1. (5 marks) (180 marks remain)

Two ferry boats travel back and forth across a river at constant speeds, turning at the banks without loss of time. They leave opposite shores at the same instant, meet for the first time 700m from one shore, continue on and turn around, and then meet for the second time 400m from the opposite shore. What is the width of the river?

2. (5 marks) (175 marks remain)

An equilateral triangle and a regular hexagon have equal perimeters. What is the ratio of the area of the triangle to the area of the hexagon in lowest terms?

3. (5 marks) (170 marks remain)

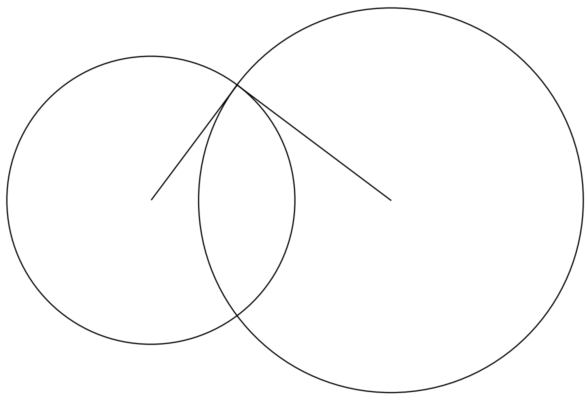
$A, B, C, D, E, F, G, H, I, J$ represent different digits. Let AB denote $10A + B$. If $x = AB + CD + EF + GH + IJ$, what is the difference between the largest possible value for x and the largest possible *odd* value for x?

4. (5 marks) (165 marks remain)

If $a + b = c$, $b + c = d$, $c + d = a$, and b is a positive integer, what is the largest possible value for $a + b + c + d$?

5. (5 marks) **(Change runner now)** (160 marks remain)

A circle of radius 3 intersects another circle of radius 4 at right angles (i.e. at the points of intersection the tangents are at right angles). What is the difference between the areas of the non-overlapping portions of the circles?



6. (5 marks) (155 marks remain)

Let

$$\frac{a}{b} = \left(1 - \frac{1}{4^2}\right)\left(1 - \frac{1}{5^2}\right)\cdots\left(1 - \frac{1}{1999^2}\right).$$

Find a/b in lowest terms (i.e. a, b have no common factors).

7. (5 marks) (150 marks remain)

How many solutions (x, y, z, w) are there to the following system of equations?

$$\begin{aligned} x+y+z+w &= 10 \\ x^2+y^2+z^2+w^2 &= 30 \\ x^3+y^3+z^3+w^3 &= 100 \\ xyzw &= 24. \end{aligned}$$

8. (5 marks) (145 marks remain)

Two opposite vertices of a 12-by-5 rectangle are brought together and the rectangle is flattened out to form a crease. Find the length of the crease.

9. (10 marks) (140 marks remain)

Find all integer values of n, $90 \le n \le 100$, that *cannot* be written in the form $n = a + b + ab$, where a and b are positive integers.

10. (10 marks) **(Change runner now)** (130 marks remain)

Let $x_n + iy_n = (1 + i\sqrt{3})^n$, where x_n and y_n are real and n is a positive integer. Find $x_{19}y_{99} + x_{99}y_{19}$.

11. (10 marks) (120 marks remain)

Find all values of x for which there is no ordered pair (x, y) satisfying the simultaneous equations

$$\begin{aligned} \frac{x^2-2xy-3y^2}{x-2y+1} &= 0 \\ \frac{x^2-4xy+3y^2}{2x-3y-2} &= 0. \end{aligned}$$

12. (10 marks) (110 marks remain)

Let a, b, c be the roots of $x^3 + qx + r = 0$. Write the *monic* polynomial in x (polynomial whose x^3 coefficient is 1) whose roots are $\frac{b+c}{a^2}, \frac{c+a}{b^2}, \frac{a+b}{c^2}$.

13. (10 marks) (100 marks remain)

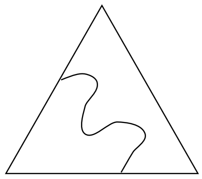
The parabola $y = ax^2 + 19x$, where a is an integer, passes through two lattice points (points with integer coordinates) in the first quadrant $(x, y \ge 0)$ whose y coordinates are *primes*. Compute the (x and y) coordinates of both these points.

14. (10 marks) (90 marks remain)

What is the greatest common divisor of all numbers of the form $p + q$, where p and q are primes whose product is 1 less than a perfect square and $p > 100$? (The greatest common divisor of a collection of numbers is the largest number that is a factor of all the numbers.)

15. (10 marks) **(Change runner now)** (80 marks remain)

What is the minimum length of a curve that bisects the area of an equilateral triangle with side length 1?



16. (10 marks) (70 marks remain)

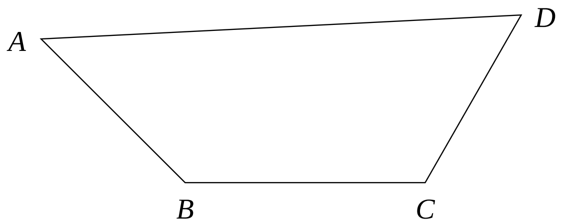
A strictly increasing sequence of positive integers contains the terms 305 and 1999. If each term after the second is equal to the sum of the two previous terms, compute the least positive value for the first term.

17. (15 marks) (60 marks remain)

Find the smallest positive integer that *cannot* be the difference between a square and a prime, if the square is greater than the prime.

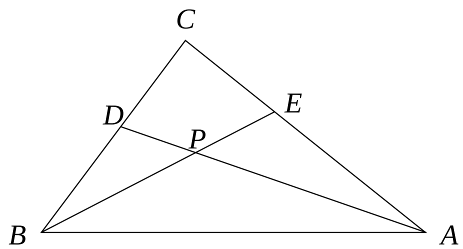
18. (15 marks) (45 marks remain)

In quadrilateral $ABCD$, $AB = 2\sqrt{6}$, $BC = 7 - 2\sqrt{3}$, $CD = 5$, $\angle B = 135°$, $\angle C = 120°$. Find AD.



19. (15 marks) (30 marks remain)

In triangle ABC, angle bisectors AD and BE intersect at P. If $BC = 3$, $CA = 5$, $AB = 7$, find BP/PE.



20. (15 marks) (15 marks remain)

If

$$(\sin 1°)(\sin 3°)\cdots(\sin 87°)(\sin 89°) = \frac{1}{2^n},$$

compute the rational number n.