Question 1 **20 marks**

A cake shop owner has the following ingredients: 5kg icing sugar, 10kg regular sugar, 6 dozen eggs and 37kg of flour. First he makes as many iced cakes as he can, then uses the remaining ingredients to make some un-iced cakes for his best customer Han, who is so one-eyed about cakes that he doesn't care about icing. Each cake is made from 500g flour, an egg and 100g regular sugar. To ice a cake requires 50g regular sugar and 100g icing sugar. How many iced cakes and how many Han-cakes will said owner make?

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What is the largest real number x such that

$$x^3 - x^2 - 17x - 15 = 0 \ ?$$

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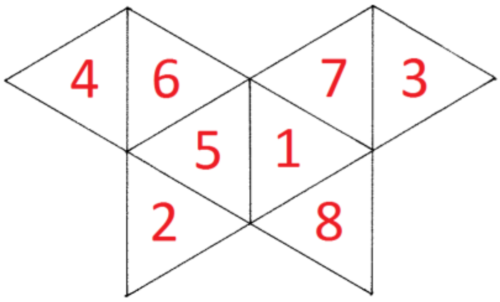
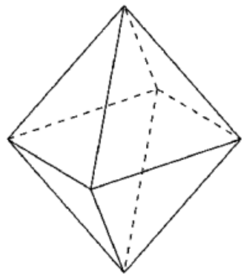
The following figure can be folded along the lines shown to form an octahedron. What is the largest sum of four numbers whose faces come together at a corner?





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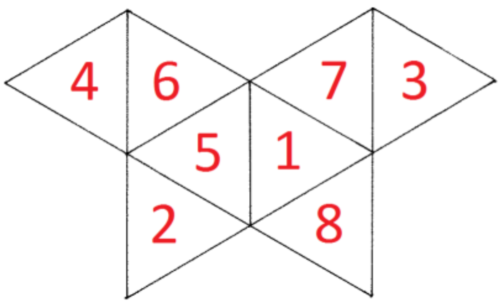

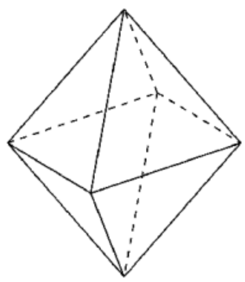

Question 4 **20 marks**

Jeff likes sheep. Jeff also likes maths. And Jeff likes to sleep! Because counting sheep before sleeping is too easy, Jeff imagines the sheep jumping over a fence and adds up the number of sheep. First one sheep jumps over the fence, then two jump over together, then three jump over together etc., until he falls asleep. Unfortunately, because he was so sleepy, he accidentally counts a group of sheep twice. If he falls asleep by the time he adds up 220 sheep, how many sheep actually jumped over that fence?

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Consider a regular 4×4 lattice of points. How many equilateral triangles have three of these points as vertices?

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Question 6 **30 marks**

Sam has won 75% of all chess games that he's ever played, and it's known that his win rates go up to 80% if he wins all 12 upcoming matches. What is the minimum number of games he must win (counting previous ones) in order to increase his win rate to 99%?

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Question 7 **30 marks**

Evil Rick has stuck a compromising picture of Calvin on a wall. Cunningly, Rick has placed the picture out of the reach of the diminutive Calvin. At full stretch, Calvin can reach a measly 1.95 metres. Rick (of decent height) has placed the picture 2.3 metres from the ground. To save his reputation from ruin, Calvin gets a chair that is 50 centimetres tall to help get the picture. In centimetres, what is the maximum distance the chair can be from the wall such that Calvin can still reach the picture?

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Question 8 **30 marks**

Yi, Han, Sam and Jeff play the game *Settlers* in the MUMS room. The *strength*, s, of a player is equal to $\frac{1}{n}$, where n is the number of letters in his name. The *adjusted strength* of a player is equal to $s - \frac{t}{12}$, where t is the number of players who target him. The probability of a player winning is proportional to his adjusted strength. Sam and Jeff sensibly target Han, while Han and Yi irrationally target Sam. What is the probability that Jeff wins?

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Question 9 **30 marks**

Looking through old ruins, you find a carving of a magic square, i.e. all rows, all columns, and both diagonals sum to the same constant. Unfortunately, all but three panels have been washed away with time and can no longer be read. What would have been the sum of the missing numbers?

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Mel recently bought a 2010 calendar, but suddenly realised that this year is already coming to an end. He will be able to use this calendar again when the days of the week match up with the dates. When will be the next year in which this occurs?

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Determine the value of

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+5)}$$

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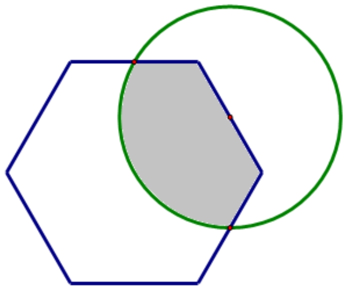
Adib draws a regular hexagon with side length 2 and then draws on top of this a circle with its centre and two boundary points positioned on the midpoints of three consecutive sides of the hexagon. What's the area of the overlapping region?



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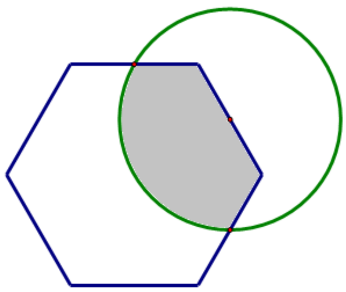

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Consider the sequence $2, 3, 5, 6, 7, 10, 11, 12, \ldots$ consisting of all positive integers that are neither squares nor cubes. What is the 2010^{th} number in the sequence?

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The non-integer solutions of $\lfloor x \rfloor \lfloor y \rfloor = x + y$, where $\lfloor x \rfloor$ is defined as the largest integer not greater than x, lies on two lines. Find the equations of these two lines.

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$$\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\cos^3 x + \sin^3 x}\,\mathrm{d}x$$

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Question 16 **50 marks**

The acute triangle $\triangle ABC$ has (positive) integer side lengths. One of these sides has length 17, and when it is set as the base of the triangle, the height of the triangle is also an integer. What are the lengths of the other two sides?

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Question 18 **50 marks**

Mark, Richard and Andrew play PASS-THE-BOMB. Richard starts with the bomb. When a player has the bomb, there is a 50% chance that the bomb will explode before they can 'pass' it. If they do manage to pass it, they pass it to one of the others, each with a 50% chance. If the bomb only blows up the person holding it, what is the probability that Richard will blow up?

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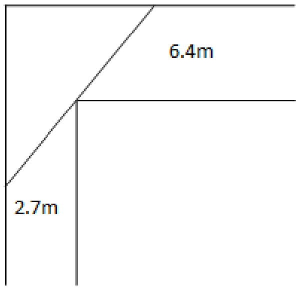
Dana is moving into a mansion, and she needs a huge whiteboard. This whiteboard is to be so heavy that it cannot be lifted; only wheeled. A certain corridor is 2.7 metres wide, and is perpendicular to a 6.4 metre wide corridor. What is the length (in metres) of the longest whiteboard that she can manoeuvre around the corner?



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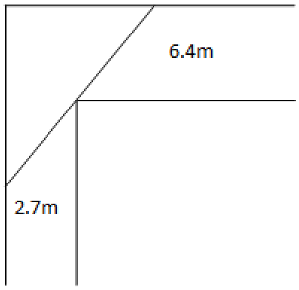

Question 20 **FINAL QUESTION!** **50 marks**

Find the largest positive integer n with no zeroes in its base 10 representation such that

$$2^{s(n)} = s(n^2)$$

where $s(n)$ is the sum of the digits of the base 10 representation of n.

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