

Question 1**10 marks**

Marathon runner Daniel has calculated that if he runs at a constant speed of 15km/h, he will cross the finish line at 10am, but if he runs at a constant speed of 10km/h, he will cross the finish line at noon. At what speed must he run (in km/h) in order to finish at 11am?

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The consecutive angles of a trapezium form an arithmetic progression. If the smallest angle is 75° , what is the largest angle?

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How many digits are in the number $4^{16}5^{25}$ (in base 10)?

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Question 4

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An 11 by 11 by 11 wooden cube is formed by gluing together 11^3 unit cubes. What is the greatest number of unit cubes that can be seen from a single point?

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CHANGE RUNNER NOW

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How many factors does N have?

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Question 6**10 marks**

Nick, David and Sally are going to race up the 99 steps that lead from their study room to the dining hall. Nick can run up 5 steps in the same time as David can run up 4 steps, which is the same time as Sally can run up 3 steps. It is agreed that Nick starts from the bottom, David starts 21 steps up and Sally 38 steps up. If they all start at the same time, in what order (from first to third) will they reach the top?

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Question 7**10 marks**

A semicircle has diameter AB and centre O . The points C and D lie on the semicircle such that $\angle BAC = 20^\circ$ and the lines AC and OD are perpendicular to each other. What is the acute angle between lines AC and BD ?

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Question 8**10 marks**

Andy and Andrew each have a bag of balls that are marked with numbers 1 through to 10. They play a game where they both randomly take a ball out of the bag, and one wins if they have the highest number. Andy decides to cheat by playing balls marked 11 and 12 into his bag. How much more likely is Andy of winning?

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Question 9**10 marks**

Let f be a function from the non-negative integers to the non-negative integers such that

$$f(mn) = mf(n) + nf(m)$$

$$f(10) = 19$$

$$f(12) = 52$$

$$f(15) = 26$$

What is the value of $f(8)$?

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where n is as large as possible.

What is the value of k ?

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Question 12**20 marks**

Chris, Julia, Lu and Ray are comparing their lameness, which only takes integer values.

Chris says: I'm twice as lame as Julia, but only $\frac{2}{3}$ Ray's lameness.

Julia says: I'm $\frac{1}{6}$ more lame than Chris, but $\frac{1}{8}$ less lame than Lu.

Lu says: I'm 50% lamer than Chris, but only 75% Ray's lameness.

Ray says: I'm 50% lamer than Julia, but only 75% Lu's lameness.

Everyone lies about their own lameness, but states other people's lameness accurately. By what factor is the most lame person lamer than the least lame person?

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Question 13**20 marks**

If

$$x + \sqrt{x^2 - 1} + \frac{1}{x - \sqrt{x^2 - 1}} = 20$$

then what is the value of

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Question 14**20 marks**

If

$$\begin{aligned}A &= (8 + 1)(8^2 + 1)(8^4 + 1)(8^8 + 1)(8^{16} + 1)(8^{32} + 1)(8^{64} + 1) \\B &= (64 + 1)(64^2 + 1)(64^4 + 1)(64^8 + 1)(64^{16} + 1)(64^{32} + 1) \\C^2 &= AB\end{aligned}$$

What is the value of $21C + 1$?**Question 14****20 marks**

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Question 15

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While road-testing his new calculator, Han notices that the figure currently displayed is still a legitimate number when the calculator is rotated by 180 degrees. If the original number decreases by 2997 when turned upside down, what is the chance that the original number was divisible by 25?

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Question 16**20 marks**

A cryptographer devises the following method for encoding positive integers. First the integer is expressed in base 5. Then a 1 to 1 correspondence is established between the digits that appear in the expression in base 5 and the elements in the set $\{A, B, C, D, E\}$. Using this correspondence, the cryptographer finds that three consecutive integers in increasing order are encoded as ADE , ADC , AAB respectively. What is the base 10 expression for the integer coded as ABC ?

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What is the area of the largest semicircle that can be contained in a unit square?

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Question 18

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A pyramid has a triangular base, each of side length 6. The other sides have length $\sqrt{15}$.
What is the volume of the pyramid?

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Question 19**20 marks**

a, b, c are positive integers that form an increasing geometric sequence. If $b - a$ is a perfect square, and $\log_6 a + \log_6 b + \log_6 c = 6$, then what is the value of $a + b + c$?

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If $a_0 = 2007$ and

$$a_n = \binom{2007}{n} a_{n-1}, \quad n > 0$$

How many powers of 2007 are in a_{2007} ?

(That is, what is the largest power of 2007 that evenly divides into a_{2007} ?)

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Question 22**30 marks**

Each of the 9 squares in a 3 by 3 grid are coloured blue or red. What is the probability that the colouring does not contain a 2 by 2 red square?

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Question 23**30 marks**

The positive real numbers x, y, z satisfy the equations

$$\begin{aligned}x^2 + xy + \frac{y^2}{3} &= 25 \\y^2 + z^2 &= 9 \\z^2 + zx + x^2 &= 16\end{aligned}$$

What is the value of $xy + 2yz + 3zx$?

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If the complex sequence of numbers z_0, z_1, z_2, \dots satisfy $z_0 = i + \frac{1}{137}$ and for $n \geq 1$,

$$z_{n+1} = \frac{z_n + i}{z_n - i}$$

then what is the value of z_{2007} ?

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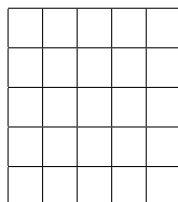
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You have some matchsticks, which can be placed on any of the sixty small edges of a 5×5 grid of squares, as shown below.

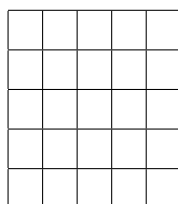


A square is called *enclosed* if one cannot reach it without crossing a matchstick, *super-enclosed* if one cannot reach it without touching an enclosed square (including corners), and *super-duper-enclosed* if one cannot reach it without touching a super-enclosed square. How many ways are there of arranging matchsticks on the grid that create a super-duper-enclosed square?

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