1. (10 marks)

   If $x = \frac{111110}{111111}$, $y = \frac{222221}{222223}$, and $z = \frac{333331}{333334}$, place $x$, $y$, and $z$ in order from largest to smallest.

2. (10 marks)

   Let $x_n$ be the $n$th odd composite number. What is $x_{x_1}$?
   *Note:* A number is composite if it is not prime, i.e. it has greater than two factors. The number 1, however, is neither composite nor prime.

3. (10 marks)

   During the refreshments served after the MUMS seminar last Friday, Jeremy ate Tim Tams for 1 minute whilst Sam ate for 9 minutes. The total number of Tim Tams consumed by the pair was 8. If, after the next seminar, they consume 11 due to Jeremy eating for 2 minutes whilst Sam eats for 3 minutes, what is Jeremy's rate of consumption in Tim Tams per minute? (Assume that they each eat at the same rate after both seminars.)

4. (10 marks)

   Norm and Dan are in a beauty contest for which the second prize is $10. Unable to decide, the judges devise a (rather silly) random experiment to determine the winner. They have two fair coins and two French coins that yield *face* with 60% probability and *pile* with 40% probability. All four coins are tossed. Norm is declared the winner if there are fewer than two heads and fewer than two *face*s. What is the percentage probability of Dan winning?

5. (10 marks) **(Change runner now)**

   Given that

   $$2\sqrt{3\left(\frac{2}{4} \cdot \frac{3}{5} \cdot \frac{4}{6} \cdot \cdots \cdot \frac{x}{y}\right)} = 1,$$

   what is $x + y$?

6. (10 marks)

   Suppose that $x + x^{-1} = 5$. What is $x^3 + x^{-3}$?

7. (10 marks)

   Geordie, slightly drunk after a scotch, collides heavily with Lori, who is superbly dressed in an outfit that sports a pearl necklace. As a result of the accident, the necklace is broken, and pearls are scattered everywhere. Lori searches frantically and manages to find 23 pearls. Lori does not know exactly how many pearls were in the necklace, but recalls that when she counted them in groups of 9, 1 was left over; and when she counted them in groups of 7, 5 were left over. Moreover, she knows that there were fewer than 100 pearls in the necklace. How many pearls were not found?

8. (10 marks)

   What is the largest power of 3 dividing $\binom{400}{100}$?
   *Note:* $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ is the number of different ways of choosing $r$ objects from $n$ of them.

9. (15 marks)                                                                 (200 marks remain)

How many different ways can five teams in a round robin completely tie? (i.e. every team wins exactly two of their four games.)

10. (15 marks) **(Change runner now)**                                         (185 marks remain)

How many positive integers, $x$, satisfy the equation

$$\left\lfloor \frac{x}{7} \right\rfloor = \left\lfloor \frac{x}{8} \right\rfloor + 1?$$

*Note:* $\lfloor x \rfloor$ denotes the greatest integer less than or equal to $x$.

11. (15 marks)                                                                (170 marks remain)

ACME has recently released a new Lie Detector which is capable of detecting lies with 90% accuracy. (By this they mean that if a person lies, the machine will beep 90% of the time, and if a person tells the truth it will stay silent 90% of the time.) Suppose that 95% of people tell the truth. Sally interviews a murder suspect and one of their answers makes the machine beep. What is the probability that the suspect is telling the truth?

12. (15 marks)                                                                (155 marks remain)

George walks down to the bottom of an escalator that is moving up and he counts 150 steps. Chaitanya walks up to the top of the escalator and counts 75 steps. If George's speed of walking (in steps per unit time) is three times Chaitanya's speed, how many steps are visible on the escalator at any given time? (Assume that this number is constant.)

13. (15 marks)                                                                (140 marks remain)

Jolene and Kuhn both have their birthdays on Valentine's day. Jolene is twice as old as Kuhn was when Jolene was as old as Kuhn was six years ago. On the other hand, Kuhn is as old as Jolene was when Kuhn was twice as old as Jolene was when Kuhn was twice as old as Kuhn was when Jolene was a quarter of her age today. Furthermore, in three years Kuhn will be half as old as Jolene will be when Kuhn is five times as old as Jolene was when Kuhn was as old as Jolene was when Kuhn was born.

How old are Jolene and Kuhn respectively?

14. (15 marks)                                                                (125 marks remain)

Evaluate

$$\sqrt{1 - \frac{1}{\sqrt{2}} + \sqrt{\frac{3}{2} + \sqrt{1 - \frac{1}{\sqrt{2}} + \sqrt{\frac{3}{2} + \sqrt{\cdots}}}}}.$$

15. (15 marks) **(Change runner now)**                                         (110 marks remain)

Let $a_1, a_2, \ldots, a_{10}$ be a permutation of the integers $1, 2, \ldots, 10$. For each permutation, form the sum

$$|a_1 - a_2| + |a_3 - a_4| + |a_5 - a_6| + |a_7 - a_8| + |a_9 - a_{10}|.$$

Find the average value of all such sums.

16. (15 marks) (95 marks remain)

How many triangles with positive area are there whose vertices are points in the *xy*-plane with integer coordinates $(x, y)$ satisfying $1 \le x \le 5$ and $1 \le y \le 5$ ?

17. (20 marks) (80 marks remain)

Consider the points $1, 1/2, 1/3, \ldots$ on the real number line. You are given five small bars, all of length $p$, which are to be placed on the number line such that all points will be covered. What is the minimum value of $p$ that will allow you to do this?
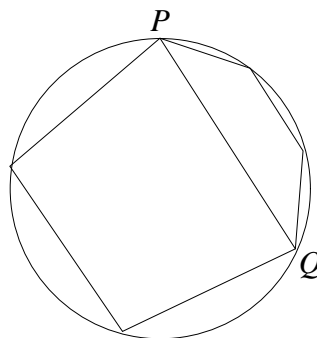
18. (20 marks) (60 marks remain)

In the right-angled triangle, $\triangle ABD$, below, $AB = 4$ and $BD = 3$. Point $C$ lies on $BD$ such that $2\angle BCA = 3\angle BAD$. Find the length of $BC$.
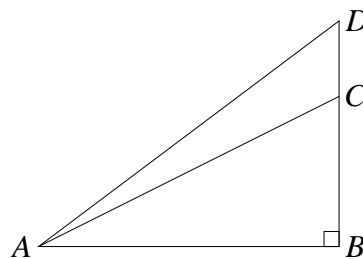


19. (20 marks) (40 marks remain)

A hexagon inscribed in a circle has three consecutive sides each of length 1 and three consecutive sides each of length 2. The chord $PQ$ of the circle divides the hexagon into two trapezoids, one with three sides each of length 1 and the other with three sides each of length 2. What is the length of $PQ$?



20. (20 marks) (20 marks remain)

A six-digit integer, $XYXYXY$, where $X$ and $Y$ are digits, is equal to five times the product of three consecutive odd integers. Determine these three odd integers.