Question 1 ***30 Marks***

First year reps, Bowan, Anand, Jason, Julie and Steven have some debts to settle. Bowan owes each of Anand and Jason a chocolate bar, Anand owes each of Jason and Julie a chocolate bar, Jason owes each of Julie and Steven a chocolate bar, Julie owes each of Steven and Bowan a chocolate bar and Steven owes each of Bowan and Anand a chocolate bar. What is the minimum total number of chocolate bars which needs to be bought to settle all the debts?

Question 1 ***30 Marks***

First year reps, Bowan, Anand, Jason, Julie and Steven have some debts to settle. Bowan owes each of Anand and Jason a chocolate bar, Anand owes each of Jason and Julie a chocolate bar, Jason owes each of Julie and Steven a chocolate bar, Julie owes each of Steven and Bowan a chocolate bar and Steven owes each of Bowan and Anand a chocolate bar. What is the minimum total number of chocolate bars which needs to be bought to settle all of the debts?

Question 2 ***30 Marks***

Find two (not necessarily distinct) positive integers such that their sum is equal to their product.

Question 2 ***30 Marks***

Find two (not necessarily distinct) positive integers such that their sum is equal to their product.

Question 3 ***30 Marks***

The number 101_b is one less than a perfect cube. What is b?

Question 3 ***30 Marks***

The number 101_b is one less than a perfect cube. What is b?

Question 4 ***30 Marks***

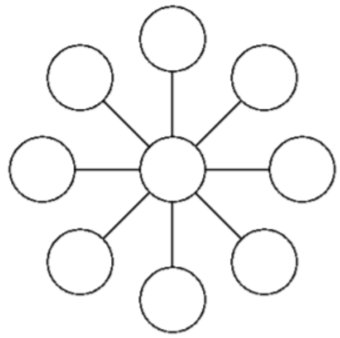
How many distinct positive factors does 2018 have?

Question 4 ***30 Marks***

How many distinct positive factors does 2018 have?

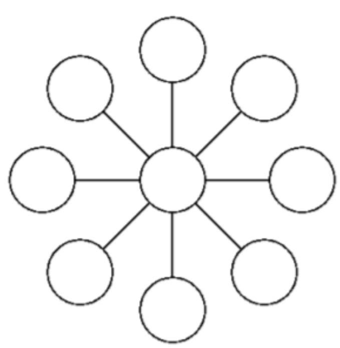
Question 5 ***CHANGE WALKER NOW*** ***30 Marks***

One rainy afternoon, Vanessa arranged the numbers 1, 4, 5, 7, 8, 9, 10, 11, 14 into the circles shown, so that every line added up to the same value. What number was in the middle?



Question 5 ***CHANGE WALKER NOW*** ***30 Marks***

One rainy afternoon, Vanessa arranged the numbers 1, 4, 5, 7, 8, 9, 10, 11, 14 into the circles shown, so that every line added up to the same value. What number was in the middle?



Question 6 ***40 Marks***

How many ways can you arrange the letters of the word MATHEMATICS?

Question 6 ***40 Marks***

How many ways can you arrange the letters of the word MATHEMATICS?

Question 7 ***40 Marks***

Find the greatest common divisor of the numbers
$A_n = 2^{3n} + 3^{(6n+2)} + 5^{(6n+2)}$ when n = 0,1,...1999

Question 7 ***40 Marks***

Find the greatest common divisor of the numbers
$A_n = 2^{3n} + 3^{(6n+2)} + 5^{(6n+2)}$ when n = 0,1,...1999

Question 8 ***40 Marks***

You are dealt 13 cards from a standard 52-card deck. What is the probability that you have at least four of the same suit?

Question 8 ***40 Marks***

You are dealt 13 cards from a standard 52-card deck. What is the probability that you have at least four of the same suit?

Question 9 ***40 Marks***

A 4L cylindrical tank has a hole drilled half way up its side, thus allowing water to leak out at 2L/min. We can fill up the tank in $2\sqrt{3}$ minutes if we pump water into the empty tank at a steady rate of v L/min, what is the value of v?

Question 9 ***40 Marks***

A 4L cylindrical tank has a hole drilled half way up its side, thus allowing water to leak out at 2L/min. We can fill up the tank in $2\sqrt{3}$ minutes if we pump water into the empty tank at a steady rate of v L/min, what is the value of v?

Question 10 ***CHANGE WALKER NOW*** ***40 Marks***

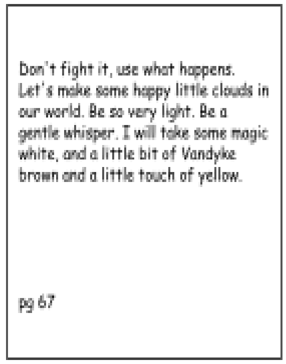
An island consists of the peak of a volcano such that one side forms a triangular shaped cliff. At the peak, there is a village and at the base of the cliff, a jetty. Due to the terrain, it is only possible to build one way roads on the island. To get supplies, a villager must travel 150 m along the edge of the cliff to the jetty. If the road along the base of the cliff is 140 m long and the village 120 m above the base, what distance is covered when ascending from the cliff base to the village from the other side? Assume roads form a triangle.

Question 10 ***CHANGE WALKER NOW*** ***40 Marks***

An island consists of the peak of a volcano such that one side forms a triangular shaped cliff. At the peak, there is a village and at the base of the cliff, a jetty. Due to the terrain, it is only possible to build one way roads on the island. To get supplies, a villager must travel 150 m along the edge of the cliff to the jetty. If the road along the base of the cliff is 140 m long and the village 120 m above the base, what distance is covered when ascending from the cliff base to the village from the other side? Assume roads form a triangle.

Question 11 ***50 Marks***

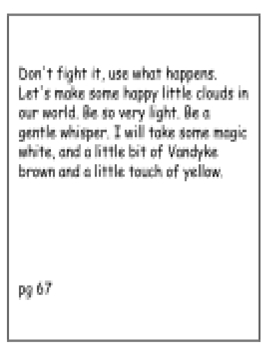
Newspapers are created by laying sheets of paper on top of each other, and then folding down the center line, creating a 'book'. A 100-sheet newspaper has its pages numbered from 1 to 400.



This sheet falls out of the newspaper. What is the sum of the numbers of the other 3 pages on the same sheet?

Question 11 ***50 Marks***

Newspapers are created by laying sheets of paper on top of each other, and then folding down the center line, creating a 'book'. A 100-sheet newspaper has its pages numbered from 1 to 400.



This sheet falls out of the newspaper. What is the sum of the numbers of the other 3 pages on the same sheet?

Question 12 ***50 Marks***

At a conference there are n mathematicians. Each of them knows exactly k participants. Find the smallest number of k such that there are at least three mathematicians that are acquainted with the other two.

Question 12 ***50 Marks***

At a conference there are n mathematicians. Each of them knows exactly k participants. Find the smallest number of k such that there are at least three mathematicians that are acquainted with the other two.

Question 13 ***50 Marks***

Find the integer solution of the equation:

$$9^x - 3^X = y^4 + 8y^3 + 4y^2 + 8y + 1$$

Question 13 ***50 Marks***

Find the integer solution of the equation:

$$9^x - 3^X = y^4 + 8y^3 + 4y^2 + 8y + 1$$

Question 14 ***50 Marks***

MUMffins are sold in packs of 5, 11, and 18. What is the largest number of donuts I cannot buy without splitting packs?

Question 14 ***50 Marks***

MUMffins are sold in packs of 5, 11, and 18. What is the largest number of donuts I cannot buy without splitting packs?

Question 15 ***CHANGE WALKER NOW*** ***50 Marks***

In a game show, there are 20 boxes. 19 boxes are empty and one has $200,000 inside.
Contestant Ivy was asked to choose one box in hopes of winning $200,000. The host then showed Ivy the contents of one of the other boxes, which was empty, and offered Ivy the chance to change their box for one of the remaining boxes. If Ivy changed boxes, what would be the probability the box chosen contained $200,000?

Question 15 ***CHANGE WALKER NOW*** ***50 Marks***

In a game show, there are 20 boxes. 19 boxes are empty and one has $200,000 inside.
Contestant Ivy was asked to choose one box in hopes of winning $200,000. The host then showed Ivy the contents of one of the other boxes, which was empty, and offered Ivy the chance to change their box for one of the remaining boxes. If Ivy changed boxes, what would be the probability the box chosen contained $200,000?

Question 16 ***60 Marks***

All possible integers are arranged in a triangle array as shown below:

1	3	6	10	15	…
2	5	9	14	…	
4	8	13	….		
7	12	…			
11	…				

Find the number of the column and the number of the row where 2002 is put.

Question 16 ***60 Marks***

All possible integers are arranged in a triangle array as shown below:

1	3	6	10	15	...
2	5	9	14	…	
4	8	13	….		
7	12	…			
11	…				

Find the number of the column and the number of the row where 2002 is put.

Question 17 ***60 Marks***

Find the smallest value for n for which there exists the positive integers x_1, x_2, .. x_n with $X_1^4 + x_2^4 ... + x_n^4 = 1998$

Question 17
60 Marks

Find the smallest value for n for which there exists the positive integers x_1, x_2, .. x_n with $X_1^4 + x_2^4 ... + x_n^4 = 1998$

Question 18 ***60 Marks***

At a tennis tournament there were twice as many girls participating than boys. Each pair of players had only one match and there were no draws. The ratio between girls winnings and boys winnings was 7/5. How many players took part in the tournament?

Question 18 ***60 Marks***

At a tennis tournament there were twice as many girls participating than boys. Each pair of players had only one match and there were no draws. The ratio between girls winnings and boys winnings was 7/5. How many players took part in the tournament?

Question 19 ***60 Marks***

Jane, Kate, and Lily are perfect logicians and always tell the truth.
They are each wearing a hat marked with a **distinct** positive integer. They know that one of the hats has a **single-digit integer**, written on it, and that the other two hats each have a divisor of written on them. Each person can only see the two numbers on the others' hats.

Jane: "I don't know my number."
Kate: "I don't know my number."
Lily: "But I know my number!"
Kate: "Now I know my number, too!"
Jane: "I still don't know my number."
What is the **sum of the numbers** written on Kate and Lily's hats?

Question 19 ***60 Marks***

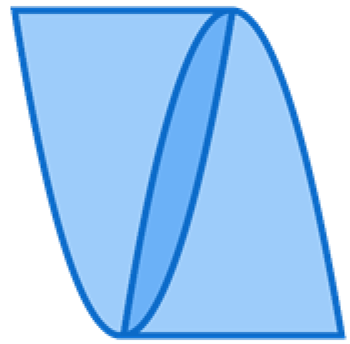
Jane, Kate, and Lily are perfect logicians and always tell the truth.
They are each wearing a hat marked with a **distinct** positive integer. They know that one of the hats has a **single-digit integer**, written on it, and that the other two hats each have a divisor of written on them. Each person can only see the two numbers on the others' hats.

Jane: "I don't know my number."
Kate: "I don't know my number."
Lily: "But I know my number!"
Kate: "Now I know my number, too!"
Jane: "I still don't know my number."

What is the **sum of the numbers** written on Kate and Lily's hats?

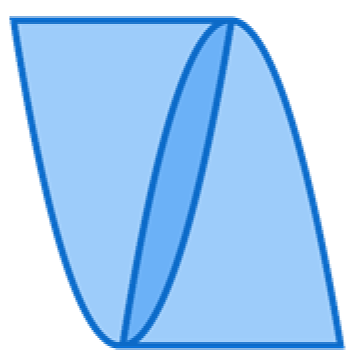
Question 20 **_CHANGE WALKER NOW_** **_60 Marks_**

A figure is constructed from two parabolas with vertical axes of symmetry and distinct vertices. The parabolas intersect through each other's vertices. The figure is contained by horizontal segments that go through the parabolas' vertices. What fraction of the figure's area is the area bounded by the two parabolas?



Question 20 **_CHANGE WALKER NOW_** **_60 Marks_**

A figure is constructed from two parabolas with vertical axes of symmetry and distinct vertices. The parabolas intersect through each other's vertices. The figure is contained by horizontal segments that go through the parabolas' vertices. What fraction of the figure's area is the area bounded by the two parabolas?



Question 21 ***70 Marks***

Consider a 1xn rectangle made from n tiles where n is odd. A pavement is a colouring of each of the n tiles with one of the 4 possible colour so that no two consecutive tiles have the same colour. What is the number of distinct symmetrical pavements? (a symmetrical pavement is a pavement for which tile symmetrical with respect to the centre have the same colour)

Question 21 ***70 Marks***

Consider a 1xn rectangle made from n tiles where n is odd. A pavement is a colouring of each of the n tiles with one of the 4 possible colour so that no two consecutive tiles have the same colour. What is the number of distinct symmetrical pavements? (a symmetrical pavement is a pavement for which tile symmetrical with respect to the centre have the same colour)

Question 22 **70 Marks**

Two points are placed randomly along the circumference of a circle. What is the probability that the chord drawn between them is longer than the radius of the circle?

Question 22 ***70 Marks***

Two points are placed randomly along the circumference of a circle. What is the probability that the chord drawn between them is longer than the radius of the circle?

Question 23 ***70 Marks***

I have four pairs of socks to be hung out side by side on a straight clothes line. The socks in each pair are identical, but the pairs themselves have different colours. How many different colour patterns can be made if no sock is allowed to be next to its mate?

Question 23 ***70 Marks***

I have four pairs of socks to be hung out side by side on a straight clothes line. The socks in each pair are identical, but the pairs themselves have different colours. How many different colour patterns can be made if no sock is allowed to be next to its mate?

Question 24 ***70 Marks***

Emma and Madeline arrive at a cafe independently at random times between 9am and 10am and each stay for m minutes. What is m if there is a 51% chance that they are in the cafe together at some moment

Question 24 ***70 Marks***

Emma and Madeleine arrive at a cafe independently at random times between 9am and 10am and each stay for m minutes. What is m if there is a 51% chance that they are in the cafe together at some moment

Question 25 ***70 Marks***

Matt is lost in the Peter Hall Building. There are 9 rooms on the top floor, arranged in a 3 by 3 grid. There are three mathematicians who have offices on the corner rooms, Anthony Mays, Joyce Zhang, Tom Wong, and Matt's supervisor Arun Ram. Arun's room is diagonally opposite Anthony's. To avoid doing his work Matt decides to see what the mathematicians are up to by going into one of their rooms. Each time he decides to leave a room, he will choose any of the doors (including the one that he came through) with equal likelihood. What is the chance he visits Anthony's room before the others?

Question 25 ***70 Marks***

Matt is lost in the Peter Hall Building. There are 9 rooms on the top floor, arranged in a 3 by 3 grid. There are three mathematicians who have offices on the corner rooms, Anthony Mays, Joyce Zhang, Tom Wong, and Matt's supervisor Arun Ram. Arun's room is diagonally opposite Anthony's. To avoid doing his work Matt decides to see what the mathematicians are up to by going into one of their rooms. Each time he decides to leave a room, he will choose any of the doors (including the one that he came through) with equal likelihood. What is the chance he visits Anthony's room before the others?