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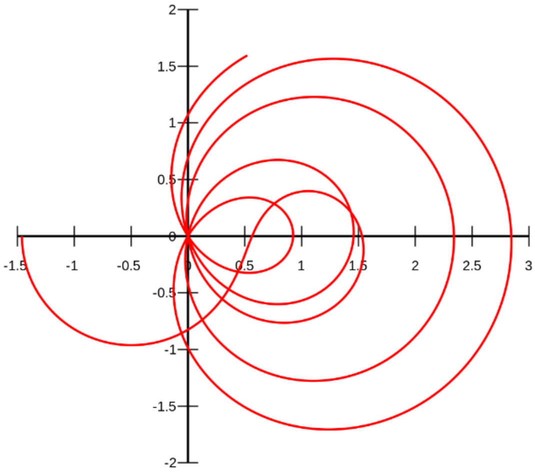
Paradox

The Magazine of the Melbourne University Mathematics and Statistics Society



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In This Edition of The Paradox

On the cover:
Left: A polar plot of the Riemann Zeta function on its critical line, $\zeta(0.5 + it)$.
Right: The Human juggernaut they call ToD. Could this man hold the answer to a 160 year old problem?

Words from the President

Hello hello everyone and welcome to the first (hopefully of many) paradox article of your 2018-2019 Committee! We have many exciting events and activities planned for you all for the remainder of this year and into next year, some of which you may have already experienced. There's been Games Night, the \relax picnic, and more or less at the time of publishing, spooks n maths, our End of Sem Celebration. Still to come this sem is the very first iteration of MUMS Study Groups during SWOTVAC!

As a committee, we're hoping to continue running a wide range of events to cater to all different kinds of maths students, and work on growing and building a diverse and inclusive MUMS community. We'd always love to hear your feedback about events we've run, and ideas for future events! Email us at mums@unimelb.edu.au, or drop by the MUMS room to chill out & chat with the committee.

Wishing you all the best for exams and see you back in 2019 for more new and exciting MUMS stuff!

Madeleine Johnson
MUMS President

From the Editor

It is a great honour to be chosen as your Paradox Editor for 2018-2019. Having been a keen reader of Paradox in the 2006-2011 era, I want to continue this proud tradition. I have taken the liberty to rename some of the sections to fit with our theme! This magazine publishes eclectic mix of articles, both mathematical and non-mathematical. Please see the back page for an explanation of different magazine sections, and a list of ideas I have compiled, for inspiration. All submissions go to mums.paradox.editor@gmail.com. Paradox articles represent the opinions of their respective authors.

For this edition, Madeleine has interviewed women and non-binary mathematics students to provide a unique perspective on their educational experiences. In recent years, underrepresentation of females and ethnic minorities in STEM disciplines has been a major focus for public institutions. There are efforts to remove social barriers, and promote acceptance of a student cohort that may have been considered atypical in the past. It is crucial that technical roles in our society are filled from the widest possible pool of eligible candidates.

This is a complex discussion, and I would love to see this explored further. Some measures to improve participation rates have been fraught with danger. Next year, a number of prominent American universities – including Harvard University – face lawsuits for Affirmative Action policies that seek to socially engineer the racial balance within the student cohort – allegedly, at the expense of Asian Americans. At other times, publicly funded bursary and scholarship opportunities that exclude a particular race or gender have been scrutinized. If you have thoughts about how Australian institutions could navigate this ethical minefield, please send me a message!

Steven Xu
Paradox Editor (Tangent Bundling Officer)

"The Narrow Margin" - Mathematical Facts!

Mathematical Tidbits: Commonly used Inequalities – *Steven Xu*

A cluster of related inequalities turn up frequently in analysis and the study of metric spaces: Jensen's, Young's, Minkowski, Holder and Cauchy-Schwartz's inequality. Here is a handy collection of short proofs. If $f'(x)$ is a continuous, strictly decreasing function over region $D = [x, x+h]$, (i.e. $f''(x) \leq 0$), we call f **concave**. Given $0 < c < 1$, compare integrands to confirm that $\int_0^h f'(x+t)dt \leq \int_0^h f'(x+ct)d\xi$. This implies:

$$\begin{aligned}
&\int_x^{x+h} f'(\xi)d\xi \leq \frac{1}{c}\int_x^{x+hc} f'(\xi)d\xi \\
&c\int_x^{x+h} f'(\xi)d\xi + f(x) \leq \int_x^{x+hc} f'(\xi)d\xi + f(x) \\
&c[f(x+h) - f(x)] + f(x) \leq f(x+hc) \\
&(1-c)f(x) + cf(x+h) \leq f(x+hc) \\
&c_1 f(x_1) + c_2 f(x_2) \leq f(c_1x_1 + c_2x_2)
\end{aligned}$$

Where we relabelled $x_1 = x$, $x_2 = x + h$, $c_1 = 1 - c$, and $c_2 = c$. We call this the **Jensen inequality**, and in general, for $\sum_{k=1}^{n} c_k = 1$, for concave f from $x_1$to x_n:

$$f\left(\sum_{k=1}^{n} c_k x_k\right) \geq \sum_{k=1}^{n} c_k\, f(x_k)$$

We prove this by induction. Label the x_k in ascending order, and assume the (n-1)-case holds:

$$f\left(\sum_{k=1}^{n-1} \frac{c_k}{1-c_n} x_k\right) \geq \sum_{k=1}^{n-1} \frac{c_k}{1-c_n} f(x_k)$$

Apply the 2-case:

$$f\left((1-c_n)\sum_{k=1}^{n-1} \frac{c_k}{1-c_n} x_k + c_n x_n\right) \geq (1-c_n) f\left(\sum_{k=1}^{n-1} \frac{c_k}{1-c_n} x_k\right) + c_n f(x_n)$$

So putting them together, we have established the n-case:

$$f\left(\sum_{k=1}^{n} c_k x_k\right) = f\left((1-c_n)\sum_{k=1}^{n-1} \frac{c_k}{1-c_n} x_k + c_n x_n\right) \geq (1-c_n)\sum_{k=1}^{n-1} \frac{c_k}{1-c_n} f(x_k) + c_n f(x_n)$$

$$= \sum_{k=1}^{n} c_k\, f(x_k)$$

In the case where $f(x)$ is **convex** – that is, $f''^{(x)} \geq 0$ – all of these inequalities are reversed. Note that $f(x) = \ln(x)$, is concave, since $f''^{(x)} = -\frac{1}{x^2} < 0$. We call positive numbers p and q, **Holder conjugates** if $\frac{1}{p} + \frac{1}{q} = 1$. The Jensen inequality tells us:

$$\ln\left(\frac{x^p}{p} + \frac{y^q}{q}\right) \geq \frac{1}{p}\ln x^p + \frac{1}{q}\ln y^q = \ln x + \ln y$$

$$\frac{x^p}{p} + \frac{y^q}{q} \geq xy$$

We call this **Young's inequality**. Suppose $\tilde{x}(s)$ and $\tilde{y}(s)$ are positive-real-valued functions $S \to \mathbb{R}$. Apply Young's Inequality to functions $x(t) := \frac{\tilde{x}(t)}{\left(\sum_{s\in S}|\tilde{x}(s)|^p\right)^{\frac{1}{p}}}$ and $y(t) := \frac{\tilde{y}(t)}{\left(\sum_{s\in S}|\tilde{y}(s)|^q\right)^{\frac{1}{q}}}$, and add these inequalities together:

$$\sum_{t\in S} \frac{y^q}{q} + \frac{x^p}{p} \geq \sum_{t\in S} yx$$

$$\frac{\sum_{t\in S}|\tilde{y}(t)|^q}{q\sum_{s\in S}|\tilde{y}(s)|^q} + \frac{\sum_{t\in S}|\tilde{x}(t)|^p}{p\sum_{s\in S}|\tilde{x}(s)|^p} \geq \sum_{t\in S} \frac{\tilde{y}(t)}{\left(\sum_{s\in S}|\tilde{y}(s)|^q\right)^{\frac{1}{q}}} \frac{\tilde{x}(t)}{\left(\sum_{s\in S}|\tilde{x}(s)|^p\right)^{\frac{1}{p}}}$$

$$1 = \frac{1}{q} + \frac{1}{p} \geq \frac{\sum_{t\in S}\tilde{y}(t)\tilde{x}(t)}{\left(\sum_{s\in S}|\tilde{y}(s)|^q\right)^{\frac{1}{q}}\left(\sum_{s\in S}|\tilde{x}(s)|^p\right)^{\frac{1}{p}}}$$

$$\left(\sum_{s\in S}|\tilde{y}(s)|^q\right)^{\frac{1}{q}}\left(\sum_{s\in S}|\tilde{x}(s)|^p\right)^{\frac{1}{p}} \geq \sum_{t\in S}\tilde{y}(t)\tilde{x}(t)$$

$$\|\tilde{y}\|_q\, \|\tilde{x}\|_p \geq \|\tilde{y}\tilde{x}\|_1$$

We call this the **Holder inequality**, where we have adopted the slick notation for the **p-norm** of function f(s):

$$\|f\|_p := \left(\sum_{s\in S} |f(s)|^p\right)^{\frac{1}{p}}$$

If S continuous set of real numbers, or some other set on which integration can be meaningfully defined, an equivalent result could be stated $\|f\|_p := \left(\int |f(s)|^p ds\right)^{\frac{1}{p}}$. The Cauchy-Schwartz inequality is the special case where p = q = 2:

$$\left(\sum_{s\in S} |\tilde{y}(s)|^2\right)^{\frac{1}{2}} \left(\sum_{s\in S} |\tilde{x}(s)|^2\right)^{\frac{1}{2}} \geq \sum\nolimits_{t\in S} \tilde{y}(t)\tilde{x}(t)$$

L^p Spaces, Minkowski Inequality

Recall that a Banach space is a vector space with a **norm** – an assignment of a real number to each vector, compatible with scaling and multiplication (among other conditions). Observe that the space of real-valued functions with finite p-norm, is indeed a vector space under pointwise function addition and scalar multiplication. This space, together with the p-norm, forms a Banach space called an **L^p Space**. The Holder inequality proves that the p-norm obeys the triangle inequality, $|f|_p + |g|_p > |f + g|_p$ – a result called the **Minkowski Inequality**; (the other norm axioms – completeness and positive-definite – are left as an exercise to the reader). Note the use of the Holder inequality to $|f|$ and $|f + g|^{p-1}$ in reaching the second line:

$$\int_{s\in S} |f + g|^p\, ds \leq \int_{s\in S} (|f| + |g|)|f + g|^{p-1} ds = \int_{s\in S} |f||f + g|^{p-1} ds + \int_{s\in S} |g||f + g|^{p-1} ds$$

$$\leq \left(\int_{s\in S} |f|^p ds\right)^{\frac{1}{p}} \left(\int_{s\in S} |f + g|^{qp-q} ds\right)^{\frac{1}{q}} + \left(\int_{s\in S} |g|^p ds\right)^{\frac{1}{p}} \left(\int_{s\in S} |f + g|^{qp-q} ds\right)^{\frac{1}{q}}$$

$$= \left[\left(\int_{s\in S} |f|^p ds\right)^{\frac{1}{p}} + \left(\int_{s\in S} |g|^p ds\right)^{\frac{1}{p}}\right] \left(\int_{s\in S} |f + g|^p ds\right)^{1-\frac{1}{p}}$$

We defined $q := \frac{p}{p-1}$ as the Holder conjugate to p, so $qp - q = p$ and $\frac{1}{q} = 1 - \frac{1}{p}$. Dividing the first and final expression by $\left(\int_{s\in S} |f + g|^p ds\right)^{1-\frac{1}{p}}$, we have:

$$\left(\int_{s\in S} |f + g|^p\, ds\right)^{\frac{1}{p}} \leq \left(\int_{s\in S} |f|^p ds\right)^{\frac{1}{p}} + \left(\int_{s\in S} |g|^p ds\right)^{\frac{1}{p}}$$

$$\|f + g\|_p \leq \|f\|_p + \|g\|_p$$

This inequality – the Minkowski inequality – confirms that the p-norm does indeed satisfy the required triangle inequality.

Archimedean Means

Given positive real numbers c_k adding to 1 ("weights"), define the p-mean of a sequence x_k of positive real numbers:

$$M(\boldsymbol{x},p) := \left[\sum_{k=1}^{n} c_k x_k^p\right]^{\frac{1}{p}}$$

And observe that it equals the ordinary weighted mean if p = 1:

$$M(\boldsymbol{x},1) = \sum_{k=1}^{n} c_k x_k$$

A weighted Harmonic Mean if p = -1:

$$M(\boldsymbol{x},-1) = \frac{1}{\sum_{k=1}^{n} \frac{c_k}{x_k}}$$

A weighted Quadratic Mean (root mean square) if p = 2:

$$M(\boldsymbol{x},2) := \left[\sum_{k=1}^{n} c_k x_k^2\right]^{\frac{1}{2}}$$

It becomes the Geometric Mean in the limit as p tends toward 0:

$$\lim_{p\to 0}\left[\sum_{k=1}^{n} c_k x_k^p\right]^{\frac{1}{p}} = \exp[\lim_{p\to 0}\frac{1}{p}\ln\sum_{k=1}^{n} c_k x_k^p]$$

$$= \exp[\lim_{p\to 0}\frac{\sum_{k=1}^{n} c_k x_k^p \ln x_k}{\sum_{k=1}^{n} c_k x_k^p}]$$

$$= \exp\left[\sum_{k=1}^{n} c_k \ln x_k\right] = \prod_{k=1}^{n} x_k^{c_k}$$

Note use of L'Hopital rule. The L'Hopital rule also shows that the limiting case where p tends to positive and negative infinity, the expression recovers the supremum and infimum respectively. One can show that Infimum < Harmonic < Geometric < Arithmetic < Quadratic Mean < Supremum, for any fixed positive-real **x** and weights c_k. To prove *all* these inequalities in one fell swoop observe that the p-mean increases with p. Differentiating using multivariable chain rule:

$$\frac{dM(\boldsymbol{x},p)}{dp} = \frac{\partial M}{\partial f}\frac{\partial f}{\partial p} + \frac{\partial M}{\partial g}\frac{\partial g}{\partial p}$$

$$= \frac{1}{p}\left[\sum_{k=1}^{n} c_k x_k^p\right]^{\frac{1}{p}-1}\sum_{k=1}^{n} c_k x_k^p \ln x_k + \left[\sum_{k=1}^{n} c_k x_k^p\right]^{\frac{1}{p}}\left[\ln\sum_{k=1}^{n} c_k x_k^p\right]\left[-\frac{1}{p^2}\right]$$

$$= \frac{1}{p^2}\left[\sum_{k=1}^{n} c_k x_k^p\right]^{\frac{1}{p}}\left[\frac{\sum_{l=1}^{n} c_l x_l^p \ln x_l^p}{\sum_{m=1}^{n} c_m x_m^p} - \ln\sum_{k=1}^{n} c_k x_k^p\right]$$

This is always positive, as:

$$\sum_{l=1}^{n} c_l x_l^p \ln x_l^p \geq \left[\sum_{m=1}^{n} c_m x_m^p\right]\ln\sum_{k=1}^{n} c_k x_k^p$$

This inequality holds by the Jensen inequality for convex functions – observe that $x_l^p \ln x_l^p$ is a convex function of x_l^p. The Holder, Jensen and Minkowski inequalities appear frequently in analysis. It is worthwhile keeping these facts in your mathematical toolbox!

Explain it Better: Fourier Series – *Stephen Zhang*

Fourier series

Introduction

Fourier analysis is a 'toolbox' of mathematical techniques for analysis of functions that has applications in a wide range of physics and engineering problems, such as crystallography and signal processing. Fourier theory builds on the core concepts of vector spaces and linear independence, as introduced in an elementary course on linear algebra.

Fourier series

We first introduce the concept of a **function space** — that is, simply a vector space in which the **vectors** are **functions**. For instance, one might consider V to be the set of all C^1 (i.e. with **continuous** derivative) functions on $[-L, L]$ subject to the condition that $f(\pm L) = 0$. It is easy to see that V is a vector space, closed under the usual operations.

Since V is a vector space, we know that there exists $\mathcal{B} = (e_1, e_2, ...)$, a **basis** for V. Importantly, in this case, $\mathcal{B}$ is **not** finite, since the function space V is of infinite dimension.

It's all well and good to know that there **exists** such a basis, but do we have any examples of such? Actually, we do!

Consider the set of functions:

$$\left\{\frac{1}{\sqrt{2}}, \sin\left(\frac{\pi n x}{L}\right), \cos\left(\frac{\pi n x}{L}\right) : n \in \mathbb{N}, x \in [-L, L]\right\}$$

We may define on V the inner product:

$$\langle f, g\rangle := \frac{1}{L}\int_{-L}^{L} f(x)g(x)dx$$

Using this inner product, we note that:

$$\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = 1$$

$$\left\langle \frac{1}{\sqrt{2}}, \sin\left(\frac{\pi n x}{L}\right)\right\rangle = \left\langle \frac{1}{\sqrt{2}}, \cos\left(\frac{\pi n x}{L}\right)\right\rangle = 0$$

$$\left\langle \sin\left(\frac{\pi n x}{L}\right), \sin\left(\frac{\pi m x}{L}\right)\right\rangle = \delta_{mn}$$

$$\left\langle \cos\left(\frac{\pi n x}{L}\right), \cos\left(\frac{\pi m x}{L}\right)\right\rangle = \delta_{mn}$$

$$\left\langle \sin\left(\frac{\pi n x}{L}\right), \cos\left(\frac{\pi m x}{L}\right)\right\rangle = 0$$

In the above, we take δ_{mn} to be the **Kronecker** delta, that is $\delta_{mn} = 1$ for $m = n$ and 0 otherwise. From this, we see that $\mathcal{B}$ is **orthonormal**.

Recall that, if $\mathcal{B} = \{e_1 e_2, ...\}$ is an orthnormal basis for V, then any element $u \in V$ may be **projected** onto basis $\mathcal{B}$ as follows:

$$f = \langle f, e_1\rangle e_1 + \langle f, e_2\rangle e_2 + ... = \sum_{k=1}^{\infty}\langle f, e_k\rangle e_k$$

Let $f \in V$ thus be any function on $[-L, L]$ with $f(\pm L) = 0$. Then, we have:

$$f(x) = \left\langle f(x), \frac{1}{\sqrt{2}}\right\rangle \frac{1}{\sqrt{2}} + \sum_{n=1}^{\infty}\left\langle f(x), \sin\left(\frac{\pi n x}{L}\right)\right\rangle \sin\left(\frac{\pi n x}{L}\right) + \sum_{n=1}^{\infty}\left\langle f(x), \cos\left(\frac{n\pi x}{L}\right)\right\rangle \cos\left(\frac{n\pi x}{L}\right)$$

$$= a_0 + \sum_{n=1}^{\infty}\left[a_n \sin\left(\frac{\pi n x}{L}\right) + b_n \cos\left(\frac{\pi n x}{L}\right)\right]$$

Where

$$a_0 = \frac{1}{\sqrt{2}}\left\langle f(x), \frac{1}{\sqrt{2}}\right\rangle = \frac{1}{\sqrt{2}}\frac{1}{L}\int_{-L}^{L} f(x)\frac{1}{\sqrt{2}}dx = \frac{1}{2L}\int_{-L}^{L} f(x)dx$$
$$a_n = \left\langle f(x), \sin\left(\frac{n\pi x}{L}\right)\right\rangle = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$
$$b_n = \left\langle f(x), \cos\left(\frac{n\pi x}{L}\right)\right\rangle = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

This is the **Fourier series** representation of the function f. The series will converge **exactly** to f as $n \to \infty$. In general, this definition of **Fourier series** can also be applied to non-C^1 functions (i.e. piecewise continuous functions), but the convergence is no longer exact, with the introduction of **Gibbs phenomena**, or aberrations close to discontinuity points that do not disappear as $n \to \infty$.

For instance, we may consider the Fourier series representation of $f(x) = x$ on $[-1, 1]$:

$$a_0 = \frac{1}{2}\int_{-1}^{1} x dx = 0$$
$$a_n = \int_{-1}^{1} x\sin(n\pi x)dx = \frac{2}{n\pi}(-1)^{n+1} \qquad \text{(integrate by parts)}$$
$$b_n = \int_{-1}^{1} x\cos(n\pi x)dx = 0 \qquad \text{(odd integrand)}$$

And so

$$f(x) = x = \sum_{n=1}^{\infty} \frac{2}{n\pi}(-1)^{n+1}\sin(n\pi x)$$

Plotting a few partial sums yields the following behaviour, for $n = 1, 10, 100, 1000$:
One can observe Gibbs phenomena (ringing) near the cusps at $x = \pm 1$.

Complex Fourier Series

One may also note that, $\{e^{i\pi nx/L}, n \in \mathbb{Z}\}$ form an orthonormal basis for functions on $\mathbb{C}$. We define an appropriate inner product for this case:

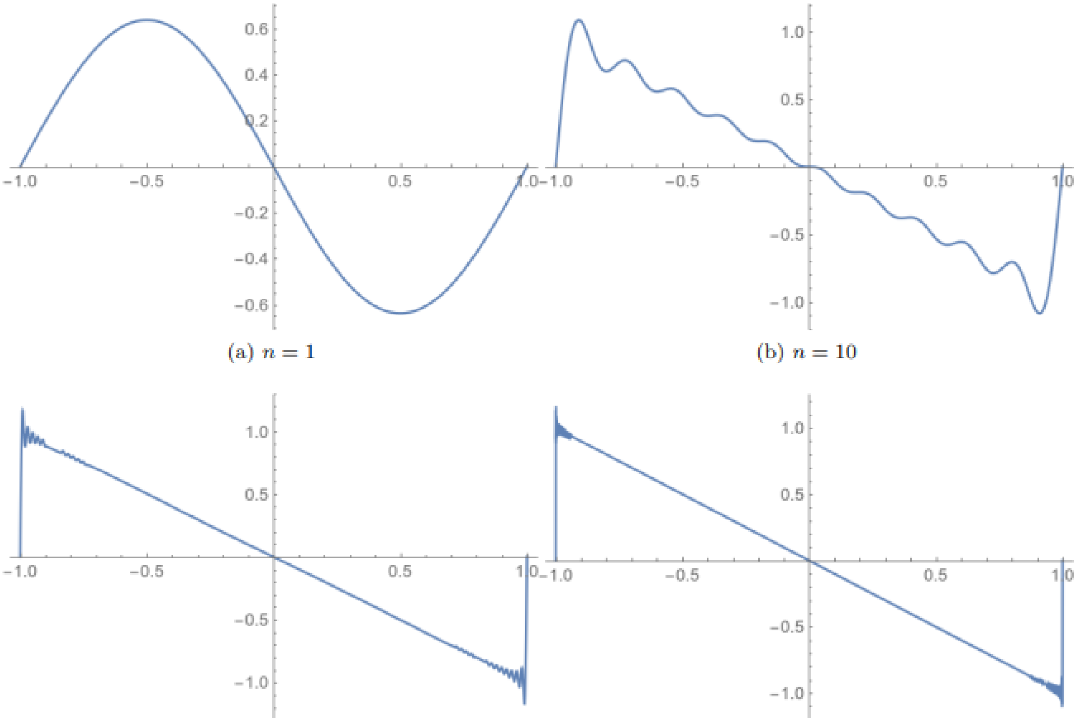
$$\langle f, g\rangle = \frac{1}{L}\int_{-L}^{L} f(x)\overline{g(x)}dx$$

Using this definition, we find that:

$$\left\langle e^{i\pi mx/L} e^{i\pi nx/L}\right\rangle = \frac{1}{\pi(m-n)}\sin(\pi(m-n)) = 0 \qquad \text{for } m \neq n$$
$$\left\langle e^{i\pi mx/L} e^{i\pi nx/L}\right\rangle = \frac{1}{2L}\int_{-L}^{L} dx = 1 \qquad \text{for } m = n$$

And so we have that, for any $f(x)$ on $[-L, L]$:

$$f(x) = \sum_{n\in\mathbb{Z}}\left\langle f(x), e^{in\pi x/L}\right\rangle e^{in\pi x/L} = \sum_{n=-\infty}^{\infty} a_n e^{in\pi x/L}$$



(a) $n = 1$ (b) $n = 10$

(c) $n = 100$ (d) $n = 1000$

with

$$a_n = \frac{1}{2L}\int_{-L}^{L} f(x)e^{in\pi x/L}dx$$

This is exactly the same thing as the sine/cosine Fourier series (exercise: show that this is the case, using Euler's formula)

"The Universal Coverage" - News and Opinion!

The Radical Ideal: Gender representation in Maths - *Madeleine Johnson*

Why do so few women study maths? In a time when feminism and gender representation are major talking points in the media and it appears that a real effort is being made across multiple arenas to address these sorts of issues, I was very surprised when I walked into a lecture theatre for the first class of my Masters of Science this semester to discover I was one of two women in the Pure Maths cohort. It seems to me that there's a big drop off in the percentage of women and non-binary people studying maths between undergrad and masters. Yet initiatives to address this are few and far between – as far as I'm aware, there are no scholarships (unlike in Engineering), mentoring programs or awareness raising campaigns run by the University, all courses of action that have produced increased percentage representation in other fields of study.

To explore this further, I interviewed four women and one non-binary person who are currently undergrad maths students here at the University of Melbourne. When I asked them about what they intended to do after completing their undergrad degree, only one of the five indicated explicitly that they were considering postgraduate studies in maths. Three indicated intention to study other related fields at a postgrad level, and one gave an "I'm not sure" response. Of the few women in my third-year maths classes last year, none have transitioned to the Masters of Science with me and all seem to have pursued either work or studies in other areas.

But why is this the case? I asked these students about whether they felt their gender had affected their studies of maths, and whether it was likely to affect their decision about studying maths in the future, and received the following responses:

"I think my gender has definitely affected my studies. I'm often not comfortable being out as non-binary in class the way I could in my previous arts degree, because I'm less sure my tutors have had the training or experience to know how to handle any issues that arise and back me up if another student is being an asshole. It's also relatively obvious when I'm being perceived as a woman vs a man: men seem more inclined to talk over me and less inclined to consider my input, whether it's intentional or not. Unfortunately, my gender will absolutely play a role in what I do after my undergrad. It's somewhat bearable when tutors and lecturers and classmates you won't see after that semester misgender you (because it's often a matter of me not having the energy to strictly enforce pronouns or provide a gender 101 to people when I'm just trying to get through the class material), but I won't be able to do postgrad without finding a niche where my gender is respected. I think there are a lot of non-binary and trans students in maths, which is why I find this stuff depressing - I'm lucky in that I've been out forever and I'm articulate and confident enough to assert myself when I choose to, but I know plenty of trans students who don't have that confidence or that grasp of English to be able to advocate for themselves in the same way."

"I don't think being female has had a big impact on my studies, but I do believe the subconscious beliefs of what certain genders should study does affect what people believe they "should" pursue. I was lucky enough to have family that encouraged me to pursue maths which offset other people's expectations of me, but a lot of people don't have that."

"I don't find gender impacts my ability to do the work. However, I do find it limits the people

who want to work with me - I tend to see students band together in their gender groups which is very exclusionary as the gender balance is so skewed in the STEM fields I'm studying in. This considerably limits the choices of who I can build meaningful relationships with which provide crucial support during studies and which will last beyond university into the professional realm.

[Do you think your gender will play a role in making decisions about what to study in the future?] Most definitely. If the field I'm going into is hostile to someone if my gender so that I will never really belong nor be afforded the same opportunities, then I have to seriously question whether it's worthwhile pursuing it knowing I'll have to constantly put up with these attitudes."

"I think identifying as female has had a subtle effect on my experiences of studying maths throughout undergrad but some of the negative aspects have become increasingly more noticeable towards later undergrad years. During my first year I remember being aware that there were less girls majoring in maths (I think I heard it was about 30%?) but at the time I didn't see that as a big issue and it didn't stop me from choosing a lot of maths-heavy subjects. People were often surprised when I said I liked maths and was planning on majoring in maths/maths-related majors. While some of these reactions I can attribute to the general population's perception of maths, when it came from guys who were also studying STEM majors it seemed that they were surprised because I don't fit the "maths-geek" stereotype which is typically thought of as male. I've noticed that the gender mis-balance gets even more extreme in third year subjects and beyond. My gender, among other things, has contributed to feeling that I don't belong in some classes.

[Do you think your gender will play a role in making decisions about what to study in the future?] It wouldn't be the main issue stopping me but it still has some influence over my decisions. In some ways also being aware that there's underrepresentation makes me even more determined to study maths or other male-dominated subjects, and though this counterbalances some of the obstacles, I'm not always sure that stubbornness will get me through."

"I don't think I've experienced explicit discrimination. However, I feel that the lack of female role models has given me a distinct sense of not really belonging in maths. Out of 15 lecturers I've had for 13 maths subjects, two have been female. I think it's really hard to feel that you belong when you don't see many people who look like you."

The main ways gender seems to have affected these students' studies, and decisions about future studies, are:

- not feeling like they belong to the maths community
- lack of representation of people at higher levels who look like them
- tutors having adequate training to ensure their classrooms are free from discrimination

Training programs are straightforward enough to implement, and, like scholarships, are a matter of funding, which is often outside the control of the School, Faculty of Science, or even the University. Yet if these responses are representative of how the majority of female and non-binary undergrad maths students feel, then it is clear that further action needs to be taken if representation is to improve. The people best equipped to identify what courses of action should be taken, or which of the existing actions being taken are the most effective, are, of course, the students affected by this:

"I would like to see a comprehensive training program regarding respecting gender diversity, not assuming pronouns, and giving tutors the tools to back up trans students who experience pushback from fellow students. There are so many trans people in this field - I think tons of tutors, academics, etc would want to support us but simply don't have the training."

"I feel there are some great organisations (eg AMSI) trying to decrease or offset the difficulties women experience, but ultimately every institution needs to make sure that they aren't being biased or negatively influencing people, intentional or otherwise.

Different stakeholders could certainly do a better job of supporting women/ non-binary people studying maths, especially with encouragement to continue maths to a high level.

Although I feel there is little direct bias, there are often few female role models in areas such as maths (I've especially noticed a lack of female lecturers, for example). This, combined with a lack of encouragement and fewer females undertaking maths, results in a lower proportion of females undertaking these studies in both undergraduate and postgraduate study.

I'd like to see more examples and visibility of women that have been successful in these areas to view as role models."

"I haven't seen many active efforts to address [gender underrepresentation] at uni. [Actions stakeholders could take include] having more female role models and addressing any active hostility and discrimination against women."

"There are some organisations doing great work in addressing some of the issues leading to gender underrepresentation in maths. It makes me smile when I see one of AMSI's ChooseMaths adverts and I hope that kind of thing can continue to encourage more people (especially girls and non-binary) to pick maths subjects in high school and university. I also love seeing the work of the women/non-binary people who manage to make it in maths being celebrated.

I think the university and in particular the School of Maths and Stats could do a lot more to support a whole range of underrepresented students by making studying maths more inclusive and accessible.

…

This maths department and maths community in general are very traditional in a lot of ways, and this tends to suit traditional students only and makes it difficult for students who don't fit the mould. And you can't really expect this cycle of underrepresentation and homogeneity to change without changes being made first from within."

"There seem to be more and more events and programs aimed at increasing representation in maths. I think this is great! [Some other actions that could be taken include] connecting young maths students with mentors who can guide them, and who can also make students feel less alone, would be helpful I think (to be honest I'm not sure if this is already happening at UniMelb). Also, the lack of scholarships for female / non-binary maths students seems like an obvious opportunity that is currently being missed at UniMelb."

There seems to be varying perceptions of the efforts currently being undertaken to address gender representation in maths, but all five of these students identified areas where more could be done to address gender representation. The main ones included:

- Better gender representation in teaching staff across mathematics subjects at all levels
- Mentoring programs for female and non-binary students
- Scholarships for female and non-binary students
- Training for teaching staff on supporting transgender and non-binary students in the classroom.

Whilst there does seem to be a concerted effort currently for having more equal gender representation across the first-year maths lecturers, other initiatives could include efforts to increase the number of female and non-binary tutors across all subjects, and teaching staff in the Accelerated Maths first year stream (where many students interested in continuing to postgraduate maths studies begin). Students who were aware of AMSI's Choose Maths program spoke favourably of it, indicating that awareness raising campaigns are effective and that other organisations should consider running them.

If you're reading this, as a female or non-binary undergrad student who likes maths but doesn't feel like they belong to this community or that there is a place for them in further studies in maths, know this: there is a place for you in the MUMS community, and people in the maths community who care about this issue and are trying to fix it. You can and should study postgrad maths if you want to, the community will be better for having you in it.

The Hairy Ball

"A mathematician is a device for turning coffee into theorems" (P. Erdos) Addendum: American coffee is good for lemmas.

An engineer thinks that their equations are an approximation to reality. A physicist thinks reality is an approximation to his equations. A mathematician doesn't care.

Old mathematicians never die; they just lose some of their functions.

Mathematicians are like Frenchmen: whatever you say to them, they translate it into their own language, and forthwith it means something entirely different. -- Goethe

Write for the Paradox!

Paradox is a magazine that publishes articles written by University of Melbourne students! This is an opportunity for you to practise your communication skills, and get valuable feedback from your peers! Here are some topics you might want to write about – this is stimulus material only, please feel free to extend outside the topics on this list, or to mix and match keywords and ideas from different entries! All submissions go to mums.paradox.editor@gmail.com

"The Narrow Margin" - Mathematical Facts!

MATHEMATICAL TIDIBITS (interesting and fun mathematical facts and theorems)
- Pick's Theorem: A curious result! How do you prove it?
- The Secretary Problem: How do you find the best secretary, if you can't go back to the ones you've already rejected? What relevance does the number, 37%, have?
- The Prisoner's Dilemma: A fantastically interesting scenario for the uninitiated! Now, what does this classic scenario in game theory teach us about taxation and shared resources? What did Nash prove about game theory?
- Bell Polynomials, Stirling Numbers: How many distinct rhyming schemes exist for a poem stanza with 5 lines? What are Bell polynomials, Bell numbers and Stirling numbers? Can we find them recursively? What are their generating functions?
- Bernoulli Polynomials: Tell the apocryphal story attributed to baby Gauss. What's the trick to sum the first n k-th powers? Present the modern (and archaic) Bernoulli polynomials, and their generating function!

EXPLAIN IT BETTER (tricky concepts in class that were taught/explained badly. Show us you can do better)
- Tips and tricks to do partial fractions quickly and accurately!
- Tips and tricks to efficiently find the right contour for complex integration!
- How do we prove Kepler's three laws of planetary motion? (Sadly, an omission from the Orbital Mechanics segment of Physics 1)
- What, exactly, is a vector? What is a tensor? Why must contravariant and covariant indices be distinguished?! Physics professors (worldwide) are notoriously bad at explaining this!!
- Discuss the three cases that arise in the Froebenius method (time constraints preclude a full discussion in the current Differential Equations subject)

"The Universal Coverage" - News and Opinion!

NEWS (recent events in math, physics, academia)
- Discuss recent developments about Mochizuki's purported proof of ABC conjecture

THE RADICAL IDEAL (opinions about matters relevant to academia, student life)

"The Connection Form" - Study and Career advice!

STUDY CORNER (tips for classes)
- What's your study routine? Do you have any tips and tricks you want to share with Paradox readers?

THE CATALOGUE (subject reviews, overviews)
- Do you have advice for classmates taking a math or other science subject you've done? How does it fit with study plan and future career goals?
- Any Breadth subjects you particularly enjoyed, that you might recommend to other students? Give it a Yelp! review…

THE PATH INTEGRAL (study plan, research interests, and future career path)
- Do you have an unconventional path into your current studies?
- Do you have unconventional plans for your future career? Share your thoughts, and the reasoning behind your plans

"The Hairy ball" - The Lighter Side of Math!

THE ARCHIVE
- Who is your favourite mathematician, physicist (or other scientist) from history? Why do you think they're awesome?
- Tell us about the early life of Galois or Newton. What was your favourite part?

THE COMEDY CORNER (jokes and memes)
- What's the funniest math joke you know? It's okay to make fun of engineers and physicists here :)
- Send in all your favourite mathematical memes! A good Loss meme is subtle…

A morphism $f \in Hom(B,C)$ is called a **monomorphism** iff it has the **universal property** that, given any object A and morphisms $g, h \in Hom(A,B)$ then $f \circ g = f \circ h \Rightarrow g = h$.

One special type of monomorphism is an **equalizer**, between morphisms $g, h \in Hom(A,B)$. An object E and morphism $e \in Hom(E,A)$ "equalizes" g and h iff $g \circ e = h \circ e$ and we require the **universal property** that, given E' and e' with the same property, $\exists!\, u \in Hom(E'E): e' = e \circ u$.