Question 1 **10 points**

Song makes and sells cupcakes. When she opens her shop on Monday she has 32 cupcakes and she sells half of them. Overnight she makes another 50% of what she has left. Then on Tuesday she sells half of her cupcakes again, and again overnight she makes another 50% of what she has leftover. How many cupcakes does she have available to sell on Wednesday?

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Question 2 **10 points**

A tennis tournament has 10 players, where each player plays each other player (a round-robin). At the end of every match, the players shake hands. At the end of the tournament, how many handshakes have there been?

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Question 3 **10 points**

Michael is a poor uni student with only \$22 to his name. At his favourite discount supermarket *MUMSworth* a box of cereal costs \$2, a packet of mi goreng noodles costs 40c, while a roast chicken costs \$4.50. If he could stretch a box of cereal to sustain him for 4 days, or a roast chicken to last him 12 days (while a packet of mi goreng will only satiate for a solitary day), how many days can he last with the money he has?

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Question 4 **10 points**

Newspapers are created by laying sheets of paper on top of each other, then folding down the center line, creating a 'book'. A 100-sheet newspaper has its pages numbered from 1 to 400. A sheet falls out of the newspaper and you notice that one of the pages is numbered 67. What is the sum of the numbers of the other 3 pages on the same sheet?

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Question 5 **10 points**

There are two high school rivals – Hillary and Donald. Wanting to trump Donald once and for all, Hillary challenges Donald to a game of Four Aces.

Four Aces is a game where there are four Aces – one of each suit – facedown. The challenger, Donald, picks two of the cards. If they are the same colour, he wins, otherwise Hillary wins.

Hillary has picked this game because she knows that she has a greater chance of winning, but what is that chance?

CHANGE WALKER NOW

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CHANGE WALKER NOW

Question 6 **10 points**

In the equation $N \times U \times (M + B + E + R) = 33$, each letter stands for a different digit $(0, 1, 2, \ldots, 9)$. How many different ways can the letters of the equation be chosen?

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Question 7 **10 points**

Samoil and Matt are stunt drivers. For the most expensive scene in an upcoming movie, the director wants a spectacular car crash. Samoil and Matt rev their engines from a starting position of 100m apart, before racing towards each other. If Samoil is travelling at 72km/h, while Matt drives at 30m/s, how far will Matt drive in metres before crashing into Samoil?

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Question 9 **10 points**

What is the last digit of 7^{2016}?

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Question 10 **10 points**

David and Ruwan each wish to buy an ice-cream which costs a whole number of dollars. However, David needs seven more dollars to buy an ice-cream, while Ruwan needs only one more dollar. They decide to buy an ice-cream together but discover they do not have enough money. How much does an ice-cream cost?

CHANGE WALKER NOW

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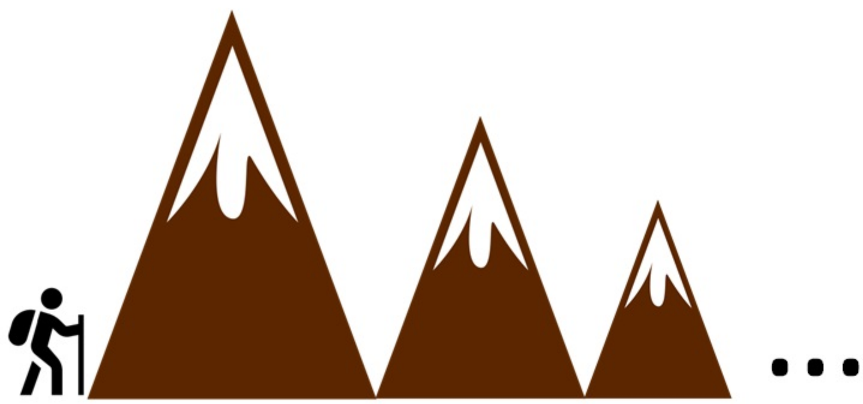
CHANGE WALKER NOW

Question 11 **20 points**

A flat adventurer traverses two-dimensional mountains, which are all in the shape of isosceles triangles and all touching base-to-base. They start from the bottom of the first mountain of height 3072m and of slope 30 degrees from the horizontal, climbs up to the top, then back down to the other end of the base. They then proceed to do the same for the next mountain, and so on, with each triangular-mountain being similar to, but half the height of, the previous one. If the last mountain is 75cm tall, then what horizontal distance of mountainous path did our adventurer travel in metres?



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Question 12 **20 points**

Two trains operate on two single-loop train track sharing a common 1km platform. One of the trains travels at 40km/h and takes 7 minutes to complete one loop of its track, while the other travels at 60km/h and takes 9 minutes (each track length includes the 1km platform). Both trains start at the front of the platform (but on opposite sides), facing the same direction, and travel continuously without stopping (they pick-up and drop-off passengers instantaneously, it's high tech). If both trains depart the front of the platform at the same time, how long (in minutes) until the fronts of both trains are on the platform simultaneously?

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Question 13 **20 points**

In how many ways can you paint exactly three edges of a cube in such a way that each face has an edge which has been painted?

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Question 14 **20 points**

Cassy and Giles, while journeying through outer space, came across a spaceship wreck and within it an injured yet amiable alien who had lost its fingers. The pair of travellers were keen to find the alien's fingers for it, but didn't know how many to look for. Just prior to crashing its ship, the alien was, naturally, doing some quick calculations on a piece of paper which Giles rescued from the wreckage. It read

$$5x^2 - 50x + 125 = 0$$
$$\therefore x = 5 \text{ or } x = 8$$

Cassy quickly deduced that the number base in which the alien was working was probably equal to the number of fingers it ought to have. How many alien-fingers were Cassy and Giles looking for?

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Question 15 **20 points**

Emma and her grandmother share the same birthday. It was found for six consecutive years that Emma's age was a divisor of her grandmother's age. What is the age difference between Emma and her grandmother in years?

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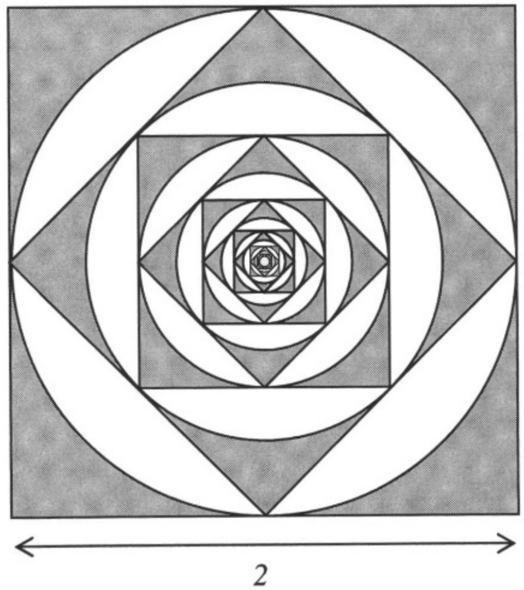
Question 16 **20 points**

What is the area of the shaded region?



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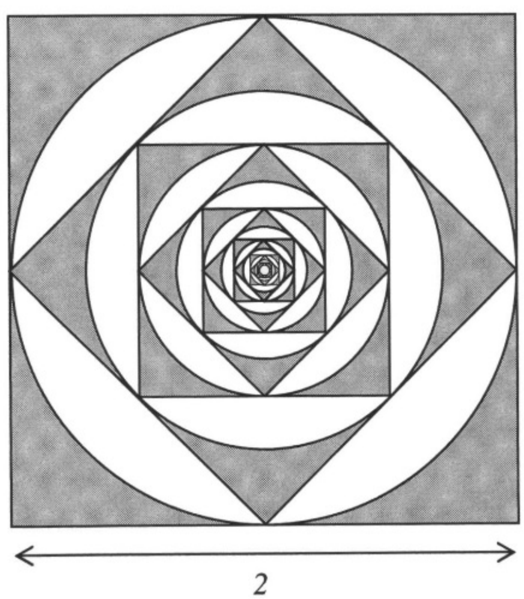

Question 17 **20 points**

In a cubic room, a spider is at one of the corners in the ceiling. A fly is resting on the floor at the midpoint of an edge on the opposite side of the room to where the spider is. If the shortest distance that the spider has to crawl in order to reach the fly is 4 metres, what is the sidelength of the room in metres?

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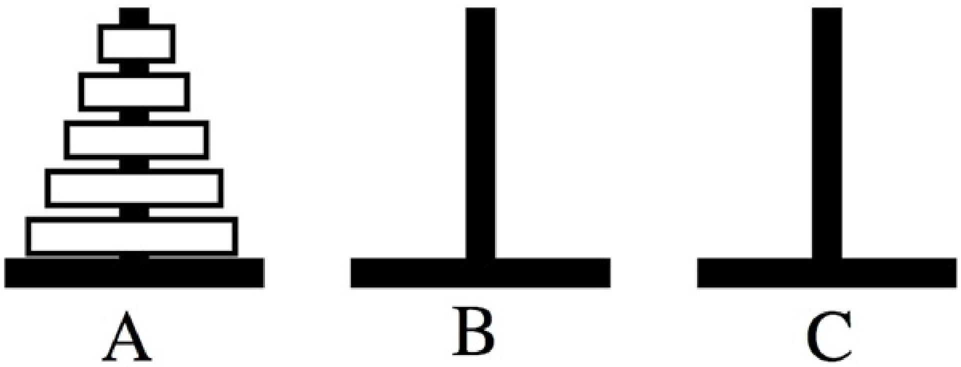
Question 18 **20 points**

In this variant of the Tower of Hanoi puzzle, the aim is to move the pile of n discs from A to C whilst maintaining the triangular shape. The three rules that must be followed are:

1. Only one disc can be moved at a time, and **only to an adjacent pole**.
2. Only the topmost disc of a pile can be moved.
3. A disc may only be placed on top of another disc which is larger than it.

What is the minimum number of moves required to solve a tower of 5?



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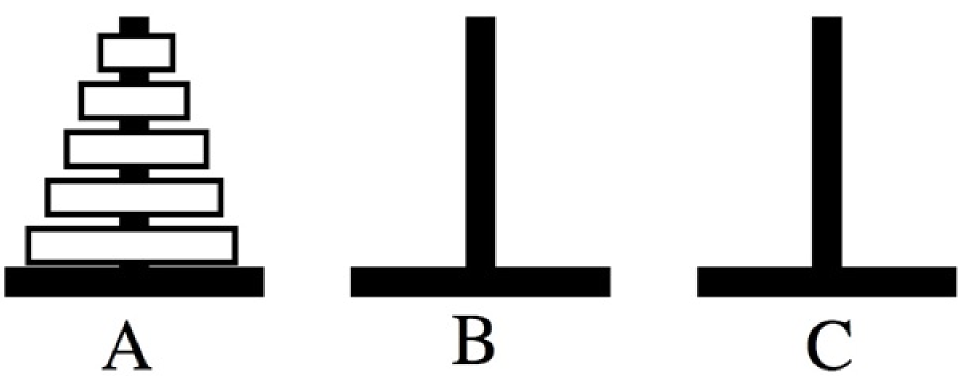

Question 19 **20 points**

James is collecting toys from the healthy cereal option *Maths-O's*. If each box contains one toy out of the four possible toys, at random, how many boxes would James have to open on average before he collects all four toys?

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Question 20 **20 points**

A Sydney-sider comes down to Melbourne and checks out a game of Australian Rules footy. They don't understand the scoring system (6 points for a goal, 1 point for a behind) and erroneously thinks that the game is scored by multiplying the number of goals by the number of behinds. Using this scoring system, how many combinations of goals and behinds are there that allow them to calculate the correct final score?

CHANGE WALKER NOW

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CHANGE WALKER NOW

Question 21 **30 points**

What is the value of

$$\frac{1}{2^0} + \frac{2}{2^1} + \frac{3}{2^2} + \ldots + \frac{n}{2^{n-1}} + \ldots ?$$

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Question 22 **30 points**

At 11.30am there are 100 large cakes on a table in preparation for a 1pm lunch. Spies inform us that one of them has been poisoned, and that the poison takes exactly one hour before it is suddenly and irrecovably lethal. In order to determine which cake is poisoned, our benevolent overlord Michael (long may he reign) decides that we should sample small slices from the cakes (doesn't take very long) and see who dies. What is the minimum number of us who need to risk our lives trying the cakes in order to uniquely identify which cake is poisoned, ensuring lunch can safely go ahead at 1pm?

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Question 23 **30 points**

Sam has two children, at least one of whom is a girl born on a Tuesday. Assuming that children are equally likely to be born on any day of the week, and that any birth has a 50% chance of producing a girl, what is the probability that Sam has two girls?

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Question 24 **30 points**

On a regular 12-hour analogue clock, as the minute hand turns so too does the hour hand rotate at a constant rate. Let any position that can be reached naturally be called *legitimate*. How many distinct legitimate positions remain legitimate when the hour hand is switched with the minute hand?

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Question 25 **30 points**

Triangle ABC has $AB = AC \neq BC$ and $\angle BAC \leq 90°$. P lies on AC, and Q lies on AB such that $AP = PQ = QB = BC$. Find the ratio of $\angle ACB$ to $\angle APQ$.

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