# 2004 Schools Maths Olympics

Melbourne University Mathematics and Statistics Society

Discovery Day, Sunday 22 August 2004

**Question 1**

Bobbi and Chris were walking up the stairs of a tower. Bobbi was constantly 52 steps ahead of Chris. When Bobbi was halfway up the stairs, she said to Chris, "When I've reached the top, you'll be three times as far as you are now." What is the number of stairs in the tower?

**Question 2**

In the lead-up to an election, the truthfulness of two leading politicians is known to follow the following pattern: John lies on Monday, Tuesday and Wednesday, and tells the truth on all other days. Mark, on the other hand, lies on Thursday, Friday and Saturday, and tells the truth on all other days. One day they both said "Yesterday was one of my lying days". On which day of the week did they say this?

**Question 3**

A square is cut along two lines parallel to a side to form three identical rectangles . If the perimeter of each rectangle is 24, then what is the area of the original square?

**Question 4**

Express

$$\frac{2^{12} + 2^{11} + 2^{10}}{2^9 + 2^8 + 2^7}$$

in simplest form.

**Question 5**

A circular table has exactly 60 chairs around it. There are $N$ people seated at this table in such a way that the next person to be seated must sit next to someone else. What is the smallest possible value of $N$?

**Question 6**

In $\Delta ABC$, $AB = 5$, $BC = 12$ and $\angle ABC = 90°$. Arcs of circles are drawn, one with centre $A$ and radius 5, the other with centre $C$ and radius 12. The two circles intersect segment $AC$ at $M$ and $N$ respectively. What is the length of $MN$?

**Question 7**

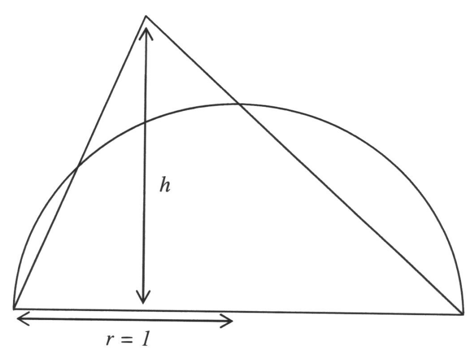
If we add 329 to the three digit number $2x4$, we get $5y3$. If $5y3$ is divisible by three, what is the greatest possible value of $x$?

**Question 8**

An urn is filled with coins and beads, all of which are either silver or gold. Twenty percent of the objects in the urn are beads. Forty percent of the coins in the urn are silver. What percentage of objects in the urn are gold coins?

**Question 9**

Consider the triangle and semicircle shown. The area of that part of the triangle lying outside the semicircle is equal to the sum of the areas of those parts of the semicircle lying outside the triangle. If the radius of the semicircle is 1, find the height of the triangle, $h$.




**Question 10**

Let $t_1 = 22$. To obtain $t_{n+1}$, we square the sum of the digits of $t_n$.
For example, $t_2 = (2+2)^2 = 16$. Find $t_{2004}$.

**Question 11**

Norm and Denise each wish to buy an ice cream which costs a whole number of dollars. However Norm needs seven more dollars to buy an ice cream, while Denise needs one more dollar. They decide to buy only one ice cream together, but discover that they do not have enough money. How much does one ice cream cost (in dollars)?

**Question 12**

In how many ways can you colour three edges of a cube green so that each face has a green edge?

**Question 13**

Express the value of this sum in simplest form:

$$\frac{1}{2\lfloor\sqrt{1}\rfloor+1}+\frac{1}{2\lfloor\sqrt{2}\rfloor+1}+\ldots+\frac{1}{2\lfloor\sqrt{100}\rfloor+1}$$

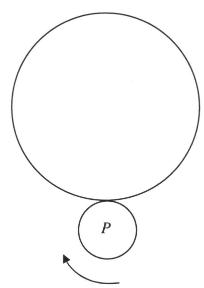
(where $\lfloor x\rfloor$ is the greatest integer less than or equal to $x$)

**Question 14**

If $t_1=1$ and $t_{n+1}=\frac{t_n}{1+t_n}$ for all positive integers $n$, what is the value of $t_{2004}$?

**Question 15**

The diameter of the small circle is a quarter of the diameter of the large circle. If the small circle rolls (without slipping) around the outside of the large circle, how many full rotations will the letter $P$ (on the small circle) undergo by the time the small circle returns to its initial position?




**Question 16**

The points $A, B, C$, and $D$ lie on a circle with centre $O$ and radius $r$. $AB=6$, $CD=8$ and $BC=DA=\sqrt{2}r$. Find $r$.

**Question 17**

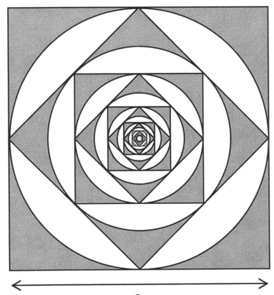
Suppose

$$f\left(\frac{1}{x}\right)+\frac{1}{x}f(-x)=2x$$

where $f$ is a function defined for all non-zero real values of $x$. What is the value of $f(2)$?

**Question 18**

Find the shaded area.




**Question 19**

You are asked to form a committee of six people from a group of 10 couples. There are five ‘happy’ couples and five ‘grumpy’ couples. A member of a ‘happy’ couple will only serve on the committee if their beloved is also on the committee, while a member of a ‘grumpy’ couple refuses outright to work on the committee if their partner is on the committee. In how many ways can you form the committee?

**Question 20**

Daniel and Joanna, whilst journeying through outer space, came across a spaceship wreck, and with it an injured yet amiable alien who had lost his fingers. The pair of travellers were keen to find the alien’s fingers for him, but did not know how many to look for. Just prior to crashing his spaceship, the alien was, naturally, doing some quick calculations on a piece of paper which Daniel rescued from the wreckage. It read:

$$\begin{aligned} 5x^2 - 50x + 125 &= 0 \\ \therefore x = 5 \quad &or \quad x = 8 \end{aligned}$$

Joanna quickly realized that the number base in which the alien was working was probably equal to the number of fingers he ought to have. How many alien-fingers were Joanna and Daniel looking for?

**Question 21**

You are given a $5 \times 5$ grid and you colour in some of its squares. A colouring is called *permissible* if for any coloured-in square, either its whole column is coloured in or its whole row is coloured in. How many *permissible* colourings are there?

**Question 22**

The polynomial $p(x) = ax^4 + bx^3 + cx^2 + bx + a$ has exactly two real roots. If $x = 29$ is one solution of $p(x) = 0$, what is the other solution of $p(x) = 0$?

**Question 23**

Circles $C_1$ and $C_2$ intersect at points $A$ and $B$. They have common tangents $CD$ and $EF$, where $C$ and $E$ lie on $C_1$, and $D$ and $F$ lie on $C_2$. The line $DA$ passes through the midpoint of segment $CE$. If $CE = 7$ and $AB = 12$, find the ratio of the radius of $C_1$ to the radius of $C_2$. Express your answer in simplest form.

**Question 24**

Find the number of $4 \times 4$ arrays with entries from $\{1, 2, 3, 4\}$ such that the sum of each row is divisible by 4 and the sum of each column is divisible by 4.

**Question 25**

In $\Delta ABC$, $AB = 360$, $BC = 507$ and $CA = 780$. $M$ is the midpoint of $AC$, $D$ is the point on $AC$ such that $BD$ bisects $\angle ABC$. $F$ is the point on $BC$ such that $BD$ and $DF$ are perpendicular. The lines $FD$ and $BM$ meet at $E$. What is the value of $\frac{DE}{EF}$?