

PARADOX

Welcome to *Paradox*, the magazine of the Melbourne University Mathematical Society. This is the first of what hopefully will be three issues for 1996. The aim of this magazine is to challenge, stimulate and entertain you with mathematics. We have a problems competition (with prize-money!), mathematical curiosities, riddles, and a look at the life of the great Leonhard Euler.

We are interested to hear your thoughts about this edition or what you would like to see in subsequent editions. Please do not hesitate to leave comments or suggestions in the *Paradox* box, situated to the left of the second and third year maths notice boards near the southern entrance of the Richard Berry Building. Thanks to all who have contributed to this issue.

Chaitanya Rao, *Paradox* Editor.

PARADOX COMPETITION

The person with the best solution to each problem will be awarded a \$5 prize, and have their solution included in the next edition of *Paradox*. Attempts to some or all of the problems can be placed in the *Paradox* box.

Problem 1: Given 13 distinct real numbers, show there exist two, say x and y , satisfying:

$$\frac{xy+1}{x-y} > \frac{7}{2}$$

Problem 2: The numbers 0, 1, 0, 1, 0 and 0 are written clockwise around the circumference of a circle. It is possible to make "moves", in each of which we add 1 to each number of a certain pair of adjacent numbers. Is it possible by means of finitely many such moves to make all the numbers on the circumference equal?

Problem 3: Let $ABCD$ be a tetrahedron with edge lengths $AB = 41$, $AC = 7$, $AD = 18$, $BC = 36$, $BD = 27$ and $CD = 13$. Find the distance between the mid-points of AB and CD .

MORE PROBLEMS

Here are some problems that are not part of the competition, but that should not stop you from having a go at them!

1. Anthony, Bob and Charles were questioned by a detective about Dolly's death by drowning.

1. Anthony said: If it was murder, Bob did it.

2. Bob said: If it was murder, I did not do it.

3. Charles said: If it was not murder, it was suicide.

4. The detective (who always tells the truth) said: If just one of these three men lied it was suicide

How did Dolly die? Was it by accident, suicide or murder?

Here's the first question from the 1959 olympiad:
 Prove that the fraction

$$\frac{21n+4}{14n+3}$$

is irreducible for every natural number n .

The questions have become much harder since! Australia has participated since 1981 and our teams have consistently managed to come in the top 20 out of about 70 countries. The Australian training programme is an extensive one, with students being identified through nation-wide competitions like the Australian Mathematics Competition (Westpac). After attending a succession of intensive training camps, they sit a series of selection exams, similar in nature to the IMO itself.

A report on their results will appear in the next issue of *Paradox*.

MATHEMATICAL CURIOSITIES

1. Here is a remarkable magic square. As with all magic squares, the sum of the numbers in any row, column or diagonal is the same. But what makes this one so special?¹

8818	1111	8188	1881
8181	1888	8811	1118
1811	8118	1181	8888
1188	8881	1818	8111

2. It is conjectured that by adding the reverse of any whole number a palindromic sum can be obtained after a finite number of steps.

For example:

$$38 + 83 = 121$$

$$139 + 931 = 1070, 1070 + 0701 = 1771$$

$$48017 + 71084 = 119101, 119101 + 101911 = 221012, 221012 + 210122 = 431134$$

Twenty-four steps are required to go from 89 to its palindromic sum - 8813200023188. How many steps are required to go from 196 to its palindromic sum?

(Hint: It would be environmentally unfriendly to try this on paper!)

THE LOST NOTE

Three women went into a hotel and were told that only one room was available and that it would cost \$30 for the night. They each paid \$10 and went to the room. Later that evening the receptionist realised that he had made an error and had overcharged the women \$5. He asked one of the other hotel staff to return the \$5 to the women. Unfortunately, this employee was not too honest. He realised that, since \$5 is not easily divisible by 3, he would keep \$2 and return \$3 to the women so that each would get back \$1. Each woman therefore only paid \$9, which totals \$27 for the room. Add to that the \$2 the employee kept and the total is only \$29. What happened to the missing \$1? Who had it? Where did it go?

¹Answer: Turning it upside down gives rise to a new magic square, with the same sums!!!

CRICKET RIDDLE

Two bowlers during the season have each taken 28 wickets for 60 runs (not bad, eh?). One bowler in the next match takes 4 wickets for 36 runs, and the other takes 1 wicket for 27 runs. Who now has the better average (runs per wicket)?

NO OFFENCE INTENDED!

A computer scientist, a mathematician, a physicist, and an engineer were travelling through Scotland when they saw a black sheep through the window of the train.

“Aha,” says the engineer, “I see that Scottish sheep are black.”

“Hmm,” says the physicist, “You mean that some Scottish sheep are black.”

“No,” says the mathematician, “All we know is that there is at least one sheep in Scotland, and that at least one side of that one sheep is black!”

“Oh, no!” shouts the computer scientist, “A special case!”

DEFINITIONS OF TERMS COMMONLY USED IN HIGHER MATHEMATICS

The following is a guide to the weary student of mathematics who is often confronted with terms which are commonly used but rarely defined. Hope you find them useful at your next maths lecture!

CLEARLY: I don't want to write down all the “in-between” steps.

TRIVIAL: If I have to show you how to do this, you're in the wrong class.

RECALL: I shouldn't have to tell you this, but for those of you who erase your memory tapes

WLOG (Without Loss Of Generality): I'm not about to do all the possible cases, so I'll do one and let you figure out the rest.

IT CAN EASILY BE SHOWN: Even you, in your finite wisdom, should be able to prove this without me holding your hand.

HINT: The hardest of several possible ways to do a proof.

BRUTE FORCE (AND IGNORANCE): Four special cases, three counting arguments, two long inductions, “and a partridge in a pair tree.”

BY A PREVIOUS THEOREM: I don't remember how it goes (come to think of it I'm not really sure we did this at all), but if I stated it right (or at all), then the rest of this follows.

LET'S TALK THROUGH IT: I don't want to write it on the board lest I make a mistake.

PROCEED FORMALLY: Manipulate symbols by the rules without any hint of their true meaning (popular in pure maths courses).

QUANTIFY: I can't find anything wrong with your proof except that it won't work if x is a moon of Jupiter (popular in applied maths courses).

PROOF OMITTED: Trust me, it's true.

EULER — THE MASTER OF ALL MATHEMATICAL TRADES

The following is a look at the life of arguably the most prolific mathematician of all time - Leonhard Euler (pronounced "oiler").

In both quantity and quality the feats of Leonhard Euler are nothing short of astounding. *Opera Omnia*, his complete works, contains 886 books and papers, filling 73 volumes. It has been estimated that if one were to collect all publications in the mathematical sciences produced over the last three-quarters of the eighteenth century, roughly one third of these were from the pen of Leonhard Euler! The frenetic pace of his production was so immense that it is reputed that there lasted a publication backlog up to 47 years after his death!

This remarkable human being was born in Basel, Switzerland in 1707. Not surprisingly, his genius shined through as a youth. His father, a Calvinist preacher, wanted his son to follow him into the church, sending him to the University of Basel to prepare him for the ministry. But Euler later obtained his father's consent to move to mathematics, using the persuasion of his mentor Johann Bernoulli.

Euler joined the St. Petersburg Academy of Science in 1727, then served as a medical lieutenant in the Russian navy from 1727 to 1730. He became professor of physics at the academy in 1730 and professor of mathematics in 1733. That year he also got married. Of the thirteen children he had, only five survived. Euler claimed that some of his greatest discoveries were made while holding a baby in his arms with other children playing around him. Euler was described by contemporaries as a kind and generous man, one who enjoyed the simple pleasures of growing vegetables and telling stories to his children. (Genius of his order does not **always** bring with it a neurotic personality!) His mathematical life was characterised by a boundless energy and enormous ingenuity. Losing sight in his right eye during the mid 1730s did not stop him - in fact he is remembered as saying:

Now I will have less distraction.

In all of his texts, Euler's exposition was quite lucid, and his mathematical notation was chosen so as to clarify, not obscure, the underlying ideas. Indeed, his mathematical writings are the first that look truly modern to today's reader; this, of course, is not because he chose a modern notation but because his influence was so pervasive that all subsequent mathematicians adopted his style, notation, and format. Moreover, he wrote with an understanding that not all his readers had his awesome ability for learning mathematics. He was never a classroom teacher but had more pedagogical influence than any modern mathematician. The philosopher Condorcet said of Euler:

He preferred instructing his pupils to the little satisfaction of amazing them.

This is quite a compliment to a person who, if he had so chosen, could surely have amazed anyone with his mathematical prowess.

Speaking of which, Euler was blessed with a memory that can only be called phenomenal. His number-theoretic investigations were aided by the fact that he had memorised not

only the first 100 prime numbers but also all of their squares, their cubes, and their fourth, fifth and sixth powers. While others were digging through tables or pulling out pencil and paper, Euler could simply recite from memory such quantities as 241^4 or 337^6 . But this was the least of his achievements. He was able to do difficult calculations mentally, some of these requiring him to retain in his head up to 50 places of accuracy! The Frenchman François Arago said that Euler calculated without apparent effort,

Just as men breathe, as eagles sustain themselves in the air.

Yet this extraordinary mind still had room for a vast collection of memorised facts, orations, and poems, including the entire text of Vergil's *Aeneid*, which Euler had committed to memory as a boy and still could recite flawlessly half a century later.

Euler's opus contains papers on acoustics, engineering, mechanics, music, astronomy, and even a three-volume treatise on optical devices such as telescopes and microscopes. It is indeed ironic, but uplifting, that much of Euler's contributions to optics were while he himself was either partially or totally blind! Some of Euler's more famous works included: *Mechanica* (1736–7): extensively presented Newtonian dynamics in the form of mathematical analysis

Lettres à une princesse d'Allemagne (Letters to a Princess of Germany) (1768–1772): a clear exposition of the basic principles of mechanics, optics, acoustics and physical astronomy

Introductio in Analysin Infinitorum (Introduction to the Analysis of the Infinite) (1748): developed the concept of function in mathematical analysis, advancing the use of infinitesimals and infinite quantities

Euler's works touched upon virtually all main branches of mathematics. He made significant inroads to the calculus of variations, graph theory, geometry, combinatorics, group theory, topology, complex analysis, differential equations and number theory. Thus, we find the Euler line in geometry, the Euler characteristic in topology, and the Euler circuit in graph theory, not to mention such entities as the Euler constant, the Euler polynomials, the Euler integrals, and so on. And even this is but half the story, for a large number of mathematical results traditionally attributed to others were in fact discovered by Euler and appear neatly tucked away amid the huge body of his work. One noted, not entirely in jest, that

There is ample precedent for naming laws and theorems for persons other than their discoverers, else half of analysis would be named for Euler.

Here are just a few of the symbols and results that are attributed to Euler:

- $f(x)$ a function of x
- e the base of natural logarithms
- π as the ratio of a circle's circumference to its diameter
- i as the square root of -1
- Σ for summation
- The remarkable equation $e^{i\theta} = \cos \theta + i \sin \theta$

— Use of integrating factors for solving differential equations.

In 1771, Euler lost most of the vision in his normal eye. Almost totally blinded and in some pain, he nonetheless continued his mathematical writings unabated, by dictating his wonderful equations and formulas to an associate. Just as deafness proved no obstacle to Ludwig van Beethoven a generation later, blindness did not reduce the flow of mathematics from Leonhard Euler. Up until he died suddenly on September 7, 1783, he had been mathematically active. Reportedly, he spent his last day playing with his grandchildren and discussing the latest theories about the planet Uranus. He is buried in St. Petersburg, which had been his home, on and off, for many happy years.

BYE!