

Question 1. (5 points)**(180 points remaining)**

What is the product of all the (positive integer) factors of 18? Express your answer as a 4 digit integer.

Question 2. (5 points)**(175 points remaining)**

A car travels from A to B at the rate of 20 km/h and then returns from B to A at the rate of 80 km/h, without stopping. What is the average speed for the round trip, in kilometres per hour?

Question 3. (5 points)**(170 points remaining)**

In MUMSland coins are shaped like regular polygons. All coins have the same side length but they vary in the number of sides they have. There are some coins that are shaped like squares, and others that are shaped like hexagons. There is another type of coin, so that the square, hexagon and the third type of coin fit together snugly at a vertex, with no hole between coins (see the diagram below).

How many sides does the third type of coin have?

Question 4. (5 points)**(165 points remaining)**

In MUMSland money comes in the currency of proofs. Proofs are divided up into smaller denominations called propositions and lemmas. 1 proof is worth 10 propositions. 1 proposition is worth 10 lemmas. There are coins in circulation worth 1 lemma, 2 lemmas, 5 lemmas, 1 proposition, 2 propositions, 5 propositions, 1 proof and 2 proofs.

Lori is about to travel on a hot air balloon to her economics tutorial but she needs to pay the fare, and must pay exactly. She knows the fare is more than 1 proof and less than 3 proofs. At least how many coins must she carry to be sure of carrying the exact correct fare?

Question 5. *Change Runner Now!* (5 points — 160 points remaining)

Sam's credit card number has 14 digits. But the number is special — the sum of every four consecutive digits is 16. The first digit is 1, the second digit is 6, and the twelfth digit is 3.

What is the seventh digit in Sam's credit card number?

Question 6. (5 points) (155 points remaining)

Geordie baked some loaves of bread today and decided to give them away in the following fashion: the first person who came received half the bread he baked plus half a loaf. The second person received half of what was left plus half a loaf, and so on. After the fifth person had left, Geordie realised that all of his bread was gone. How many loaves of bread did Geordie bake?

Question 7. (5 points) (150 points remaining)

What is the smallest non-negative value of the following expression?

$$\pm 1^3 \pm 2^3 \pm 3^3 \pm 4^3 \pm 5^3$$

Question 8. (5 points)**(145 points remaining)**

A trapezium is divided into four triangles by its diagonals. The areas of the top and bottom triangles (adjacent to the parallel sides) are 25 and 49 square light years respectively. Find the area of the trapezium in square light years.

Question 9. (10 points)**(135 points remaining)**

Joe loved cantaloupes. He put a spherical one in his cubical cantaloupe fridge and it just fit — touching all of the fridge's (very thin) sides and lid.

However an alien civilization angry at human worship of cantaloupes destroyed all cantaloupe plantations on Earth. But Joe was prepared for an escalation of anti-cantaloupist violence and had purchased a cantaloupe-fridge cryogenic suspension device. To preserve his cantaloupe for future generations, Joe put the fridge inside the spherical suspension chamber and it just fit.

The chamber wasn't very well packed, so Joe decided to remove the fridge, puree all his cantaloupes (all identical) and pour the puree directly into the chamber. How many pureed cantaloupes can Joe fit in the cantaloupe suspension chamber? (The answer need not be a whole number, and should be expressed exactly).

Question 10. *Change Runner Now!* (10 points — 125 points remaining)

Claire and her grandson were born on the same day of the year. One year on that day Claire noted that her age had become an integral multiple of her grandson's age and furthermore, this phenomenon would be repeated for the following five birthdays as well. How old was Claire when she made these observations?

Question 11. (10 points) (115 points remaining)

In Norm's favourite maths textbook, on each page the page number appears once, starting from page 1 (numbers are written in decimal notation). 642 digits are used in Norm's favourite textbook to number pages. How many pages are in Norm's favourite textbook?

Question 12. (10 points) (105 points remaining)

$WXYZ$ is a square of side length 30. V is a point on XY and P is a point inside the square with PV perpendicular to XY . $PW = PZ = PV - 5$. What is PV ?

Question 13. (10 points)**(95 points remaining)**

$PQRS$ is a rectangle in which $PQ = 2PS$. T and U are the midpoints of PS and PQ respectively. QT and US intersect at V . What is the area of $QRSV$ divided by the area of PQT ? Express your answer as a fraction in simplest terms.

Question 14. (10 points)**(85 points remaining)**

Jolene has a 13 pound measuring weight that broke into three pieces as the result of a freak tsunami. When the pieces were subsequently weighed, it was found that the weight of each piece was a whole number of pounds and that using a two-pan balance, the three pieces could be used to weigh every integral weight between 1 and 13 pounds. What were the weights of the pieces in pounds, in increasing order?

Question 15. *Change Runner Now!* (10 points — 75 points remaining)

Let a , b and c denote the angles of elevation of a tower measured at horizontal distances of 100, 200 and 300 metres from the tower, respectively. If $a + b + c$ is a right angle, find the height of the tower in metres.

Question 16. (10 points) (65 points remaining)

A mathematician had nine children when he died — quadruplets aged 17, quadruplets aged 18, and one nineteen year old – all born on the same day of the year. His assets, totalling \$100,000, according to his will, were to be divided and put into super-speculative interest bearing trusts. The trust fund compounded the amounts for each child annually until they turned 21, upon which they could access the amount.

As each child turned 21 they were overjoyed to discover that each individual inheritance had compounded to the same amount of \$400,000. The interest rate had remained constant over the entire period.

If you had put \$1 into the fund a year ago, how many dollars would you have gained in the last year? Express your answer exactly.

Question 17. (15 points)**(50 points remaining)**

How many pairs of natural numbers (u, v) are there such that $1 \leq u \leq v$, with the property that the least common multiple of u and v is 2000?

Question 18. (15 points)**(35 points remaining)**

The positive fractions can be written out in the following pattern, occupying an infinite number of positions:

$$\frac{1}{1}, \frac{2}{1}, \frac{1}{2}, \frac{3}{1}, \frac{2}{2}, \frac{1}{3}, \frac{4}{1}, \frac{3}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{1}, \frac{4}{2}, \frac{3}{3}, \frac{2}{4}, \frac{1}{5}, \dots$$

For instance the number $\frac{1}{3}$ occupies position number 6 and the number $\frac{2}{6} (= \frac{1}{3})$ occupies position number 42.

Consider the first five positions occupied by $\frac{1}{2}$, or by fractions equal to $\frac{1}{2}$. What is the sum of the numbers of these five positions?

Question 19. (15 points)

(20 points remaining)

Find all positive integers n such that $2^{200} + 2^4 + 2^n - 2^{103}$ is a perfect square.

Question 20. (20 points)

FINAL QUESTION!

Let $x = (4 + \sqrt{17})^{714}$. It is known that the decimal representation of x begins 3480003819317.... There are infinitely many digits after the decimal point and N digits before the decimal point. What are the $(N - 1)^{\text{th}}$ and N^{th} digits of x after the decimal point, in order?