PARADOX

Welcome to the O'Week edition of *Paradox,* the probably triannually published magazine of the Melbourne University Mathematics and Statistics Society. *Paradox* aims to provide challenges, entertainment and occasional pieces of useful information. Readers are invited to make contributions or comments, which can be e-mailed to us at `paradox@mundoe.maths.mu.oz.au` or placed in the *Paradox* box, near the southern entrance to the Richard Berry Building.

Many thanks to those who helped with this issue, especially Sally Miller, Frances Ip, Lawrence Ip, Chaitanya Rao and Tony Wirth.

Vanessa Teague, *Paradox* editor.

MELBOURNE UNIVERSITY MATHEMATICS AND STATISTICS SOCIETY

If you are studying mathematics or statistics this year, then you are automatically a member of the Melbourne University Mathematics and Statistics Society (MUMS). MUMS is a student society dedicated to providing interesting mathematical and statistical events for students. We arrange talks, videos and other activities for all members, and even give away some food and drinks occasionally (the O'Week barbecue, for example). Anyone is welcome at the events we advertise.

The main activity on the MUMS calendar is the annual Maths Olympics, a unique competition which requires a combination of intellectual prowess and physical agility. Entrants compete in teams of five. The main aim is to solve as many of the problems as possible within the time limit. Being able to run back and forth in the theatre at high speeds while carrying the answers helps too. It's only seven months away, so find a team and start training. Of course, if you see people running up and down the stairs in Theatre A, you will know not to laugh.

The MUMS Annual General Meeting will be held at 1pm on Friday 21st March, in Theatre A, Richard Berry Building.

For more information see either the MUMS noticeboard in the first year learning centre or the MUMS homepage at `http://www.maths.mu.oz.au/~lip/mums/mums.html`.

O'WEEK BARBECUE

All mathematics and statistics students are invited to the O'Week barbecue. Free sausages, salads and vegie burgers will be provided.

1pm, Thursday 27 February, 1997, Old Geology Courtyard

UNDERGRADUATE SEMINAR SERIES

One of MUMS' activities this semester will be an undergraduate seminar series. About once every two weeks, somewhere in the Richard Berry Building, there will be a talk or video presented by a student or lecturer from this department. The main aim of the series is to provide all undergraduate students with an interesting glimpse of some mathematics and statistics. Topics to be covered include: the economics and dynamics of exchange rates, cryptography, data compression, operations research and number theory.

THE INAUGURAL UNDERGRADUATE SERIES SEMINAR
FERMAT'S LAST THEOREM
PRESENTED BY FRANK CALEGARI

Pierre de Fermat (1601–1665) was a lawyer and amateur mathematician and philosopher. In about 1637, he annotated his copy (now lost) of Bachet's translation of Diophantus' Arithmetika with the following statement:

> Cubem autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos, et generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere: cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caparet.

In English, and using modern terminology, the paragraph above reads as:

> There are no positive integers x, y, z such that $x^n + y^n = z^n$ for $n > 2$. I've found a remarkable proof of this fact, but there is not enough space in the margin to write it.

Fermat never published a proof of this statement. It came to be known as Fermat's Last Theorem (FLT) not because it was his last piece of work, but because it is the last remaining statement in the posthumous list of Fermat's works that needed to be proven or independently verified. All others have either been shown to be true or disproven long ago.

A couple of years ago, a mathematician at Princeton, Andrew Wiles, dramatically revealed a proof in a marathon seminar at Cambridge. The proof was later found to be flawed, but Wiles managed to patch up the proof in a satisfactory manner about a year later.

Frank Calegari will present a talk on Fermat's Last theorem on Wednesday 12 March at 1:10 in Theatre B, Richard Berry Building.

MUMS T-SHIRT COMPETITION

MUMS is looking for a design for a T-shirt. The design must be somewhat mathematical and statistical in nature, contain the phrase "Melbourne University Mathematics and Statistics Society" and must be something that people would actually want to buy and wear—nothing vulgar, please!!! All designs will be judged according to originality, imagination and creativity, colour (no limitations) and mathematical content (it must contain some). The winning designer will be awarded **$50**. Partial prizes will be given to people who provide particularly imaginative ideas or suggestions (so you don't have to enter a completed work of art). All entries must be in by the end of March, 1997. So all of you with a creative bent may want to use your energy productively and win some money along the way. Entries may be placed in the *Paradox* box or mailed to:

Melbourne University Mathematics and Statistics Society
Department of Mathematics and Statistics
The University of Melbourne
Parkville VIC 3052

MATHEMATICS MENTOR SCHEME

Welcome to all 121 and 211 students who have joined the mathematics mentor scheme. We hope that mentors and mentees alike enjoy this opportunity to meet some other mathematics students. The initial aim of the scheme is to help first year students to settle into the University, and this will probably be facilitated by a group meeting some time in O'Week, followed by weekly or fortnightly gatherings. If you have any queries, talk to Dr Kerry Landman, on 9344 6762.

SOME PROBLEMS FOR PRIZEMONEY

Paradox will award a prize to the first-year student who enters the best solution to any of these problems. We will also print the solutions with the winners' names in the next edition. You can leave your solutions in the *Paradox* box or e-mail them to us at `paradox@mundoe.maths.mu.oz.au`. All solutions must be received by the end of April, 1997. Numbers after the problem number refer to the amount of money offered.

1. (\$10) Points A and B have Cartesian coordinates $(0, -10)$ and $(2, 0)$ respectively. Find the point C on the parabola $y = x^2$ which minimises the area of $\triangle ABC$ and determine this area.
2. (\$15) How many six digit numbers (base 10 of course) contain each of their digits more than once? For example, 234342 and 334343 are counted but 232331 is not.
3. (\$15) How can one arrange six matchsticks so that each one is touching every other? The arrangement must be constructible and reasonably stable in reality.

SOME EASIER (BUT LESS LUCRATIVE) PROBLEMS

The MUMS treasurer forbids the editor from giving away more than \$90 in one edition, so no pecuniary reward is offered for these problems. However, the first person to submit a correct solution to each problem will receive the honour and glory associated with having their solution published in this illustrious periodical and, better yet, we will give them a year's subscription to *Paradox* absolutely free!

1. Prove that given five integers, one can choose three of them whose sum is divisible by three.
2. A man walks into a bar, orders a drink, and starts chatting with the bartender. After a while, he learns that the bartender has three children. "How old are your children?" he asks. "Well," replies the bartender, "the product of their ages is 72." The man thinks for a moment and then says, "That's not enough information." "All right," continues the bartender, "if you go outside and look at the building number posted over the door to the bar, you'll see the sum of the ages." The man steps outside, and after a few moments he re-enters and declares, "Still not enough!" The bartender smiles and says, "My youngest loves strawberry ice cream."

   How old are the children?

THE TOP SIX REASONS FOR NOT HAVING DONE YOUR MATHS HOMEWORK

If you're bored after thirteen years of "My dog ate it," you could try:

- I accidentally divided by zero and my paper burst into flames.
- Isaac Newton's birthday.
- I could only get arbitrarily close to my textbook. I couldn't actually reach it.
- I have the proof, but there isn't enough room to write it in this margin.
- I was watching the World Series and I got tied up trying to prove that it converged.
- I took time out to snack on a doughnut and a cup of coffee. I spent the rest of the night trying to figure out which one to dunk.

SOME MORE MATHEMATICAL HUMOUR

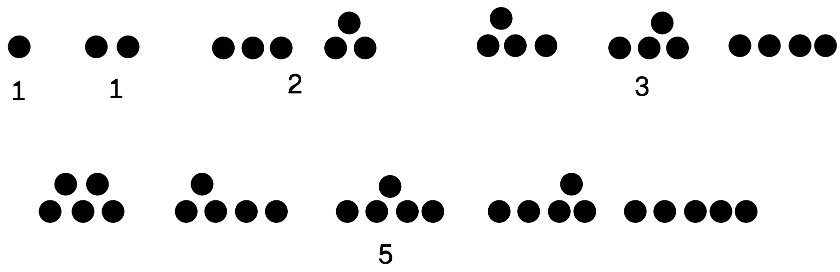
A mathematician, a physicist, a computer scientist and an engineer are given the same problem: prove that all odd numbers greater than two are prime. They proceed:
Mathematician: 3 is prime, 5 is prime, 7 is prime, 9 is not prime. Counterexample — the claim is false.
Physicist: 3 is prime, 5 is prime, 7 is prime, 9 is an experimental error, 11 is prime, ...
Computer Scientist: 3 is prime, 5 is prime, 7 is prime, 7 is prime, 7 is prime, ...
Engineer: 3 is prime, 5 is prime, 7 is prime, 9 is prime, 11 is prime, ...

THE STRONG LAW OF SMALL NUMBERS

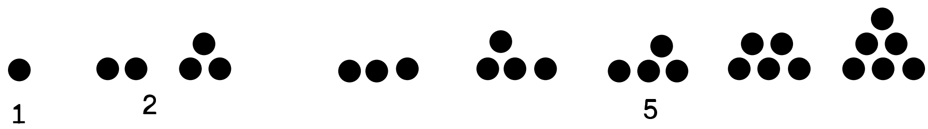
Often sequences of small numbers appear to exhibit some pattern, but when larger numbers of the sequence are calculated it is clear that this was merely coincidental. Here are a few examples of patterns which appear to hold, at least for the first few numbers. Some of them are generally true, and some eventually break down. Your task is to decide which are which. We have provided answers at the end.

1. Consider the sequence $\{a_n\}$, where a_n equals one plus the sum of the first n terms of the Fibonacci sequence[1]. It begins $a_1 = 1+1 = 2, a_2 = 1+1+1 = 3, a_3 = 1+1+1+2 = 5, a_4 = 1+1+1+2+3 = 8, a_5 = 13$. Is a_n always the $(n+2)$th Fibonacci number?
2. Consider numbers of the form $2^p - 1$, where p is prime. Are all such numbers prime? For example $2^2 - 1 = 3, 2^3 - 1 = 7, 2^5 - 1 = 31, 2^7 - 1 = 127$. Such primes are called Mersenne primes.
3. This sequence $\{a_n\}$ begins 1,1,2,3,5,8. Here a_n equals number of ways in which you can arrange n coins in horizontal rows so that all the coins in each row touch and every coin above the bottom row touches two coins in the row below it. (See diagram on next page). Does it continue to be the Fibonacci sequence?

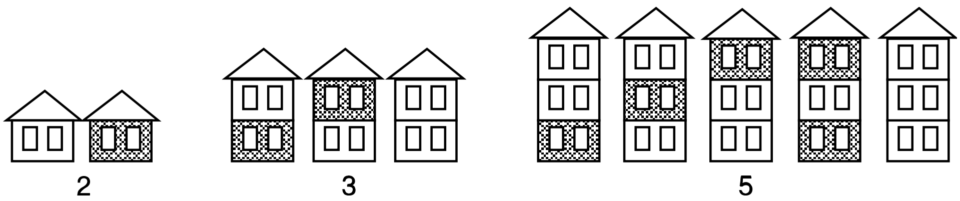
[1]The Fibonacci sequence is 1,1,2,3,5,8,13,21,..., where each number is the sum of the two numbers which precede it.

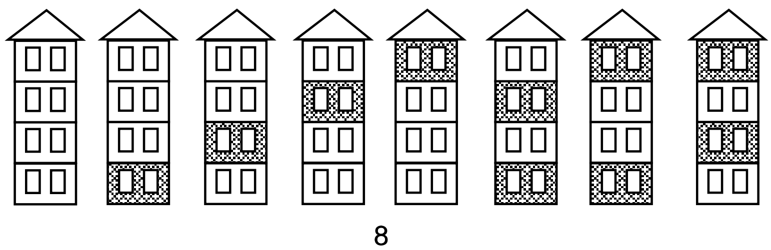


4. The same as the last example, except that n is the number of coins in the bottom row. Does it continue to list every second Fibonacci number?



5. The final sequence is $2, 3, 5, 8, \ldots, a_n$. Here a_n is the number of ways of painting a mathematics and statistics building of n floors with purple and green paint so that no two adjacent floors are painted purple (they may, however, be green). Shaded areas in the figure represent purple paint, unshaded areas green.





Is this the Fibonacci sequence?

ANSWERS

1. Yes. Suppose we know that $a_n = F_{n+2}$ for the first k numbers in the sequence, where F_n denotes the nth Fibonacci number. (so $k \geq 4$ from the examples listed). Then

$$\begin{aligned} a_{k+1} &= (1 + F_1 + F_2 + \ldots + F_k) + F_{k+1} \\ &= a_k + a_{k-1} \end{aligned}$$

Hence each element of the sequence is the sum of the two before it, so the numbers continue to be Fibonacci, by mathematical induction. (For 121 students who haven't heard of mathematical induction, stay tuned. You will learn about it in a couple of weeks.)

2. No. For example, $2^{11} - 1 = 23 \times 89$ is not prime.

3. No. The sequence continues 12,18,26,. . . .

4. Yes. Imagine arranging coins on the n coins in the bottom row. There are a_{n-1} ways of arranging them with $n-1$ coins in the second row. There are $2a_{n-2}$ ways of arranging them with $n-2$ coins in the second row, because there are two ways a second row of $n-2$ coins can be positioned. (See diagram).

In the same way, there are $3a_{n-3}$ ways of arranging them with $n-3$ coins in the second row, and so on. There is one way of arranging the coins with no coins in the second row. Therefore, $a_n = a_{n-1} + 2a_{n-2} + 3a_{n-3} + \ldots + (n-1)a_1 + 1$
Note that for Fibonacci Numbers

$$\begin{aligned} F_{2n-1} &= F_{2n-2} + F_{2n-3} \\ &= F_{2n-2} + (F_{2n-3} - F_{2n-5}) + (F_{2n-5} - F_{2n-7}) + (F_{2n-7} - F_{2n-9}) + \ldots + (F_3 - F_1) + 1 \\ &= F_{2n-2} + F_{2n-4} + F_{2n-6} + \ldots + F_2 + 1 \\ &= F_{2n-2} - F_{2n-4} + 2(F_{2n-4} - F_{2n-6}) + 3(F_{2n-6} - F_{2n-8}) + \ldots + (n-2)(F_4 - F_2) \\ &\quad +(n-1)F_2 + 1 \\ &= F_{2n-3} + 2F_{2n-5} + 3F_{2n-7} + \ldots + (n-2)F_3 + (n-1)F_1 + 1 \end{aligned}$$

So the sequence $\{a_n\}$ continues to list the Fibonacci numbers.

5. Yes. Consider an n-floor building. The top floor can be either green or purple. If it is green, the remaining $n-1$ floors can be coloured in a_{n-1} ways. If it is purple, the $(n-1)$th floor must be green, and the remaining $n-2$ floors can be coloured in a_{n-2} ways. So $a_n = a_{n-1} + a_{n-2}$, implying that the sequence continues to be Fibonacci.

REFERENCES

1. "Fibonacci Forgeries," *Scientific American,* 272 #5 (May 1995), pp. 82–85.

2. Gardner, Martin. "Mathematical Games: patterns in primes are a clue to the strong law of small numbers," *Scientific American,* 243 #6 (Dec 1980), pp. 18–28.

3. Guy, Richard. "The Strong Law of Small Numbers," from *The lighter side of mathematics* ed. Guy and Woodrow. pp. 265–280.