Schools Maths Olympics Questions 2000

1. (5 marks)

   John has loaned all his money: to Dan he gave a third, to Norm a fourth, and to George he gave \$200. How many dollars did John loan in total?

2. (5 marks)

   If the point $(x, -4)$ lies on the straight line joining the points $(0, 8)$ and $(-4, 0)$ in the xy-plane, find x.

3. (5 marks)

   How many integers between 5^2 and 5^3 are not divisible by 5?

4. (5 marks)

   Two runners are running a 60 metre race, each at a constant speed. If runner B has run 40 metres when runner A has run 50 metres, how far behind (in metres) will runner B be when runner A finishes?

5. (5 marks)

   The areas of three faces of a rectangular box are 3, 6 and 8 square metres. What is its volume in cubic metres?

6. (5 marks)

   Louise and Caroline throw dice (one each). What is the probability that Louise throws a higher number than Caroline? Write your answer as a fraction in reduced form.

7. (5 marks)

   What is the sum of all the real solutions x of

   $$\sqrt[4]{x} = \frac{12}{7 - \sqrt[4]{x}}?$$

8. (5 marks)

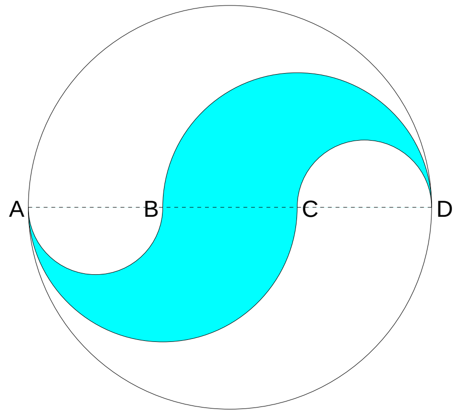
   What is the sum of the digits of the number $2^{1999} \times 5^{2001}$ written out in full?

9. (10 marks)

   If one hose can fill a 6 litre bucket in 1 minute, and another hose can fill a 2 litre bucket in 3 minutes, how many seconds would it take both hoses together to fill a 4 litre bucket?

10. (10 marks)

In the figure below points B and C divide diameter AD of the circle into thirds. Semicircular arcs are formed with diameters AB, AC, BC and BD as shown. If the area of the circle is 36π, find the shaded area.



11. (10 marks)

A circular disc with diameter D is placed on an 8 by 8 chessboard with width D so that the centres coincide. How many of the chessboard squares are completely covered by the disc?

12. (10 marks)

Simplify:

$$\sin^2 2^\circ + \sin^2 4^\circ + \sin^2 6^\circ + \ldots + \sin^2 88^\circ$$

13. (10 marks)

$$x^2 + \Box x - \Box 1 = (x + \Box\Box)(x - \Box)$$

In the above equation each box stands for a non-zero digit (1 to 9). What is the sum of the five missing digits?

14. (10 marks)

How many sequences of positive integers are there where the sum of the terms is 5? Note that the sequence "$1, 2, 2$" is considered to be different from "$2, 1, 2$".

15. (10 marks)

Jeremy had his 16th birthday on New Year's Day 2000. Being an obsessive factorizer, he was quick to notice that his age is a factor of the present year (that is, 16 is a factor of 2000). What will his age be when this next occurs?

16. (10 marks)

There are two spherical balls of different sizes lying in two corners of a rectangular room, each touching two walls and the floor. If there is a point on each ball which is 5 centimetres from each wall which that ball touches, and 10 centimetres from the floor, find the sum of the diameters of the balls, in centimetres.

17. (15 marks)

Let ABC be a triangle with a right-angle at B. Let M and N be the midpoints of AB and AC respectively. If the lines BN and CM are perpendicular and $BC = 1$, what is the value of CM^2?

18. (15 marks)

> They counted some rats in despair,
> The number was a third of a square.
> If a quarter were slain,
> Just a cube would remain.
> How many at least must be there?

19. (20 marks)

An ordered pair (b, c) of integers, each of which has absolute value less than or equal to five, is chosen at random, with each such ordered pair having an equal likelihood of being chosen. What is the probability that the equation $x^2 + bx + c = 0$ will *not* have distinct positive real roots? Write your answer as a fraction in reduced form.

20. (20 marks)

In base A the expanded fraction F_1 becomes $.141414\ldots$, and the expanded fraction F_2 becomes $.414141\ldots$. In base B fraction F_1, when expanded, becomes $.272727\ldots$, while fraction F_2 becomes $.727272\ldots$. What is $A + B$ (in base 10)?