Melbourne
University
Maths +
Stats Society

Schools Maths Olympics 2020

Melbourne University Maths and Stats Society

Welcome!

- Hello and welcome to MUMS Schools Maths Olympics 2020!
- When you enter the Zoom please set your name as your school's name (initials or short version), 1 or 2 (if your school has two teams), and then your name. For example: `fitzroyhs 2 Hannah`
- Have one member from your team log into the Google Classroom to sign your team in `https://classroom.google.com/c/MTQzMTUwNzE3NjYw?cjc=afldpb4` or use the class code **afldpb4**. Please note: the team member who does the sign in will be responsible for submitting your team's answers.

Viewing the questions

- We will be using Google Classroom to distribute the questions. See previous slide for the link/code.
- Each round is listed as an assignment containing a pdf file and a Google Form.
- The pdf file contains the questions for that round as slides.
- While in your breakout rooms we suggest having one team member share their screen to display the question slides (open the pdf, click share in zoom, then select the window with your pdf viewer).
- You could use the annotate feature in Zoom to draw/type on the slides; we suggest taking a screenshot or saving your solution so you can see it later when you submit your team's answers.

Submitting your team's answers

- In the assignment page on the Google Classroom there is a Google Form where you can submit your answers for that round.
- Please only submit your team's answers with a single account in the Google Classroom.
- Once you have filled in the form and submitted your team's answers, use the "mark as done" button on the assignment page to let us know you have finished that round.
- We will then release the next round to your team.

Further details

- There is no time limit on each round but keep in mind the 1 hour overall time limit of the competition.
- Your team need to have submitted the current round's answers before we will release the next round to your team.
- We will mark the first answer form submission from your team, so please make sure it is complete and correctly filled before submitting.
- Please give the exact answer unless specified otherwise (e.g. $\sqrt{2}$ not 1.414...) and give any fractions in their simplest form (e.g. 1/2 not 3/6).

Questions and scoring

- Questions in round 1 are worth 25 points each.
- Questions in round 2 are worth 30 points each.
- Questions in round 3 are worth 35 points each.
- There are 8 questions in each round.
- The questions will get progressively more difficult through the rounds.
- If teams draw based on score then the tie-breaker will be the time their answers were submitted in the google form.

Before we begin...

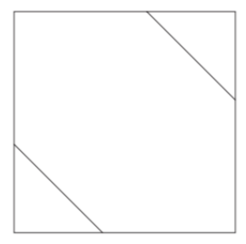
Any Questions?

Round 1

Question 1 *(25 points)*

What is the smallest number that is divisible by 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13?

Question 2 *(25 points)*



An irregular hexagon with all sides of equal length is placed inside a square of side length 1, as shown above (not to scale). What is the length of one of the hexagon sides?

Question 3 *(25 points)*

There are 24 questions in the SMO.

Team A finishes all questions after 40 minutes. At the same time, team B still have 4 more questions to go.

When team Bob finishes all questions, team C still have 4 more questions left.

Assuming the speed of doing questions is constant for each team, how long does it take for team C to finish all questions?

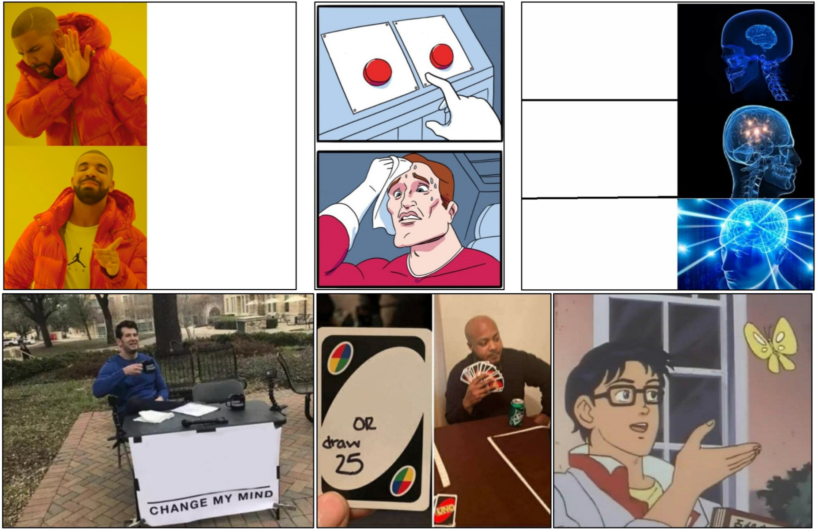
(Express your answer with one decimal place.)

Question 4 *(25 points)*

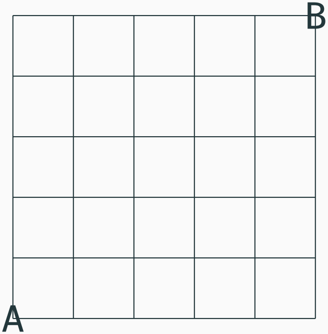
The University of Melbourne recycles paper cups for use in its canteens. Seven used cups are needed to make one new cup, which can then be used. If there are currently 2251 used cups, how many new cups can possibly be made?

Question 5 *(25 points)*

Fill in 5 of the meme templates with maths-related jokes



Question 6 *(25 points)*



How many ways to go from A to B, if you can only go up or to the right by 1 unit each time?

Question 7 *(25 points)*

Gauss has been counting his days in lockdown. However, he has stayed at home for so long that he is losing it a bit. When he counts, he always forgets to count multiples of 7 and any number that contains 7 in its digits.

$$1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 16, 18\ldots$$

Today, Gauss concluded that he has been at home for 218 days. How many days has Gauss actually been at home for?

Question 8 *(25 points)*

There are 100 closed lockers and 100 students in a school. Student 1 enters the school and opens lockers $1, 2, 3, \ldots$. Student 2 enter the school and closes lockers $2, 4, 6\ldots$. Student 3 enters and opens/closes lockers $3, 6, 9, \ldots$. Continuing in this fashion, student 100 enters the school and either opens or closes their locker. After all 100 students have entered the school, how many lockers are left open?

Round 2

Question 9 *(30 points)*

A circle of radius 2, centred on the origin, is drawn on a grid of points with integer coordinates, Let n be the number of grid points that lie within or on the circle. What is the smallest amount the radius needs to increase by for there to be $2n - 5$ grid points within or on the circle?

Question 10 *(30 points)*

It has been shown (or proven impossible) that every number, n, between 1 and 100, can be written as the sum of three cubes:

$$x^3 + y^3 + z^3 = n,$$

where x, y and z are integers.

When $n = 3$, the trivial solution is $(x, y, z) = (1, 1, 1)$. Find another solution.

Question 11 *(30 points)*

The following numbers are written on a blackboard:

$$\frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{100}$$

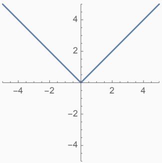
Each time, I take any two of the numbers on the blackboard - let's call them x and y. I erase x and y, and then add the number $x \cdot y - x - y + 2$ to the blackboard.

I repeat this process until only 1 number is left on the blackboard. What is this last number?

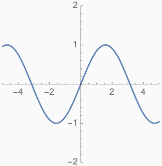
(Remember to express this number in simplest fraction form)

Question 12 *(30 points)*

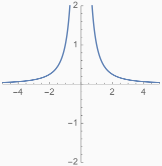
Take a team photo where each member picks a different one of these graphs and creates the shape with their arms[1]:



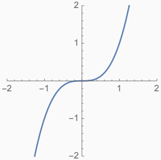
(a) $|x|$



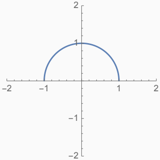
(b) $\sin x$



(c) x^{-2}



(d) x^3

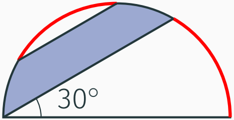


(e) $\sqrt{1 - x^2}$

[1] If someone doesn't have a camera, we will accept a picture of themselves drawn in the whiteboard on zoom (share options → whiteboard)

Question 13 *(30 points)*

If the two red arcs in the image below have the same length, then what fraction of the semicircle is shaded in blue?

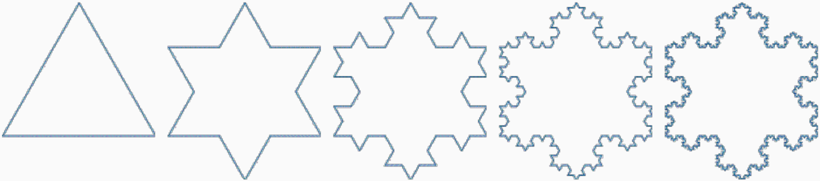


Question 14 *(30 points)*

Find the exact value of

$$\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+\ldots}}}}$$

Question 15 *(30 points)*



Starting with an equilateral triangle with an area of 1.

As shown above, each time we add a smaller equilateral triangle to the middle third of each side of the previous shape.

If we repeat this process forever, what is the area of the resultant shape?

Question 16 *(30 points)*

Pick a whole number. Add one. Square the answer. Multiply the answer by 4. Subtract 3.

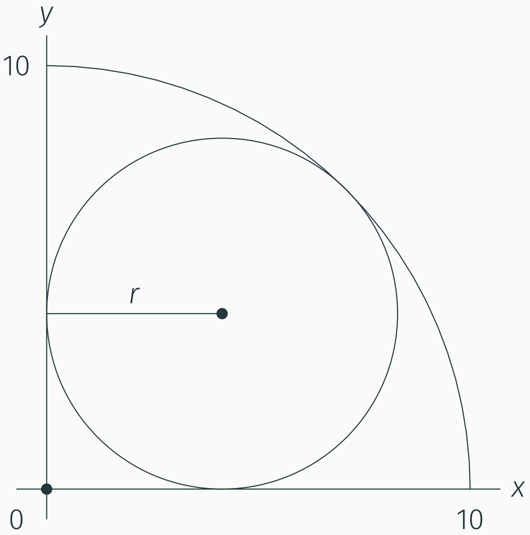
Which of the following statements are true regardless of which starting number is chosen?

1. The final answer is odd.
2. The final answer is one more than a multiple of three.
3. The final answer is one more than a multiple of eight.
4. The final answer is not prime.
5. The final answer is not one less than a multiple of three.

Round 3

Question 17 *(35 points)*

What is the exact radius of the circle?



Question 18 *(35 points)*

The factorial of a positive integer n, denoted by $n!$, is the product of all positive integers less than or equal to n. For example,

$$4! = 4 \times 3 \times 2 \times 1 = 24.$$

Calculate the value of

$$\frac{(10! + 9!)(8! + 7!)(6! + 5!)(4! + 3!)(2! + 1!)}{(10! - 9!)(8! - 7!)(6! - 5!)(4! - 3!)(2! - 1!)}.$$

Question 19 *(35 points)*

Write a short poem about a concept in mathematics.

Question 20 *(35 points)*

The number of *pairs of positive integers* x, y which solve the equation

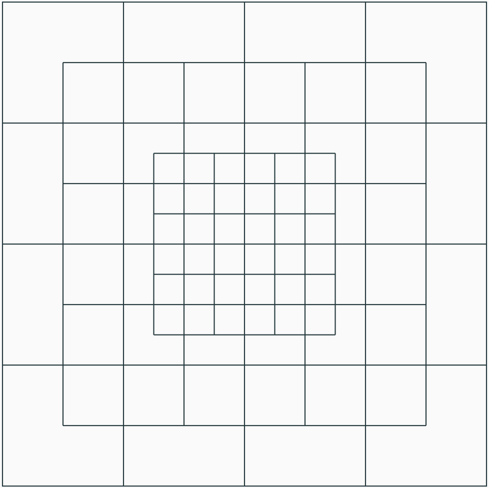
$$x^3 + 6x^2y + 12xy^2 + 8y^3 = 2^{30}$$

is

$$(a)\ 0; \quad (b)\ 2^6; \quad (c)\ 2^9 - 1; \quad (d)\ 2^{10} + 2.$$

Question 21 *(35 points)*

How many squares are in the diagram below?



Question 22 *(35 points)*

The natural numbers are entered in a grid in successive diagonals, as shown.

16				
11				
7	12			
4	8	13		
2	5	9	14	
1	3	6	10	15

The number 9 has the grid position $(3, 2)$, 16 has the grid position $(1, 6)$. Find the grid position of 1000.

Question 23 *(35 points)*

Six classmates are swapping gifts at an end-of-term celebration. For each pair of students, they can exchange gifts once at most. In each round of exchange, the two students send each other a gift.

Given that these 6 classmates have performed 13 rounds of exchange, what are the possible numbers of classmates that have received exactly 4 gifts?

Question 24 *(35 points)*

A box contains a single variable polynomial of finite order, which has been labelled $f(x)$. When you input any x into the box, you we be shown the value of $f(x)$.

What is the minimum number of inputs needed to determine every coefficient of the polynomial?