either to exchange your box with your friend's, not knowing how much it contains, or to keep the one which you already possess. What should you do?

If you keep your box, you will definitely earn \$10. You know that your friend's box either contains \$20 or \$5 and that there is a probability of $\frac{1}{2}$ either way. Thus, the expected value of money in his/her box is $\frac{1}{2} \times \$20 + \frac{1}{2} \times \$5 = \$12.5$, so it is in your advantage to swap. The same reasoning shows that whatever amount your friend starts with, it is in her/his advantage to swap also.

In general, if you start with $\$x$ and know that your friend has either double or half that amount, you are certain of $\$x$ if you don't swap and you expect $\$1.25x$ if you do. If this process were repeated many times, you would be 1.25 times richer if you swapped every time than if you abstained from doing so. But the same reasoning shows that your friend is, in the long run, better off swapping too. How can this be?

A proof that $\pi = 0$[1]

For all values of θ,

$$\cos\theta = \cos(2\pi + \theta),$$

and

$$\sin\theta = \sin(2\pi + \theta).$$

Therefore

$$\cos\theta + i\sin\theta = \cos(2\pi + \theta) + i\sin(2\pi + \theta),$$

and

$$(\cos\theta + i\sin\theta)^i = [\cos(2\pi + \theta) + i\sin(2\pi + \theta)]^i. \tag{1}$$

Recall from De Moivre's theorem that $(\cos x + i\sin x)^n = \cos nx + i\sin nx$. Hence (1) can be written in the form

$$\cos i\theta + i\sin i\theta = \cos i(2\pi + \theta) + i\sin i(2\pi + \theta). \tag{2}$$

[1] Eugene P. Northrop, *Riddles in Mathematics*, Pelican Books, 1964, pp. 210, 211.

Now apply Euler's formula, $\cos x + i \sin x = e^{ix}$, to both sides of (2). We obtain

$$e^{-\theta} = e^{-2\pi-\theta}.$$

Dividing both sides of this expression by $e^{-2\pi-\theta}$,

$$e^{2\pi} = 1.$$

But e^x has the value 1 only when x is zero. Hence $2\pi = 0$, and $\pi = 0$.

Another paradox

What is 'the smallest positive integer not expressible in fewer than twenty-five syllables?'
Note: We have just expressed it in 24 syllables!

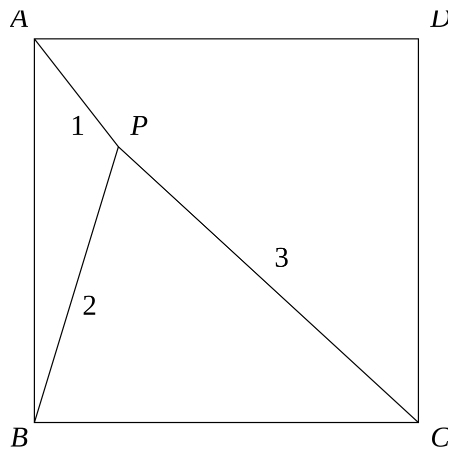
Problems

The following are some problems for prize-money. The person who submits the best (i.e. clearest and most elegant) solution to each problem will be awarded the sum of money indicated beside the problem number, and will have their solution published in the next edition of *Paradox*. Solutions may be emailed to `paradox@ms.unimelb.edu.au`. (LaTeX format would be appreciated though not demanded.) If you do not have access to email then drop in a hard copy of your solution to the MUMS pigeon-hole near the Maths and Stats Office in the Richard Berry building.

1. (\$10) Let $f(n)$ be the least common multiple of the first n natural numbers. Show that $f(n)$ is not bounded by any polynomial of n.

2. (\$5) Let $A_1A_2\ldots A_n$ be a regular n-gon inscribed in the unit circle. Show that $|A_1A_2| \cdot |A_1A_3| \cdot \cdots \cdot |A_1A_n| = n$.

3. (\$10) P is a point inside a square $ABCD$ such that $PA = 1$, $PB = 2$, and $PC = 3$. How large is $\angle APB$?



4. (\$10) Using only sin, cos, tan, arcsin, arccos, and arctan keys on a calculator, show that starting from 0, pressing some finite sequence of buttons will yield any positive rational number q. (Functions are in terms of radians, and assume that the calculator has infinite precision.)

5. (\$10) For each positive integer n, determine a set of n distinct positive integers such that no subset of them adds up to a perfect square.

6. (\$10) Find the smallest integer $n > 4$ such that there exists a set of n people such that any two acquainted people have no common acquaintance and any pair of unacquainted people have exactly two common acquaintances. (Acquaintance is a symmetrical relation; if A knows B, then B knows A.)

Unsolved problems

If you consider it beneath your dignity to solve a problem for which a solution has already been found, here are some which are as yet unsolved.

1. Can every positive even integer be expressed as the sum of two primes?
   i.e. $4 = 2 + 2$, $6 = 3 + 3$, $8 = 5 + 3$, $10 = 5 + 5 = 7 + 3$, $12 = 7 + 5$, $14 = 7 + 7$, $16 = 11 + 5$, etc.

2. Start with any positive integer x_0. If it is even, $x_1 = \frac{1}{2}x_0$, otherwise $x_1 = 3x_0 + 1$. Continue similarly. (i.e. $x_{n+1} = \frac{1}{2}x_n$ for x_n even, and $x_{n+1} = 3x_n + 1$ for x_n odd.) Must the process always end with 1 (i.e. no cycles other than 4,2,1,4,2,1...)?
   e.g. 15, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1.

3. Is $\pi + e$ irrational?

4. Find five positive integers such that the product of any two is one less than a perfect square. ($\{1, 3, 8, 120\}$ is a set of four, but does a set of five exist?)

Chinese mathematics and the volume of a sphere

The contributions made by the ancient Chinese to mathematics receive little recognition. Largely because many of their discoveries were either preceded by similar discoveries in the West, or were rediscovered at a later stage by Western mathematicians. However, this does not mean that they are of no interest or relevance to the modern reader. The contrary is true rather, due to the contrasting methodology adopted by Eastern and Western mathematicians. Moreover, the study of the development Chinese mathematics, and indeed of the mathematics of any ancient civilisation, provides one with considerable insight into the historical development of mathematics as a whole.

Continued page 8

Paradox **April centrefold:** π

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The *observant* reader may have noticed errors in this decimal expansion of π. As a matter of fact, four non-consecutive digits have been deliberately altered. Prize-money is being offered for the first person to specify the location of these incorrect digits, and what their correct values should be. Answers must be submitted to our email address.

10¢ will be rewarded for the first incorrect digit,
\$1 will be rewarded for the second incorrect digit,
\$2 will be rewarded for the third incorrect digit, and
\$5 will be rewarded for the fourth incorrect digit.

From page 5

The emphasis of Chinese mathematics, rather than being on proofs, is on algorithms. Indeed, Chinese mathematical texts consist of problems and solutions, or what could be termed as "worked solutions" by the modern student. (Though presumably, students were given oral instruction in the underlying principles.) A good example of the differing approaches is on the calculation of the volume of a sphere, V, which we know as

$$V = \frac{4}{3}\pi r^3,$$

where r is the radius of the sphere.

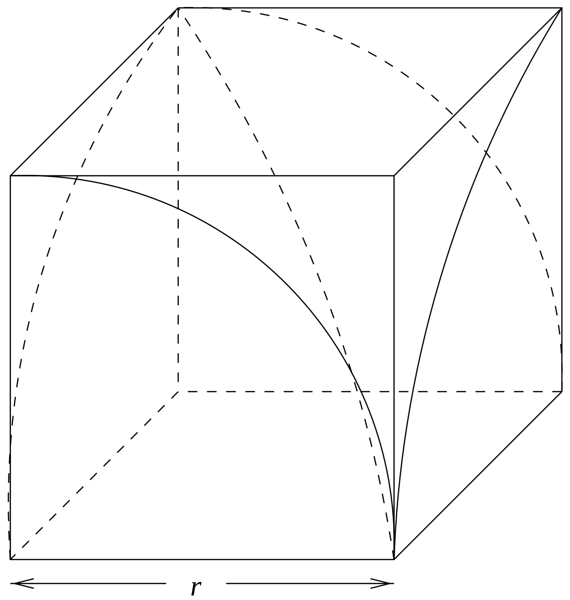
This result was first proved by Archimedes (287?-212 BC) and published in his work, *On the sphere and cylinder, Book I.* Translated into modern English, he stated:

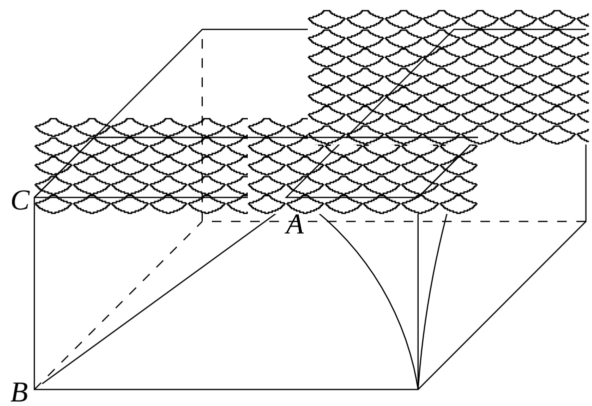
> Any sphere is equal to four times the cone whose base is equal to the greatest circle in the sphere and whose height is equal to the radius of the sphere.
> — Archimedes, *Of the circle and sphere, Book I*, Proposition 34.

Archimedes proved this by contradiction, i.e. by showing that it is not possible for a sphere not to satisfy this condition. (For details, see *The works of Archimedes* [1].)

The same result was calculated several centuries later by the Chinese mathematician Zǔ Chōngzhī (429-500 AD) and his son Zǔ Gèng. (*Note:* Chinese names will be denoted by Mandarin Chinese Pinyin.)

Zǔ Chōngzhī was an astronomer and mathematician in the period of the North and South Dynasties of considerable repute. On top of his contributions to mathematics, he made significant contributions to astronomy (most notably the Dà Míng calendar which he constructed), engineering, and the theory of music. Another of his notable achievements in mathematics is the determination of π to seven decimal places. Unfortunately, the details of this calculation, and most of his other works, have been lost. Zǔ Gèng continued much of the work of his father.





The method utilised by the pair can be briefly summarised as follows. Consider a cube of side-length r, where r is the radius of the sphere. This is $\frac{1}{8}$ the cube circumscribing the sphere. Cut the cube with two cylinders, of radius and height r, one through the face of the cube, and the other through the side.

The shape formed by the intersection of two cylinders which circumscribe a sphere, and whose axes intersect perpendicularly, was dubbed "two square umbrellas" (*móuhé fānggài*) by the mathematician Liú Huī. The shape obtained above, by the intersection of the two cylinders and the cube, is $\frac{1}{8}$ the "two square umbrellas", and shall be referred to as "part umbrellas" for convenience.

Take a cross-section of the divided cube at a height h. Let a be the length AC. Then by Pythagoras' theorem (the analogous result in Chinese mathematics is known as the Gōugŭ theorem),

$$a^2 = r^2 - h^2.$$

Also, $a^2 =$ the area of the cross-section of the "part umbrella". Hence the area of the shaded area, S, is given by

$$S = r^2 - a^2 = r^2 - (r^2 - h^2) = h^2$$

which is true for all $h \in [0, r]$.

Consider an upside-down square-based pyramid of height and side-length r. Then for all $h \in [0, r]$, S is equal to the cross-sectional area of the pyramid which equals h^2. The pair reasoned that since the cross-sectional areas were equal at the same height, the volumes must be equal.

The volume of the pyramid is $\frac{1}{3}$ the volume of the cube. Hence the volume of the "part umbrella" is $\frac{2}{3}$ the volume of the cube, and so the volume of the "two square umbrellas" is $\frac{2}{3}$ the volume of eight cubes. Furthermore, since the ratio of the cross-sectional area of the "two square umbrellas" to that of the sphere is simply the ratio of a circle to a square,

$$V = \frac{\pi}{4} \cdot \frac{2}{3} 8r^3 = \frac{4}{3}\pi r^3.$$

— Desmond Lun

References

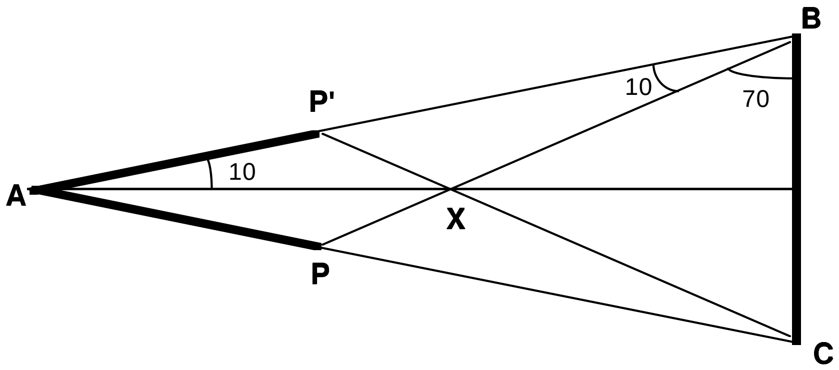
[1] T. L. Heath (ed.), *The works of Archimedes*, Dover Publications, Inc., New York, 1953.

[2] Lĭ Yăn and Dù Shíràn, translated by John N. Crossley and Anthony W. C. Lun, *Chinese mathematics: A concise history*, Oxford University Press, Oxford, 1987.

[3] Yoshio Mikami, *The development of mathematics in China and Japan*, Chelsea Publishing Company, New York, 1974.

Solutions to last issue's problems

Problem: In triangle ABC, $AB = AC$ and $\angle BAC = 20°$. P is a point on AC such that $AP = BC$. Find $\angle PBC$.
Solution 1: By use of trigonometry and approximation to 10 decimal places, angle PBC looks as though it will be 70 degrees.

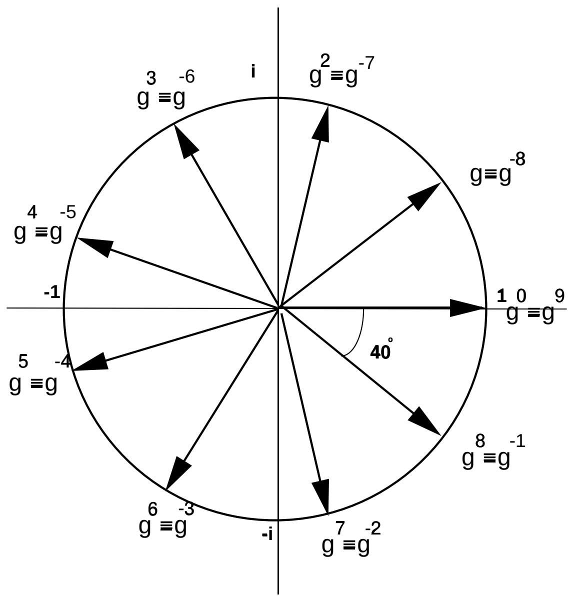


This is the case *if and only if* $\triangle AXB$ is *isosceles if and only if* X is the centre of the *circumcircle* for $\triangle ABC$ *if and only if* $|AX| = |BX|$.
We will use Pythagoras and an equation to show the latter.

Without loss of generality let us scale $\triangle ABC$ so that the equal side length of $\triangle ABC$ is 1. Let $x = \sin 10° = \cos 80°$. We will show that $8x^3 - 6x + 1 = 0$. Let

$$\zeta = e^{\frac{2\pi i}{9}} (= e^{i40°})$$

giving $\cos 80° = x = \frac{\zeta^2 + \zeta^{-2}}{2}$. Diagrammatically, we have:



Consider

$$\begin{aligned}
& 8(y-\frac{\zeta+\zeta^{-1}}{2})(y-\frac{\zeta^2+\zeta^{-2}}{2})(y-\frac{\zeta^4+\zeta^{-4}}{2}) \\
= & [2y-(\zeta+\zeta^{-1})][2y-(\zeta^2+\zeta^{-2})][2y-(\zeta^4+\zeta^{-4})] \\
= & 8y^3-4y^2(\zeta+\zeta^{-1}+\zeta^2+\zeta^{-2}+\zeta^4+\zeta^{-4}) \\
& +2y(\zeta^6+\zeta^2+\zeta^{-2}+\zeta^{-6}+\zeta^5+\zeta^3+\zeta^{-3}+\zeta^{-5}+\zeta^3+\zeta^1+\zeta^{-1}+\zeta^{-3}) \\
& -(\zeta^7+\zeta^5+\zeta^3+\zeta^1+\zeta^{-1}+\zeta^{-3}+\zeta^{-5}+\zeta^{-7}) \\
= & 8y^3-4y^2(\zeta^0+\zeta^1+\zeta^2+\zeta^3+\zeta^4+\zeta^5+\zeta^6+\zeta^7+\zeta^8-\{\zeta^0+\zeta^3+\zeta^6\}) \\
& +2y(\zeta^0+\zeta^1+\zeta^2+\zeta^3+\zeta^4+\zeta^5+\zeta^6+\zeta^7+\zeta^8+2\{\zeta^0+\zeta^3+\zeta^6\}-3\zeta^0) \\
& -(\zeta^0+\zeta^1+\zeta^2+\zeta^3+\zeta^4+\zeta^5+\zeta^6+\zeta^7+\zeta^8-\zeta^0) \\
= & 8y^3-6y+1
\end{aligned}$$

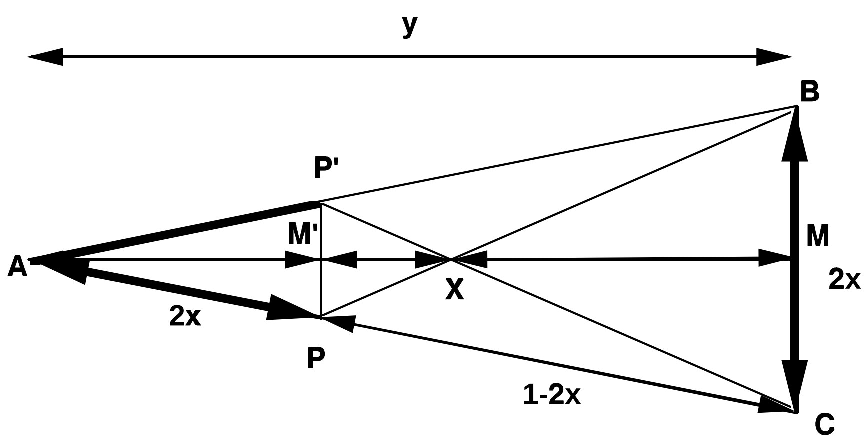
Clearly x is a root of $8(y-\frac{\zeta+\zeta^{-1}}{2})(y-\frac{\zeta^2+\zeta^{-2}}{2})(y-\frac{\zeta^4+\zeta^{-4}}{2})$ and so $8x^3-6x+1=0$.

This equation may also be derived from

$$\sin 10^\circ = \cos 80^\circ$$

and using the double angle formulae for sin and cos on the RHS until angles of 10 are obtained. The resulting degree 8 polynomial may be factorised (using for example *Mathematica* into 2 linear terms and 2 degree 3 terms. Substitution shows that x is a solution to the cubic term $8y^3-6y+1$.

By Pythagoras $(y=)|AM|=\sqrt{1-x^2}$. Now $\triangle APP' \sim \triangle ABC$ therefore $|PP'| = (2x)|BC| = (2x)^2$, $AM' = (2x)AM = 2xy$ and $M'M = (1-2x)y$.
Also $\triangle XPP' \sim \triangle XBC$ so $|XM'| = (2x)|XM|$ hence $|XM'| = \frac{2x}{1+2x}|M'M| = \frac{(2x)(1-2x)}{1+2x}y$ and $|XM| = \frac{1-2x}{1+2x}y$.



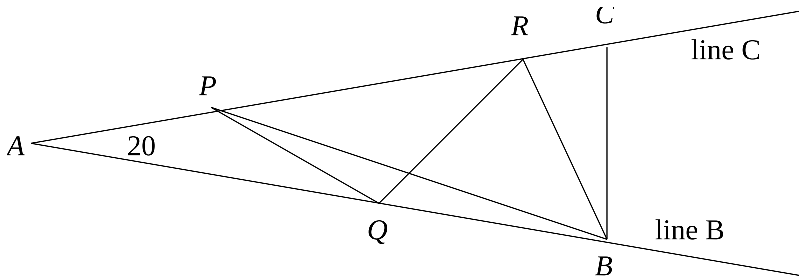
Now $\quad |AX|^2-|BX|^2=0$

$$\begin{aligned}
&\Leftrightarrow \quad (2xy + 2x\frac{1-2x}{1+2x}y)^2 - ((\frac{1-2x}{1+2x}y)^2 + x^2) = 0 \\
&\Leftrightarrow \quad (\frac{4xy}{1+2x})^2 - (\frac{(1-2x)^2y^2 + x^2(1+2x)^2}{(1+2x)^2}) = 0 \\
&\Leftrightarrow \quad (\frac{1}{1+2x})^2\{16x^2y^2 - (1-2x)^2y^2 - x^2(1+2x)^2\} = 0 \\
&\Leftrightarrow \quad \{16x^2(1-x^2) - (1-2x)^2(1-x^2) - x^2(1+2x)^2\} = 0 \\
&\Leftrightarrow \quad -16x^4 - 8x^3 + 12x^2 + 4x - 1 = 0 \\
&\Leftrightarrow \quad (-2x-1)(8x^3 - 6x + 1) = 0,
\end{aligned}$$

which is known to be true (with $x = \sin 10°$) from above. Thus the proposition that angle $PBC = 70°$ follows.

— David Coulson

Solution 2: This is a geometric solution to the problem.



Begin with two lines, at an angle of 20° to each other. Mark a point P along line C and construct isosceles triangles down the lines.

Thus $BC = BR = QR = PQ = AP$.

$$\begin{aligned}
&\angle AQP = \angle QAP = 20° \\
&\angle QRP = \angle QPR = 2\angle AQP = 40° \\
&\angle BQR = \angle QBR = 2\angle QRP - \angle AQP = 60° \\
&\angle RCB = \angle BRC = 2\angle QBR - \angle QRP = 80° \\
&\angle ABC = 180° - \angle BAC - \angle ACB = 80°
\end{aligned}$$

Thus $\triangle ABC$ is the isosceles triangle in the problem. Now $\triangle BQR$ is equilateral (all angles 60°). Thus $QB = QR = QP$. $\Rightarrow \triangle QBP$ is isosceles. $\Rightarrow \angle QBP = \angle QPB$.

$$\begin{aligned}
\angle BQP &= \angle BQR + \angle RQP \\
&= 60° + 100° \\
&= 160°
\end{aligned}$$

Thus

$$\begin{aligned}
\angle QBP &= \frac{180^\circ - \angle BQP}{2} \\
&= \frac{180^\circ - 160^\circ}{2} = 10^\circ \\
\Rightarrow \angle PBC &= \angle ABC - \angle QBP \\
&= 80^\circ - 10^\circ = 70^\circ
\end{aligned}$$

— Kuhn Ip

Working space

In the past, *Paradox* has been conspicuously deficient of any space for working out solutions to problems contained within it. This year, we are committed to bringing you the highest quality working space available. Enjoy ...

Do you wish that at trivia nights they didn't ask who won the Oscar for Best Actor in 1952 for his role in *High Noon* but instead asked about *Good Will Hunting*? Or rather than ask about Raoul Julia and the Addams Family, they wanted to know for which fractal objects his namesake Gaston is famous? Or the manner of Galois' death?

No?
Well, anyway, the Melbourne University Mathematics and Statistics Society

MUMS

is holding a

TRIVIA AFTERNOON

Friday 1st May 1:30 – 3:15pm

Theatre B, Richard Berry Building

This promises to be an afternoon of some hilarity interspersed with white-knuckled competition. There will be about sixty (60) questions. Questions will be general but have more than usual emphasis on scientific trivia. Food and drinks will be provided.

Entry Information: ENTRY IS FREE AND OPEN TO ANYONE WITH SOME CONNECTION TO MATHS AND STATS. The competition is limited to fifteen (15) teams. Each team should have six (6) members.

Entry is by writing the names of team members, a name for the team and a contact number or email address on a piece of paper and placing that paper in the MUMS pigeonhole beside the Maths and Stats Office. Rules of competition will be announced on the day. Prizes will include book vouchers for at least the top two teams.

For further information, contact:

Tony Wirth	`awirth@ms.unimelb.edu.au`	9344-4021
Adam Cagliarini	`aac@ms.unimelb.edu.au`	

Entries close once the fifteen team spots have been filled,

So get your team in today!